



Faculté de Technologie
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L1 (ST)

**Corrigé de Série N1 Maths II
2020/2021**

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1 Série N1 Maths II

Exercice 1.1. Calculer les primitives suivantes :

1. $\int \frac{4x-4}{x^2-2x+3} dx$.
On a,

$$\begin{aligned} \int \frac{4x-4}{x^2-2x+3} dx &= 2 \int \frac{2x-2}{x^2-2x+3} dx \\ &= 2 \ln |x^2-2x+3| + c, \quad c \in \mathbb{R}. \end{aligned}$$

L'utilisation de $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$.

2. $\int \frac{1+\ln x}{x} dx$.
On a,

$$\begin{aligned} \int \frac{1+\ln x}{x} dx &= \int \left(\frac{1}{x} + \frac{\ln x}{x} \right) dx \\ &= \int \frac{1}{x} dx + \int \frac{\ln x}{x} dx \\ &= \int \frac{1}{x} dx + \frac{1}{2} \int 2 \frac{1}{x} \ln x dx \\ &= \ln |x| + \frac{1}{2} (\ln x)^2 + c, \quad c \in \mathbb{R}. \end{aligned}$$

L'utilisation de $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$ et $\int f'(x) f^n(x) dx = \frac{f^{n+1}(x)}{n+1} + c$.

3. $\int \cos(5x) dx$.
On a,

$$\begin{aligned} \int \cos(5x) dx &= \frac{1}{5} \int 5 \cos(5x) dx \\ &= \frac{1}{5} \sin(5x) + c, \quad c \in \mathbb{R}. \end{aligned}$$

L'utilisation de $\int f'(x)g(f(x))dx = G(f(x)) + c$, où G est la primitive de g .

4. $\int (2x-2)\sqrt{x^2-2x+3} dx$.
On a,

$$\begin{aligned} \int (2x-2)\sqrt{x^2-2x+3} dx &= \int (2x-2)(x^2-2x+3)^{\frac{1}{2}} dx \\ &= \frac{(x^2-2x+3)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c, \quad c \in \mathbb{R} \\ &= \frac{2}{3} (x^2-2x+3)^{\frac{3}{2}} + c, \quad c \in \mathbb{R}. \end{aligned}$$

L'utilisation de $\int f'(x)f^n(x)dx = \frac{f^{n+1}(x)}{n+1} + c$.

Exercice 1.2. Trouver les primitives suivantes :

1. *Intégration par parties*

(a) $\int x^3 \ln(3x)dx$.

On pose

$$\begin{aligned} u(x) = \ln(3x) &\Rightarrow u'(x) = \frac{(3x)'}{3x} = \frac{3}{3x} = \frac{1}{x}, \\ v'(x) = x^3 &\Rightarrow v(x) = \frac{x^4}{4}, \end{aligned}$$

et l'on intègre par parties, ce qui donne :

$$\begin{aligned} \int x^3 \ln(3x)dx &= \frac{x^4}{4} \ln(3x) - \int \frac{x^4}{4} \frac{1}{x} dx \\ &= \frac{x^4}{4} \ln(3x) - \frac{1}{4} \int x^3 dx \\ &= \frac{x^4}{4} \ln(3x) - \frac{1}{16} x^4 + c, \quad c \in \mathbb{R}. \end{aligned}$$

(b) $\int (2x + 1)e^x dx$.

On pose

$$\begin{aligned} u(x) = (2x + 1) &\Rightarrow u'(x) = 2, \\ v'(x) = e^x &\Rightarrow v(x) = e^x, \end{aligned}$$

et l'on intègre par parties, ce qui donne :

$$\begin{aligned} \int (2x + 1)e^x dx &= (2x + 1)e^x - \int e^x 2 dx \\ &= (2x + 1)e^x - 2e^x + c, \quad c \in \mathbb{R} \\ &= (2x - 1)e^x + c, \quad c \in \mathbb{R}. \end{aligned}$$

(c) $\int (x + 1)\sqrt{2x + 1} dx$.

