Résumé Chapitre IV

Équation Généralisée de Lagrange

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = -\frac{\partial \mathcal{D}}{\partial \dot{q}} + \mathcal{F}.$$

Translations: $q = x, y, z...$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = -\frac{\partial \mathcal{D}}{\partial \dot{x}} + F.$$

Rotations: $q = \theta, \varphi, ...$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = -\frac{\partial \mathcal{D}}{\partial \dot{\theta}} + \mathcal{M}.$$

Équation du Mouvement		**Solution Permanente**
$\ddot{q} + 2\lambda\dot{q} + \omega_0^2 q = (F_0/a)\cos\Omega t$	$\longrightarrow$	$q(t) = A\cos(\Omega t + \phi)$
$\ddot{q} + 2\lambda\dot{q} + \omega_0^2 q = (F_0/a)\sin\Omega t$	$\longrightarrow$	$q(t) = A\sin(\Omega t + \phi)$

Calcul de A et ϕ à l'aide de la Représentation Complexe

$$F_0\cos\Omega t \longrightarrow F_0 e^{j\Omega t}$$
$$q(t) = A\cos(\Omega t + \phi) \longrightarrow \underline{q}(t) = \underline{A}e^{j\Omega t}$$
$$\ddot{\underline{q}} + 2\lambda\dot{\underline{q}} + \omega_0^2\underline{q} = (F_0/a)e^{j\Omega t} \Longrightarrow$$

$$\underline{A} = \frac{(F_0/a)}{\omega_0^2 - \Omega^2 + j2\lambda\Omega}. \qquad A = \frac{(F_0/a)}{\sqrt{\left(\omega_0^2 - \Omega^2\right)^2 + 4\lambda^2\Omega^2}}. \qquad \tan\phi = -\frac{2\lambda\Omega}{\omega_0^2 - \Omega^2}.$$

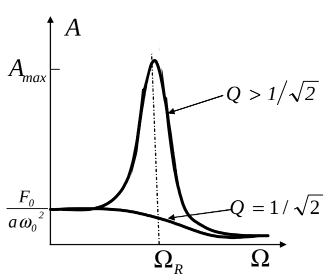
Pulsations de Résonance

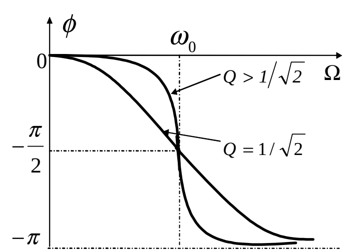
$$\frac{\partial A}{\partial \Omega} = 0 \Longrightarrow \Omega_R = \sqrt{\omega_0^2 - 2\lambda^2}. \text{ (résonance d'amplitude)}$$
$$\tan\phi = -\infty \Longrightarrow \Omega = \omega_0. \text{ (résonance de phase: } \phi = -\frac{\pi}{2}\text{)}$$

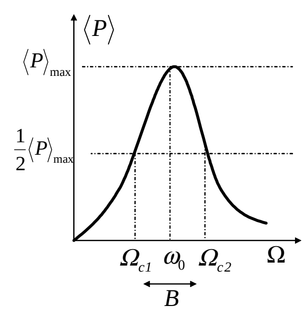
Amplitude Maximale et Puissance Moyenne

$$A_{\max} = \frac{F_0/a}{\sqrt{4\lambda^2\omega_0^2 - 4\lambda^4}}. \quad \langle \mathcal{P} \rangle = \frac{\Omega^2\lambda\left(F_0^2/a\right)}{\left(\omega_0^2 - \Omega^2\right)^2 + 4\lambda^2\Omega^2}. \quad \langle \mathcal{P} \rangle_{\max} = \frac{F_0^2}{4\lambda a}$$

Graphes







Pulsations de Coupure, Bande Passante, et Facteur de Qualité $(\lambda \ll \omega_0)$

$$\Omega_{c1} \approx \omega_0 - \lambda. \quad \Omega_{c2} \approx \omega_0 + \lambda. \quad B = \Omega_{c2} - \Omega_{c2} = 2\lambda. \quad Q = \omega_0/B$$