

Exercice N°1 :

➤ Addition : $C = A + B$ $C = \begin{pmatrix} 7 & -2 & 10 \\ -4 & -6 & -6 \end{pmatrix}$

➤ Produit par un scalaire : $C = 5 \times B$ $C = \begin{pmatrix} 15 & -10 & 30 \\ 10 & -35 & -30 \end{pmatrix}$

➤ Déterminant : $\det A = \begin{vmatrix} 3 & -2 \\ 2 & -7 \end{vmatrix} = -21 + 4 = -17$

$$\det B = \begin{vmatrix} 1 & 0 & -1 \\ -2 & 3 & 4 \\ 0 & 1 & 1 \end{vmatrix} = 1 \begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} - 0 \begin{vmatrix} -2 & 4 \\ 0 & 1 \end{vmatrix} - 1 \begin{vmatrix} -2 & 3 \\ 0 & 1 \end{vmatrix} = -1 - 0 + 2 = 1$$

➤ Transposition : ${}^t A = \begin{pmatrix} 3 & 2 \\ -2 & -7 \\ 6 & -6 \end{pmatrix}$ ${}^t B = \begin{pmatrix} 4 & -6 \\ 0 & 1 \\ 4 & 0 \end{pmatrix}$

➤ Produit : $C = A \times B$ $\begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 4 & 5 & 4 \end{pmatrix}$

$$\begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & -6 \end{pmatrix} = \begin{pmatrix} 1+2+12 & 2+4+15 & 3+6+12 \\ 4-5-24 & 8-10-30 & 12-15-24 \end{pmatrix} = \begin{pmatrix} 15 & 21 & 21 \\ -25 & -32 & -27 \end{pmatrix} = C$$

$$C = A \times B = \begin{pmatrix} 3 & 7 & 6 \\ -1 & -2 & -3 \\ 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 & 7 & 6 \\ -3 & -7 & -6 \\ 0 & 0 & 0 \end{pmatrix} = C$$

➤ Inversion : $A = \begin{pmatrix} 3 & 7 & 6 \\ -8 & -7 & 8 \\ 3 & -2 & 4 \end{pmatrix}$ L'inverse de A est : $A^{-1} = \frac{1}{\det A} {}^t C$

$$\det A = \begin{vmatrix} 3 & 7 & 6 \\ -8 & -7 & 8 \\ 3 & -2 & 4 \end{vmatrix} = 3 \begin{vmatrix} -7 & 8 \\ -2 & 4 \end{vmatrix} - 7 \begin{vmatrix} -8 & 8 \\ 3 & 4 \end{vmatrix} + 6 \begin{vmatrix} -8 & -7 \\ 3 & -2 \end{vmatrix} = 578$$

$$\left(\begin{array}{c|c|c} \begin{pmatrix} -7 & 8 \\ -2 & 4 \\ 7 & 6 \\ -2 & 4 \\ 7 & 6 \\ -7 & 8 \end{pmatrix} & \begin{pmatrix} -8 & 8 \\ 3 & 4 \\ 3 & 6 \\ 3 & 4 \\ 3 & 6 \\ -8 & 8 \end{pmatrix} & \begin{pmatrix} -8 & -7 \\ 3 & -2 \\ 3 & 7 \\ 3 & -2 \\ 3 & 7 \\ -8 & -7 \end{pmatrix} \end{array} \right) = \begin{pmatrix} -12 & -56 & 37 \\ 40 & -6 & -27 \\ 98 & 72 & 35 \end{pmatrix}$$

$$\begin{pmatrix} -12 & -56 & 37 \\ 40 & -6 & -27 \\ 98 & 72 & 35 \end{pmatrix} \times \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} = \begin{pmatrix} -12 & 56 & 37 \\ -40 & -6 & 27 \\ 98 & -72 & 35 \end{pmatrix} = C$$

$${}^t C = \begin{pmatrix} -12 & -40 & 98 \\ 56 & -6 & -72 \\ 37 & 27 & 35 \end{pmatrix} \quad \text{En en déduit : } A^{-1} = \begin{pmatrix} -\frac{12}{578} & -\frac{40}{578} & \frac{98}{578} \\ \frac{56}{578} & -\frac{6}{578} & -\frac{72}{578} \\ \frac{37}{578} & \frac{27}{578} & \frac{35}{578} \end{pmatrix}$$

Exercice N°2 :

$$A = \begin{pmatrix} 13 & -8 & -3 \\ -8 & 10 & -1 \\ -3 & -1 & 11 \end{pmatrix} \quad {}^t A = \begin{pmatrix} 13 & -8 & -3 \\ -8 & 10 & -1 \\ -3 & -1 & 11 \end{pmatrix}$$

$$A \cdot {}^t A = \begin{pmatrix} 242 & -181 & -64 \\ -181 & 165 & 3 \\ -64 & 3 & 131 \end{pmatrix} = {}^t A \cdot A$$

Exercice N°3 :

$$\text{Tr}(A) = 15 + 55 - 33 = 37$$

$$C = A \times B = \begin{pmatrix} 3070 & 30 & -80 \\ -1582 & -56 & 408 \\ -1338 & 349 & -260 \end{pmatrix} \quad \text{Tr}(C) = 3070 - 56 - 260 = 2754$$

Exercice N°4 :

$$\begin{aligned} 13a - 8b - 3c &= 20 \\ -8a + 10b - c &= -5 \\ -3a - b + 11c &= 0 \end{aligned}$$

$$X = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 13 & -8 & -3 \\ -8 & 10 & -1 \\ -3 & -1 & 11 \end{pmatrix} \quad X = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad B = \begin{pmatrix} 20 \\ -5 \\ 0 \end{pmatrix}$$

$$\text{On a : } A \cdot X = B \quad \text{On tire : } X = A^{-1} \cdot B$$