On pose

$$\begin{aligned} u(x) = (x + 1) &\Rightarrow u'(x) = 1, \\ v'(x) = \sqrt{2x + 1} &\Rightarrow v(x) = \frac{1}{3}(2x + 1)^{\frac{3}{2}}, \end{aligned}$$

et l'on intègre par parties, ce qui donne :

$$\begin{aligned}
 \int (x+1)\sqrt{2x+1} dx &= (x+1)\frac{1}{3}(2x+1)^{\frac{3}{2}} - \int \frac{1}{3}(2x+1)^{\frac{3}{2}} dx \\
 &= \frac{(x+1)}{3}(2x+1)^{\frac{3}{2}} - \frac{1}{3} \int (2x+1)^{\frac{3}{2}} dx \\
 &= \frac{(x+1)}{3}(2x+1)^{\frac{3}{2}} - \frac{1}{6} \int 2(2x+1)^{\frac{3}{2}} dx \\
 &= \frac{(x+1)}{3}(2x+1)^{\frac{3}{2}} - \frac{1}{6} \cdot \frac{2}{\frac{5}{2}} (2x+1)^{\frac{5}{2}} + c, \quad c \in \mathbb{R} \\
 &= \frac{(x+1)}{3}(2x+1)^{\frac{3}{2}} - \frac{1}{15}(2x+1)^{\frac{5}{2}} + c, \quad c \in \mathbb{R}.
 \end{aligned}$$

2. Intégration par changement de variables

(a) $\int \frac{e^{(2x+1)}}{2+5e^{(2x+1)}} dx.$
 En posant

$$t = e^{(2x+1)} \Rightarrow dt = 2e^{(2x+1)} dx \Rightarrow dx = \frac{1}{2e^{(2x+1)}} dt \Rightarrow dx = \frac{1}{2t} dt,$$

on obtient

$$\begin{aligned}
 \int \frac{e^{(2x+1)}}{2+5e^{(2x+1)}} dx &= \int \frac{t}{2+5t} \frac{1}{2t} dt \\
 &= \frac{1}{2} \int \frac{1}{2+5t} dt \\
 &= \frac{1}{10} \int \frac{5}{2+5t} dt \\
 &= \frac{1}{10} \ln |2+5t| + c, \quad c \in \mathbb{R} \\
 &= \frac{1}{10} \ln |2+5e^{(2x+1)}| + c, \quad c \in \mathbb{R}.
 \end{aligned}$$

(à la fin on a remplacé $t = e^{(2x+1)}$).

(b) $\int \frac{1}{1+\sqrt{x+1}} dx.$
 En posant

$$t = \sqrt{x+1} \Rightarrow dt = \frac{1}{2\sqrt{x+1}} dx \Rightarrow dx = 2\sqrt{x+1} dt \Rightarrow dx = 2t dt,$$

on obtient

$$\begin{aligned}
 \int \frac{1}{1 + \sqrt{x+1}} dx &= \int \frac{1}{1+t} 2t dt \\
 &= 2 \int \frac{t}{1+t} dt \\
 &= 2 \int \frac{t+1-1}{1+t} dt \\
 &= 2 \int \left(\frac{t+1}{1+t} + \frac{-1}{1+t} \right) dt \\
 &= 2 \int 1 dt - 2 \int \frac{1}{1+t} dt \\
 &= 2t - 2 \ln |1+t| + c, \quad c \in \mathbb{R} \\
 &= 2\sqrt{x+1} - 2 \ln |1 + \sqrt{x+1}| + c, \quad c \in \mathbb{R}.
 \end{aligned}$$

(à la fin on a remplacé $t = \sqrt{x+1}$).

(c) $\int \frac{3}{2x(\ln x)^2} dx$.
En posant

$$t = \ln x \Rightarrow dt = \frac{1}{x} dx,$$

on obtient

$$\begin{aligned}
 \int \frac{3}{2x(\ln x)^2} dx &= \frac{3}{2} \int \frac{1}{(\ln x)^2} \frac{1}{x} dx \\
 &= \frac{3}{2} \int \frac{1}{t^2} dt \\
 &= \frac{3}{2} \int t^{-2} dt \\
 &= \frac{3}{2} \frac{t^{-2+1}}{-2+1} dt \\
 &= \frac{-3}{2} \frac{1}{t} + c, \quad c \in \mathbb{R} \\
 &= \frac{-3}{2 \ln x} + c, \quad c \in \mathbb{R}.
 \end{aligned}$$

(à la fin on a remplacé $t = \ln x$).

