

$$y = \frac{\iint_S y \, dudy}{(\text{Area}(S))} \quad (0,1)$$

in coordinates polaires

$$y_0 = \frac{1}{2\pi} \iint_S r^2 \sin \theta \, dr \, d\theta \quad (0,1)$$

$$= \frac{1}{2\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2} + \frac{\pi}{6}} \left[ \int_1^3 r^2 \sin \theta \, dr \right] d\theta \quad (0,1)$$

$$= \frac{26}{6\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2} + \frac{\pi}{6}} \sin \theta \, d\theta \quad (0,2)$$

$$= \frac{-26}{6\pi} \cos \theta \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2} + \frac{\pi}{6}} \quad (0,2)$$

$$= \frac{-26}{6\pi} \left( \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) - \cos\frac{\pi}{3} \right) \quad (0,2)$$

$$= \frac{-26}{6\pi} \left( -\sin\frac{\pi}{6} - \cos\frac{\pi}{3} \right) = \frac{26}{6\pi} = \frac{13}{3\pi}$$