Mécanique Quantique II (2013-2014)
Solution Interrogation 2

I- Pour les kets :

$$|A\rangle = \frac{1}{\sqrt{2}}(e^{-i\theta}|1\rangle + |2\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ 1 \end{pmatrix}, |B\rangle = \frac{1}{\sqrt{2}}(e^{-i\theta}|1\rangle - |2\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ -1 \end{pmatrix}$$

1) Relations de d'orthogonalité

$$\langle A|A\rangle = \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta} & 1 \end{pmatrix}\tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ 1 \end{pmatrix} = 1 = \langle B|B\rangle$$

et

$$\langle B|A\rangle = \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta} & -1 \end{pmatrix}\tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ 1 \end{pmatrix} = 0$$

2) Relation de fermeture

$$|A\rangle\langle A| + |B\rangle\langle B| = \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ 1 \end{pmatrix}\tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta} & 1 \end{pmatrix} + \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ -1 \end{pmatrix}\tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta} & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

3) Expression dans la nouvelle base

$$|1\rangle = \tfrac{e^{i\theta}}{\sqrt{2}}(|A\rangle + |B\rangle), |2\rangle = \tfrac{1}{\sqrt{2}}(|A\rangle - |B\rangle)$$

$$\rightarrow |\psi\rangle = 2i\tfrac{e^{i\theta}}{\sqrt{2}}(|A\rangle + |B\rangle) + \tfrac{1}{\sqrt{2}}(|A\rangle - |B\rangle) = \tfrac{1}{\sqrt{2}}(2e^{i\theta}i + 1)|A\rangle + \tfrac{1}{\sqrt{2}}(2e^{i\theta}i - 1)|B\rangle$$

4) Calcul de la norme

$$\langle\psi|\psi\rangle = \begin{pmatrix} -2i & 1 \end{pmatrix}\begin{pmatrix} 2i \\ 1 \end{pmatrix} = 5 = \begin{pmatrix} \tfrac{1}{\sqrt{2}}(-2e^{-i\theta}i + 1) & \tfrac{1}{\sqrt{2}}(-2e^{-i\theta}i - 1) \end{pmatrix}\begin{pmatrix} \tfrac{1}{\sqrt{2}}(2e^{i\theta}i + 1) \\ \tfrac{1}{\sqrt{2}}(2e^{i\theta}i - 1) \end{pmatrix} = 5$$

II- Pour les kets :

$$|A\rangle = \frac{1}{\sqrt{2}}(e^{-i\theta}|1\rangle - |2\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ -1 \end{pmatrix}, |B\rangle = \frac{1}{\sqrt{2}}(e^{-i\theta}|1\rangle + |2\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ 1 \end{pmatrix}$$

1) Relations de d'orthogonalité

$$\langle B|B\rangle = \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta} & 1 \end{pmatrix}\tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ 1 \end{pmatrix} = 1 = \langle A|A\rangle$$

Et

$$\langle A|B\rangle = \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta} & -1 \end{pmatrix}\tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ 1 \end{pmatrix} = 0$$

2) Relation de fermeture

$$|A\rangle\langle A| + |B\rangle\langle B| = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta} & -1 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

3) Expression dans la nouvelle base

$$|1\rangle = \frac{e^{i\theta}}{\sqrt{2}}(|B\rangle + |A\rangle), |2\rangle = \frac{1}{\sqrt{2}}(|B\rangle - |A\rangle)$$

$$\rightarrow |\psi\rangle = \frac{e^{i\theta}}{\sqrt{2}}(|B\rangle + |A\rangle) + \frac{1}{\sqrt{2}}2i(|B\rangle - |A\rangle) = \frac{1}{\sqrt{2}}(e^{i\theta} - 2i)|A\rangle + \frac{1}{\sqrt{2}}(e^{i\theta} + 2i)|B\rangle$$

4) Calcul de la norme

$$\langle\psi|\psi\rangle = \begin{pmatrix} 1 & -2i \end{pmatrix}\begin{pmatrix} 1 \\ 2i \end{pmatrix} = 5 = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(e^{-i\theta} + 2i\right) & \frac{1}{\sqrt{2}}\left(e^{-i\theta} - 2i\right) \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}}\left(e^{i\theta} - 2i\right) \\ \frac{1}{\sqrt{2}}\left(e^{i\theta} + 2i\right) \end{pmatrix} = 5$$

III- Pour les kets :

$$|A\rangle = \frac{1}{\sqrt{2}}(|1\rangle + e^{i\theta}|2\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\theta} \end{pmatrix}, |B\rangle = \frac{1}{\sqrt{2}}(|1\rangle - e^{i\theta}|2\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -e^{i\theta} \end{pmatrix}$$

1) Relations de d'orthogonalité

$$\langle A|A\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & e^{-i\theta} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\theta} \end{pmatrix} = 1 = \langle B|B\rangle$$

Et

$$\langle B|A\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -e^{-i\theta} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\theta} \end{pmatrix} = 0$$

2) Relation de fermeture

$$|A\rangle\langle A| + |B\rangle\langle B| = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\theta} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & e^{-i\theta} \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -e^{i\theta} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -e^{-i\theta} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

3) Expression dans la nouvelle base

$$|1\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle), |2\rangle = \frac{e^{-i\theta}}{\sqrt{2}}(|A\rangle - |B\rangle)$$

$$\rightarrow |\psi\rangle = 2i\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle) + \frac{e^{-i\theta}}{\sqrt{2}}(|A\rangle - |B\rangle) = \frac{1}{\sqrt{2}}(2i + e^{-i\theta})|A\rangle + \frac{1}{\sqrt{2}}(2i - e^{-i\theta})|B\rangle$$

4) Calcul de la norme

$$\langle\psi|\psi\rangle = \begin{pmatrix} -2i & 1 \end{pmatrix}\begin{pmatrix} 2i \\ 1 \end{pmatrix} = 5 = \begin{pmatrix} \frac{1}{\sqrt{2}}(-2i + e^{i\theta}) & \frac{1}{\sqrt{2}}(-2i - e^{i\theta}) \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}}(2i + e^{-i\theta}) \\ \frac{1}{\sqrt{2}}(2i - e^{-i\theta}) \end{pmatrix} = 5$$