Exercice 1.3. Intégrer les fractions rationnelles suivantes :

1. $\int \frac{x^3+4x^2+6x-3}{x^2+2x+1} dx.$

1-ère étape : effectuer la division euclidienne :

$$\frac{x^3 + 4x^2 + 6x - 3}{x^2 + 2x + 1} = x + 2 + \left(\frac{x - 5}{x^2 + 2x + 1} \right).$$

2-ème étape : d'écouter ($\frac{x-5}{x^2+2x+1}$) en fractions simples :

$$\begin{aligned} \frac{x-5}{x^2+2x+1} &= \left(\frac{x-5}{(x+1)^2} \right) \\ &= \frac{A}{(x+1)} + \frac{B}{(x+1)^2} \\ &= \frac{A(x+1) + B}{(x+1)^2} \\ &= \frac{Ax + (A+B)}{(x+1)^2} \\ &= \frac{1x + (-5)}{(x+1)^2}, \end{aligned}$$

par identification, on obtient :

$$\begin{cases} A = 1 \\ A + B = -5 \end{cases} \Rightarrow A = 1, B = -6,$$

d'où

$$\frac{x-5}{(x+1)^2} = \frac{1}{x+1} + \frac{-6}{(x+1)^2}.$$

Alors,

$$\frac{x^3 + 4x^2 + 6x - 3}{x^2 + 2x + 1} = x + 2 + \left(\frac{x-5}{x^2+2x+1} \right) = x + 2 + \left(\frac{1}{x+1} + \frac{-6}{(x+1)^2} \right).$$

3-ème étape : intégrer :

$$\begin{aligned} \int \frac{x^3 + 4x^2 + 6x - 3}{x^2 + 2x + 1} dx &= \int \left(x + 2 + \frac{x-5}{x^2+2x+1} \right) dx \\ &= \int \left(x + 2 + \frac{1}{x+1} + \frac{-6}{(x+1)^2} \right) dx \\ &= \int (x+2) dx + \int \frac{1}{x+1} dx - 6 \int \frac{1}{(x+1)^2} dx \\ &= \frac{x^2}{2} + 2x + \ln |x+1| - 6 \frac{-1}{(x+1)} + c, c \in \mathbb{R} \\ &= \frac{x^2}{2} + 2x + \ln |x+1| + \frac{6}{(x+1)} + c, c \in \mathbb{R}. \end{aligned}$$

2. $\int \frac{x-1}{x^2+x+1} dx$. On a

$$\begin{aligned}
 \int \frac{x-1}{x^2+x+1} dx &= \frac{1}{2} \int \frac{2x-2}{x^2+x+1} dx \\
 &= \frac{1}{2} \int \frac{2x+1-3}{x^2+x+1} dx \\
 &= \frac{1}{2} \int \left(\frac{2x+1}{x^2+x+1} + \frac{-3}{x^2+x+1} \right) dx \\
 &= \frac{1}{2} \int \left(\frac{2x+1}{x^2+x+1} \right) dx + \frac{1}{2} \int \left(\frac{-3}{x^2+x+1} \right) dx \\
 &= \frac{1}{2} \int \left(\frac{2x+1}{x^2+x+1} \right) dx - \frac{3}{2} \int \left(\frac{1}{x^2+x+1} \right) dx \\
 &= \frac{1}{2} \ln |x^2+x+1| - \frac{3}{2} \int \left(\frac{1}{x^2+x+1} \right) dx
 \end{aligned}$$

