Representation simultanée individus_caractères

Individu $\longrightarrow$ ${}^tX_i$ de $\mathbb{R}^p$ $\longrightarrow$ $W_i^{(r)}$ sur Δu_r

$\longrightarrow W^{(r)} = X u_r = X\left(\frac{1}{\sqrt{\lambda_r}}\, {}^tX s_r\right) = \frac{1}{\sqrt{\lambda_r}}\, X\,{}^tX s_r = \sqrt{\lambda_r}\, s_r$

projection ou representation de ind. sur

ind $\longrightarrow \sqrt{\lambda_r}\, s_r$ sur Δs_r

variable $\longrightarrow v^r = {}^tX s_r$

Representation des individus et caractères sur Δu_r

Individus $W^{(r)} = X u_r$.

variables. $V^r = {}^tX s_r = {}^tX\left(\frac{1}{\sqrt{\lambda_r}} X u_r\right) = \frac{1}{\sqrt{\lambda_r}}\, {}^tX X u_r = \sqrt{\lambda_r}\, u_r$

Les formules de s_r et u_r sont dits formule de transition.

Reconstitution des données

Les formules de transition

$$\begin{cases} \forall r \quad r \geq 1 & u_r = \dfrac{1}{\sqrt{\lambda_r}}\, {}^tX s_r. \\ \forall r \geq 1 & s_r = \dfrac{1}{\sqrt{\lambda_r}}\, X u_r. \end{cases}$$

Proposition

$$X = \sum_{k=1}^{p} \sqrt{\lambda_k}\, s_k\, {}^tu_k.$$

Lemme.

Si $P = [u_1, \ldots, u_m]$ ou u_i est le vect prop de la matrice ${}^tX\, X$ alors. P = matrice orthogonale.

En effet :

$$P = [u_1, \ldots, u_m] \Rightarrow {}^tP = \begin{bmatrix} {}^tu_1 \\ \vdots \\ {}^tu_m \end{bmatrix}$$

$$\underset{(m,p)\,(p,m)}{{}^tP\,P} = \begin{bmatrix} {}^tu_1 \\ \vdots \\ {}^tu_m \end{bmatrix} [u_1, \ldots, u_m]$$

$$= \begin{bmatrix} {}^tu_1 u_1 & {}^tu_1 u_2 & \cdots & {}^tu_1 u_m \\ {}^tu_2 u_1 & {}^tu_2 u_2 & \cdots & {}^tu_2 u_m \\ \vdots & & & \\ {}^tu_m u_1 & {}^tu_m u_2 & \cdots & {}^tu_m u_m \end{bmatrix} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

$${}^tP\,P = I_m.$$

Demonstration

On a : $s_k = \dfrac{1}{\sqrt{\lambda_k}} X u_k$.

$$\Rightarrow s_k\, {}^tu_k = \frac{1}{\sqrt{\lambda_k}} X u_k\, {}^tu_k$$

$$\sum \sqrt{\lambda_k}\, s_k\, {}^tu_k = X\, u_k\, {}^tu_k$$

$$\Rightarrow \sum_{k=1}^{p} \sqrt{\lambda_k}\, s_k\, {}^tu_k = X \sum_{k=1}^{p} u_k\, {}^tu_k$$

Il rest à montrer $\sum_{k=1}^{p} u_k\, {}^tu_k = I_p$.

$\langle u_i, u_j \rangle = \delta_{ij}$

$u_k = \begin{pmatrix} u_k^1 \\ \vdots \\ u_k^p \end{pmatrix} \in \mathbb{R}^p$ $\qquad u_k^j = j^{\text{eme}}$ composante de u_k.

$$\sum_{j=1}^{p} (u_k^j)^2 = 1 \qquad (\|u_k\|^2 = 1)$$

sur la diagonale de la matrice $u_k\,{}^t u_k$

$$\begin{pmatrix} (u_k^1)^2 & \times & \cdots & \times \\ \times & \ddots & & \vdots \\ \vdots & & \ddots & \times \\ \times & \cdots & & (u_k^p)^2 \end{pmatrix}$$

$\sum_{k=1}^{p} u_k\,{}^t u_k$ sur la diagonale

$$\begin{pmatrix} \sum_{k=1}^{p} (u_k^1)^2 & \times & \cdots & \times \\ \times & \ddots & & \vdots \\ \vdots & & \ddots & \times \\ \times & \cdots & \times & \sum_{k=1}^{p} (u_k^p)^2 \end{pmatrix}$$