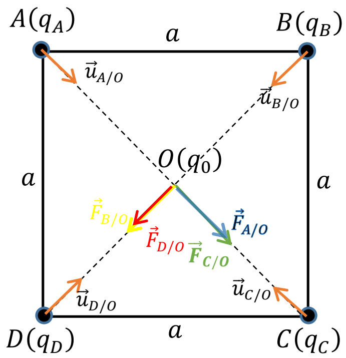
Exercice 01

D'après l'énonce on a:

$q_A = 2q$, $q_B = q$, $q_C = -2q$ et $q_D = -q$

La force électrostatique appliquée sur la charge q_O sera la somme des forces appliquées par les quatre charges placées aux sommet du carré sur cette charge, donc :



$$\vec{F}(O) = \vec{F}_{A/O} + \vec{F}_{B/O} + \vec{F}_{C/O} + \vec{F}_{D/O}$$

$$OA = OB = OC = OD = a/\sqrt{2}$$

$$\vec{u}_{A/O} = \cos\left(\frac{\pi}{4}\right)\vec{\imath} - \sin\left(\frac{\pi}{4}\right)\vec{\jmath} = 1/\sqrt{2}\vec{\imath} - 1/\sqrt{2}\vec{\jmath}\ ;\ \vec{u}_{B/O} = -\cos\left(\frac{\pi}{4}\right)\vec{\imath} - \sin\left(\frac{\pi}{4}\right)\vec{\jmath} = -1/\sqrt{2}\vec{\imath} - 1/\sqrt{2}\vec{\jmath}$$

$$\vec{u}_{C/O} = -\cos\left(\frac{\pi}{4}\right)\vec{\imath} + \sin\left(\frac{\pi}{4}\right)\vec{\jmath} = -1/\sqrt{2}\vec{\imath} + 1/\sqrt{2}\vec{\jmath}\ ;\ \vec{u}_{D/O} = \cos\left(\frac{\pi}{4}\right)\vec{\imath} + \sin\left(\frac{\pi}{4}\right)\vec{\jmath} = 1/\sqrt{2}\vec{\imath} + 1/\sqrt{2}\vec{\jmath}$$

$$\vec{F}_{A/O} = \frac{kq_Aq_0}{AO^2}\vec{u}_{A/O} = \frac{2kqq_0}{(a/\sqrt{2})^2}\left(\frac{1}{\sqrt{2}}\vec{\imath} - \frac{1}{\sqrt{2}}\vec{\jmath}\right) = \frac{2\sqrt{2}kqq_0}{a^2}(\vec{\imath} - \vec{\jmath})$$

$$\vec{F}_{B/O} = \frac{kq_Bq_0}{BO^2}\vec{u}_{B/O} = \frac{kqq_0}{(a/\sqrt{2})^2}\left(-\frac{1}{\sqrt{2}}\vec{\imath} - \frac{1}{\sqrt{2}}\vec{\jmath}\right) = \frac{\sqrt{2}kqq_0}{a^2}(-\vec{\imath} - \vec{\jmath})$$

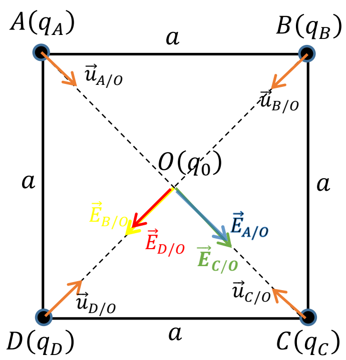
$$\vec{F}_{C/O} = \frac{kq_Cq_0}{CO^2}\vec{u}_{C/O} = \frac{-2kqq_0}{(a/\sqrt{2})^2}\left(-\frac{1}{\sqrt{2}\vec{\imath}} + \frac{1}{\sqrt{2}}\vec{\jmath}\right) = \frac{2\sqrt{2}kqq_0}{a^2}(\vec{\imath} - \vec{\jmath})$$

$$\vec{F}_{D/O} = \frac{kq_Dq_0}{DO^2}\vec{u}_{D/O} = \frac{-kqq_0}{(a/\sqrt{2})^2}\left(\frac{1}{\sqrt{2}}\vec{\imath} + \frac{1}{\sqrt{2}}\vec{\jmath}\right) = \frac{\sqrt{2}kqq_0}{a^2}(-\vec{\imath} - \vec{\jmath})$$

$\vec{F}(O) = \vec{F}_{A/O} + \vec{F}_{B/O} + \vec{F}_{C/O} + \vec{F}_{D/O} = \frac{2\sqrt{2}kqq_0}{a^2}(\vec{\imath} - \vec{\jmath}) + \frac{\sqrt{2}kqq_0}{a^2}(-\vec{\imath} - \vec{\jmath}) + \frac{2\sqrt{2}kqq_0}{a^2}(\vec{\imath} - \vec{\jmath}) + \frac{\sqrt{2}kqq_0}{a^2}(-\vec{\imath} - \vec{\jmath}) = \frac{2\sqrt{2}kqq_0}{a^2}(\vec{\imath} - 3\vec{\jmath})$