On calcule l'intégrale $\int \frac{1}{x^2+x+1} dx$,

$$\begin{aligned}
 x^2+x+1 &= x^2 + 2\frac{1}{2}x + \frac{1}{4} - \frac{1}{4} + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \\
 &= \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{3}{4} \left(\frac{\left(\frac{2x+1}{2}\right)^2}{\frac{3}{4}} + 1 \right) \\
 &= \frac{3}{4} \left(\left(\frac{2x+1}{\sqrt{3}} \right)^2 + 1 \right)
 \end{aligned}$$

on pose

$$t = \frac{2x+1}{\sqrt{3}} \Rightarrow dt = \frac{2}{\sqrt{3}} dx \Rightarrow \frac{\sqrt{3}}{2} dt = dx,$$

puis

$$\begin{aligned}
 \int \frac{1}{x^2 + x + 1} dx &= \int \frac{1}{\frac{3}{4} \left(\left(\frac{2x+1}{\sqrt{3}} \right)^2 + 1 \right)} dx \\
 &= \frac{4}{3} \int \frac{1}{\left(\left(\frac{2x+1}{\sqrt{3}} \right)^2 + 1 \right)} dx \\
 &= \frac{4}{3} \int \frac{1}{t^2 + 1} \frac{\sqrt{3}}{2} dt \\
 &= \frac{2}{\sqrt{3}} \int \frac{1}{t^2 + 1} dt \\
 &= \frac{2}{\sqrt{3}} \arctan(t) + c, \quad c \in \mathbb{R} \\
 &= \frac{2}{\sqrt{3}} \arctan\left(\frac{2x+1}{\sqrt{3}}\right) + c, \quad c \in \mathbb{R}.
 \end{aligned}$$

à la fin

$$\begin{aligned}
 \int \frac{x-1}{x^2+x+1} dx &= \frac{1}{2} \ln |x^2+x+1| - \frac{3}{2} \int \left(\frac{1}{x^2+x+1} \right) dx \\
 &= \frac{1}{2} \ln |x^2+x+1| - \frac{3}{2} \frac{2}{\sqrt{3}} \arctan\left(\frac{2x+1}{\sqrt{3}}\right) + c, \quad c \in \mathbb{R} \\
 &= \frac{1}{2} \ln |x^2+x+1| - \sqrt{3} \arctan\left(\frac{2x+1}{\sqrt{3}}\right) + c, \quad c \in \mathbb{R}.
 \end{aligned}$$

3. $\int_4^6 \frac{7x+6}{x^2-x-6} dx.$

On calcule d'abord $\int \frac{7x+6}{x^2-x-6} dx.$

On décompose d'abord $\frac{7x+6}{x^2-x-6}$ en éléments simples :

$$\begin{aligned}
 \frac{7x+6}{x^2-x-6} &= \frac{7x+6}{(x+2)(x-3)} \\
 &= \frac{A}{x+2} + \frac{B}{x-3} \\
 &= \frac{A(x-3) + B(x+2)}{(x+2)(x-3)} \\
 &= \frac{Ax - 3A + Bx + 2B}{(x-1)(x+1)} \\
 &= \frac{(A+B)x + (-3A+2B)}{(x+1)(x-1)} \\
 &= \frac{7x+6}{x^2-1}
 \end{aligned}$$

par identification, on obtient :

$$\begin{cases} (A + B) = 7 \\ (-3A + 2B) = 6 \end{cases} \Rightarrow \begin{cases} 3A + 3B = 21 \\ (-3A + 2B) = 6 \end{cases} \Rightarrow A = \frac{8}{5}, B = \frac{27}{5},$$

d'où

$$\frac{7x + 6}{x^2 - x - 6} = \frac{7x + 6}{(x + 2)(x - 3)} = \frac{\frac{8}{5}}{x + 2} + \frac{\frac{27}{5}}{x - 3} = \frac{8}{5(x + 2)} + \frac{27}{5(x - 3)}.$$

Puis,

$$\begin{aligned} \int \frac{7x + 6}{x^2 - x - 6} dx &= \int \frac{8}{5(x + 2)} + \frac{27}{5(x - 3)} dx \\ &= \int \frac{8}{5(x + 2)} dx + \int \frac{27}{5(x - 3)} dx \\ &= \frac{8}{5} \int \frac{1}{(x + 2)} dx + \frac{27}{5} \int \frac{1}{(x - 3)} dx \\ &= \frac{8}{5} \ln |x + 2| + \frac{27}{5} \ln |x - 3| + c, \quad c \in \mathbb{R}. \end{aligned}$$

Donc

$$\begin{aligned} \int_4^6 \frac{7x + 6}{x^2 - x - 6} dx &= \left[\frac{8}{5} \ln |x + 2| + \frac{27}{5} \ln |x - 3| \right]_4^6 \\ &= \left[\frac{8}{5} \ln |6 + 2| + \frac{27}{5} \ln |6 - 3| \right] - \left[\frac{8}{5} \ln |4 + 2| + \frac{27}{5} \ln |4 - 3| \right] \\ &= \frac{8}{5} \ln |8| + \frac{27}{5} \ln |3| - \frac{8}{5} \ln |6| - \frac{27}{5} \ln |1| \\ &= \frac{8}{5} \ln(8) + \frac{27}{5} \ln(3) - \frac{8}{5} \ln(6) \\ &= \frac{8}{5} \ln(4 * 2) + \frac{27}{5} \ln(3) - \frac{8}{5} \ln(3 * 2) \\ &= \frac{8}{5} \ln(4) + \frac{8}{5} \ln(2) + \frac{27}{5} \ln(3) - \frac{8}{5} \ln(3) - \frac{8}{5} \ln(2) \\ &= \frac{8}{5} \ln(4) + \frac{27}{5} \ln(3) - \frac{8}{5} \ln(3). \end{aligned}$$

Exercice 1.4.

1. Intégrer les fonctions trigonométriques suivantes :

(a) $\int \sin^2 x \cos^2 x dx$.

On utilise les deux formules suivantes :

$$\cos(2x) = 2 \cos^2(x) - 1 \Rightarrow \cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x)$$

et

$$\cos(2x) = 1 - 2 \sin^2(x) \Rightarrow \sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x)$$

d'où

$$\begin{aligned} \sin^2 x \cos^2 x &= \left(\frac{1}{2} - \frac{1}{2} \cos(2x)\right) \left(\frac{1}{2} + \frac{1}{2} \cos(2x)\right) \\ &= \frac{1}{2}(1 - \cos(2x)) \frac{1}{2}(1 + \cos(2x)) \\ &= \frac{1}{4}(1 - \cos^2(2x)) \\ &= \frac{1}{4}\left(1 - \left(\frac{1}{2} + \frac{1}{2} \cos(4x)\right)\right) \\ &= \frac{1}{4}\left(1 - \frac{1}{2} - \frac{1}{2} \cos(4x)\right) \\ &= \frac{1}{4}\left(\frac{1}{2} - \frac{1}{2} \cos(4x)\right) \\ &= \frac{1}{8}(1 - \cos(4x)), \end{aligned}$$

donc

$$\begin{aligned} \int \sin^2 x \cos^2 x dx &= \int \frac{1}{8}(1 - \cos(4x)) dx \\ &= \frac{1}{8} \int (1 - \cos(4x)) dx \\ &= \frac{1}{8}\left(x - \frac{1}{4} \sin(4x)\right) + c, \quad c \in \mathbb{R} \\ &= \frac{1}{8}x - \frac{1}{32} \sin(4x) + c, \quad c \in \mathbb{R}. \end{aligned}$$

(b) $\int \sin(3x) \cos(4x) dx$.

On a,

$$\begin{aligned}
 \int \sin(3x) \cos(4x) dx &= \int \frac{1}{2} (\sin((3+4)x) + \sin((3-4)x)) dx \\
 &= \int \frac{1}{2} (\sin(7x) + \sin(-x)) dx \\
 &= \int \frac{1}{2} \sin(7x) dx + \int \frac{1}{2} \sin(-x) dx \\
 &= \frac{-1}{14} \cos(7x) dx + \frac{1}{2} \cos(-x) + c, \quad c \in \mathbb{R} \\
 &= \frac{-1}{14} \cos(7x) dx + \frac{1}{2} \cos(x) + c, \quad c \in \mathbb{R}. \quad (\cos(-x) = \cos(x)).
 \end{aligned}$$

(c) $\int_0^{3\pi} \cos^2 x \sin^3 x dx$.

On calcule d'abord $\int \cos^2 x \sin^3 x dx$.