N.B. Il est aussi possible déduire $\vec{F}_{C/O}$ et $\vec{F}_{D/O}$ directement à partir de $\vec{F}_{A/O}$et de $\vec{F}_{C/O}$

2- Calcul de champ électrique :



1ere methode :

$$\vec{F}(O) = q_0\vec{E}(O) \rightarrow \vec{E}(O) = \frac{\vec{F}(O)}{q_0} = \frac{2\sqrt{2}kq}{a^2}(\vec{\imath} - 3\vec{\jmath})$$

2mee methode :

$$\vec{E}(O) = \vec{E}_{A/O} + \vec{E}_{B/O} + \vec{E}_{C/O} + \vec{E}_{D/O}$$

$$\vec{E}_{A/O} = \frac{kq_A}{AO^2}\vec{u}_{A/O} = \frac{2kq}{(a/\sqrt{2})^2}\left(\frac{1}{\sqrt{2}}\vec{\imath} - \frac{1}{\sqrt{2}}\vec{\jmath}\right) = \frac{2\sqrt{2}kq}{a^2}(\vec{\imath} - \vec{\jmath})$$

$$\vec{E}_{B/O}=\frac{kq_B}{BO^2}\vec{u}_{B/O}=\frac{kq}{(a/\sqrt{2})^2}\left(-\frac{1}{\sqrt{2}}\vec{\imath}-\frac{1}{\sqrt{2}}\vec{\jmath}\right)=\frac{\sqrt{2}kqq_0}{a^2}(-\vec{\imath}-\vec{\jmath})$$

$$\vec{E}_{C/O}=\frac{kq_C}{CO^2}\vec{u}_{C/O}=\frac{-2kq}{(a/\sqrt{2})^2}\left(-\frac{1}{\sqrt{2}}\vec{\imath}+\frac{1}{\sqrt{2}}\vec{\jmath}\right)=\frac{2\sqrt{2}kqq_0}{a^2}(\vec{\imath}-\vec{\jmath})$$

$$\vec{E}_{D/O}=\frac{kq_Dq_0}{DO^2}\vec{u}_{D/O}=\frac{-kqq_0}{(a/\sqrt{2})^2}\left(\frac{1}{\sqrt{2}}\vec{\imath}+\frac{1}{\sqrt{2}}\vec{\jmath}\right)=\frac{\sqrt{2}kqq_0}{a^2}(-\vec{\imath}-\vec{\jmath})$$

$$\vec{E}(O)=\frac{2\sqrt{2}kq}{a^2}(\vec{\imath}-\vec{\jmath})+\frac{\sqrt{2}kq}{a^2}(-\vec{\imath}-\vec{\jmath})+\frac{2\sqrt{2}kq}{a^2}(\vec{\imath}-\vec{\jmath})+\frac{\sqrt{2}kq}{a^2}(-\vec{\imath}-\vec{\jmath})=\frac{2\sqrt{2}kq}{a^2}(\vec{\imath}-3\vec{\jmath})$$

3 Le potentiel :

$$V(O)=V_A+V_B+V_C+V_D$$

$$V_A=\frac{kq_A}{OA}=\frac{2\sqrt{2}Kq}{a}$$

$$V_B=\frac{kq_B}{OB}=\frac{\sqrt{2}Kq}{a}$$

$$V_C=\frac{kq_C}{OC}=-\frac{2\sqrt{2}Kq}{a}$$

$$V_D=\frac{kq_D}{OD}=-\frac{\sqrt{2}Kq}{a}$$

$$V(O)=0$$

4 L'énergie potentiel de la charge q_0

$$E_p=q_0\,V(O)=0$$

5 *L'énergie interne du système* (on va la notée U_p)

$$U_p=\sum_{i}^{4}\sum_{j>i}^{4}\frac{kq_iq_j}{r_{ij}}=\frac{kq_Aq_B}{AB}+\frac{kq_Aq_C}{AC}+\frac{kq_Aq_D}{AD}+\frac{kq_Bq_C}{BC}+\frac{kq_Bq_D}{BD}+\frac{kq_Cq_D}{CD}$$

$$U_p=\frac{2Kq^2}{a}-\frac{4Kq^2}{a\sqrt{2}}-\frac{2Kq^2}{a}-\frac{2Kq^2}{a}-\frac{Kq^2}{a\sqrt{2}}+\frac{2Kq^2}{a}=-\frac{5Kq^2}{\sqrt{2}a}$$