On a,

$$\begin{aligned}
 \int \sin^3 x \cos^2 x dx &= \int \sin^2 x \cos^2 x \sin x dx \\
 &= \int (1 - \cos^2 x) \cos^2 x \sin x dx,
 \end{aligned}$$

on pose $t = \cos x$ d'où $dt = -\sin x dx$, on obtient

$$\begin{aligned}
 \int (1 - \cos^2 x) \cos^2 x \sin x dx &= \int -(1 - \cos^2 x) \cos^2 x (-\sin x) dx \\
 &= \int -(1 - t^2)t^2 dt \\
 &= \int (t^4 - t^2) dt \\
 &= \frac{t^5}{5} - \frac{t^3}{3} + c, \quad c \in \mathbb{R} \\
 &= \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + c, \quad c \in \mathbb{R}.
 \end{aligned}$$

Et

$$\begin{aligned}
 \int_0^{3\pi} \cos^2 x \sin^3 x dx &= \left[\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} \right]_0^{3\pi} \\
 &= \left[\frac{\cos^5(3\pi)}{5} - \frac{\cos^3(3\pi)}{3} \right] - \left[\frac{\cos^5 0}{5} - \frac{\cos^3 0}{3} \right] \\
 &= \frac{\cos^5(\pi)}{5} - \frac{\cos^3(\pi)}{3} - \frac{\cos^5 0}{5} + \frac{\cos^3 0}{3} \\
 &= \frac{-1}{5} - \frac{-1}{3} - \frac{1}{5} + \frac{1}{3} \\
 &= \frac{-2}{5} + \frac{2}{3} = \frac{4}{15}.
 \end{aligned}$$

2. Considérons les primitives suivantes :

$$I = \int \frac{\sin x}{\sin x + \cos x} dx \quad \text{et} \quad J = \int \frac{\cos x}{\sin x + \cos x} dx.$$

(a) Calculer $I + J$ et $I - J$.

Alors,

$$\begin{aligned}
 I + J &= \int \frac{\sin x}{\sin x + \cos x} dx + \int \frac{\cos x}{\sin x + \cos x} dx \\
 &= \int \left(\frac{\sin x}{\sin x + \cos x} + \frac{\cos x}{\sin x + \cos x} \right) dx \\
 &= \int \frac{\sin x + \cos x}{\sin x + \cos x} dx \\
 &= \int 1 dx \\
 &= x + c_1, \quad c_1 \in \mathbb{R}.
 \end{aligned}$$

Et

$$\begin{aligned}
 I - J &= \int \frac{\sin x}{\sin x + \cos x} dx - \int \frac{\cos x}{\sin x + \cos x} dx \\
 &= \int \left(\frac{\sin x}{\sin x + \cos x} - \frac{\cos x}{\sin x + \cos x} \right) dx \\
 &= \int \frac{\sin x - \cos x}{\sin x + \cos x} dx \\
 &= - \int \frac{-\sin x + \cos x}{\sin x + \cos x} dx \\
 &= - \ln | \sin x + \cos x | + c_2, \quad c_2 \in \mathbb{R}.
 \end{aligned}$$

(b) En déduire I et J .

On a,

$$\begin{aligned} \begin{cases} I + J = x \\ I - J = -\ln |\sin x + \cos x| \end{cases} &\Rightarrow \begin{cases} I = x - J \\ 2I = x - \ln |\sin x + \cos x| \end{cases} \\ &\Rightarrow \begin{cases} J = x - I \\ I = \frac{x - \ln |\sin x + \cos x|}{2} \end{cases} \\ &\Rightarrow \begin{cases} I = \frac{x}{2} - \frac{\ln |\sin x + \cos x|}{2} \\ J = x - \left(\frac{x}{2} - \frac{\ln |\sin x + \cos x|}{2} \right) \end{cases} \\ &\Rightarrow \begin{cases} I = \frac{x}{2} - \frac{\ln |\sin x + \cos x|}{2} + c_3, \quad c_3 \in \mathbb{R} \\ J = \frac{x}{2} + \frac{\ln |\sin x + \cos x|}{2} + c_4, \quad c_4 \in \mathbb{R}. \end{cases} \end{aligned}$$

Exercice 1.5. Trouver les primitives suivantes :

1. $\int (x+1)\sqrt{x^2+2x+5} dx$.

On a,

$$\begin{aligned} \int (x+1)\sqrt{x^2+2x+5} dx &= \frac{1}{2} \int (2x+2)(x^2+2x+5)^{\frac{1}{2}} dx \\ &= \frac{1}{2} \frac{(x^2+2x+5)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c, \quad c \in \mathbb{R} \\ &= \frac{1}{3} (x^2+2x+5)^{\frac{3}{2}} + c, \quad c \in \mathbb{R}. \end{aligned}$$

L'utilisation de $\int f'(x)f^n(x)dx = \frac{f^{n+1}(x)}{n+1} + c$.

2. $\int \sin x e^{\cos x} dx$.

On a,

$$\begin{aligned} \int \sin x e^{\cos x} dx &= - \int -\sin x e^{\cos x} dx \\ &= -e^{\cos x} + c, \quad c \in \mathbb{R}. \end{aligned}$$

L'utilisation de $\int f'(x)e^{f(x)} dx = e^{f(x)} + c$.

3. $I = \int \sin(2x)e^{3x} dx$.

On pose

$$\begin{aligned} u(x) = \sin(2x) &\Rightarrow u'(x) = 2 \cos(2x), \\ v'(x) = e^{3x} &\Rightarrow v(x) = \frac{1}{3} e^{3x}, \end{aligned}$$

et l'on intègre par parties, ce qui donne :

$$\begin{aligned} I &= \sin(2x)\frac{1}{3}e^{3x} - \int 2\cos(2x)\frac{1}{3}e^{3x}dx \\ &= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{3}\int \cos(2x)e^{3x}dx \end{aligned}$$

Le calcul de $\int e^x \cos(x)dx$:
on pose

$$\begin{aligned} u(x) = \cos(2x) &\Rightarrow u'(x) = -2\sin(2x), \\ v'(x) = e^{3x} &\Rightarrow v(x) = \frac{1}{3}e^{3x}, \end{aligned}$$

et l'on intègre une deuxième fois par parties, ce qui donne :

$$\begin{aligned} \int \cos(2x)e^{3x}dx &= \cos(2x)\frac{1}{3}e^{3x} - \int -2\sin(2x)\frac{1}{3}e^{3x}dx \\ &= \frac{1}{3}\cos(2x)e^{3x} + \frac{2}{3}\int \sin(2x)e^{3x}dx \end{aligned}$$

d'où,

$$\begin{aligned} I &= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{3}\int \cos(2x)e^{3x}dx \\ &= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{3}\left(\frac{1}{3}\cos(2x)e^{3x} + \frac{2}{3}\int \sin(2x)e^{3x}dx\right) \\ &= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{9}\cos(2x)e^{3x} - \frac{4}{9}\int \sin(2x)e^{3x}dx \\ &= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{9}\cos(2x)e^{3x} - \frac{4}{9}I \\ \Rightarrow I + \frac{4}{9}I &= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{9}\cos(2x)e^{3x} \\ \Rightarrow \frac{13}{9}I &= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{9}\cos(2x)e^{3x} \\ \Rightarrow I &= \frac{9}{13}\left(\frac{1}{3}\sin(2x)e^{3x} - \frac{2}{9}\cos(2x)e^{3x}\right) + c, c \in \mathbb{R} \\ \Rightarrow I &= \frac{3}{13}\sin(2x)e^{3x} - \frac{2}{13}\cos(2x)e^{3x} + c, c \in \mathbb{R}. \end{aligned}$$

4. $\int \frac{\sin x}{\cos^4 x} dx$.
On a,

$$\begin{aligned} \int \frac{\sin x}{\cos^4 x} dx &= \int \sin x \cos^{-4} x dx \\ &= - \int -\sin x \cos^{-4} x dx \\ &= -\frac{\cos^{-4+1} x}{-4+1} + c, c \in \mathbb{R} \\ &= -\frac{\cos^{-3} x}{-3} + c, c \in \mathbb{R} \\ &= \frac{1}{3 \cos^3 x} + c, c \in \mathbb{R}. \end{aligned}$$

L'utilisation de $\int f'(x)f^n(x)dx = \frac{f^{n+1}(x)}{n+1} + c$.

*On calcule aussi par changement de variable :
en posant*

$$t = \cos x \Rightarrow dt = -\sin x dx,$$

on obtient

$$\begin{aligned} \int \frac{\sin x}{\cos^4 x} dx &= \int \frac{1}{\cos^4 x} \sin x dx \\ &= - \int \frac{1}{\cos^4 x} (-\sin x) dx \\ &= - \int \frac{1}{t^4} dt \\ &= - \int t^{-4} dt \\ &= -\frac{t^{-4+1}}{-4+1} + c, c \in \mathbb{R} \\ &= -\frac{t^{-3}}{-3} + c, c \in \mathbb{R} \\ &= \frac{1}{3t^3} + c, c \in \mathbb{R} \\ &= \frac{1}{3 \cos^3 x} + c, c \in \mathbb{R}. \end{aligned}$$

(à la fin on a remplacé $t = \cos x$).

5. $\int \cos x \sin^4 x dx$.

On a,

$$\begin{aligned} \int \cos x \sin^4 x dx &= \frac{\sin^{4+1} x}{4+1} + c, c \in \mathbb{R} \\ &= \frac{\sin^5 x}{5} + c, c \in \mathbb{R}. \end{aligned}$$

L'utilisation de $\int f'(x)f^n(x)dx = \frac{f^{n+1}(x)}{n+1} + c$.

On calcule aussi par changement de variable :

en posant

$$t = \sin x \Rightarrow dt = \cos x dx,$$

on obtient

$$\begin{aligned} \int \cos x \sin^4 x dx &= \int \sin^4 x \cos x dx \\ &= \int t^4 dt \\ &= \frac{t^5}{5} + c, c \in \mathbb{R} \\ &= \frac{\sin^5 x}{5} + c, c \in \mathbb{R}. \end{aligned}$$

(à la fin on a remplacé $t = \sin x$).

6. $\int \frac{\cos x}{1-\sin^2 x} dx$.

en posant

$$t = \sin x \Rightarrow dt = \cos x dx,$$

on obtient

$$\begin{aligned} \int \frac{\cos x}{1-\sin^2 x} dx &= \int \frac{1}{1-\sin^2 x} \cos x dx \\ &= \int \frac{1}{1-t^2} dt \end{aligned}$$

On décompose d'abord $\frac{1}{1-t^2}$ en éléments simples :

$$\begin{aligned} \frac{1}{1-t^2} &= \frac{1}{(1-t)(1+t)} \\ &= \frac{A}{1-t} + \frac{B}{1+t} \\ &= \frac{A(1+t) + B(1-t)}{(1-t)(1+t)} \\ &= \frac{A + At + B - Bt}{(1-t)(1+t)} \\ &= \frac{(A-B)t + (A+B)}{(1-t)(1+t)} \\ &= \frac{0t + 1}{(1-t)(1+t)} \end{aligned}$$

par identification, on obtient :

$$\begin{cases} (A-B) = 0 \\ (A+B) = 1 \end{cases} \Rightarrow \begin{cases} A = B \\ (A+B) = 1 \end{cases} \Rightarrow A = \frac{1}{2}, B = \frac{1}{2},$$

d'où

$$\frac{1}{1-t^2} = \frac{1}{(1-t)(1+t)} = \frac{\frac{1}{2}}{1-t} + \frac{\frac{1}{2}}{1+t} = \frac{1}{2(1-t)} + \frac{1}{2(1+t)}.$$

Puis,

$$\begin{aligned} \int \frac{1}{1-t^2} dx &= \int \left(\frac{1}{2(1-t)} + \frac{1}{2(1+t)} \right) dx \\ &= \int \frac{1}{2(1-t)} dx + \int \frac{1}{2(1+t)} dx \\ &= \frac{-1}{2} \ln |1-t| + \frac{1}{2} \ln |1+t| + c, \quad c \in \mathbb{R} \\ &= \frac{-1}{2} \ln |1 - \sin x| + \frac{1}{2} \ln |1 + \sin x| + c, \quad c \in \mathbb{R} \end{aligned}$$

(à la fin on a remplacé $t = \sin x$). Donc

$$\int \frac{\cos x}{1 - \sin^2 x} dx = \frac{-1}{2} \ln |1 - \sin x| + \frac{1}{2} \ln |1 + \sin x| + c, \quad c \in \mathbb{R}.$$