Physics 218 Exam 1

with Solutions

Spring 2011, Sections 513-515,526,528

Fill out the information below but do not open the exam until instructed to do so!

Name :__________________________

Signature :__________________________

Student ID :__________________________

E-mail :__________________________

Section # :__________________________

Rules of the exam:

1. You have the full class period to complete the exam.
2. Formulae are provided on the last page. You **may NOT** use any other formula sheet.
3. When calculating numerical values, be sure to keep track of units.
4. You may use this exam or come up front for scratch paper.
5. Be sure to **put a box around your final answers** and clearly indicate your work to your grader.
6. Clearly erase any unwanted marks. No credit will be given if we can't figure out which answer you are choosing, or which answer you want us to consider.
7. Partial credit can be given only if your work is clearly explained and labeled.
8. **All work must be shown to get credit for the answer marked. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer.**

Put your initials here after reading the above instructions:__________

Table to be filled by graders only!

Part	Score
Part 1 (15)	
Part 2 (20)	
Part 3 (20)	
Part 4 (20)	
Bonus (5) (Actually 6 points)	
Exam Total	

Part 1: Basic ideas of units, conversions, and vectors.

Problem 1.1: (2p) What system of units is used in this course? What are the basic units of mass, length, and time of that system ?

International System (SI), Kilogram, meter, seconds.

Problem 1.2: Pascal, Bar and psi are units of pressure defined as:

1 Pa (Pascal) $= 10^{-5}$ bar

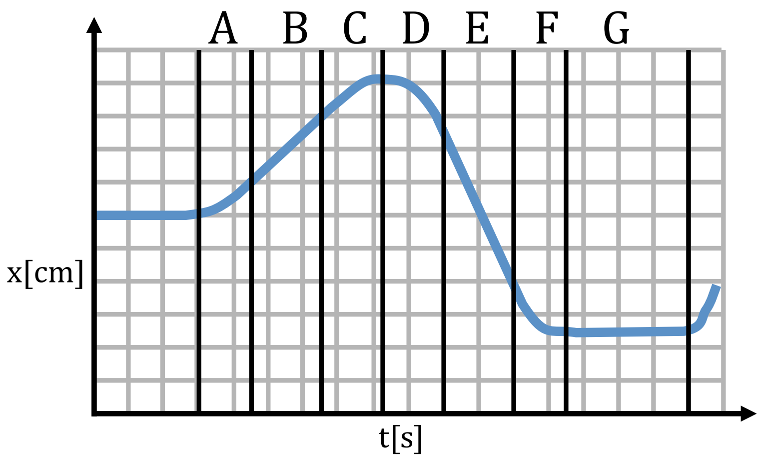
1 Bar (bar) $= 145\ 10^{-6}$ psi

Question 1.2.1: (4p) Express 290 psi in units of Pascals.

$$290\ psi = 290\ \frac{Bar}{145\ 10^{-6}} = \frac{290}{145\ 10^{-6}}\frac{Pa}{10^{-5}} = 2\ \ 10^{11}\ Pa$$

Problem 1.3: The following plot shows the position x as a function of time

Question 1.3.1: (7p) For each time range A,B,C…I, fill the table below writing in each cell whether the velocity and acceleration are <0, >0, or =0.



Region	Velocity	Acceleration
A	>0	>0
B	>0	=0
C	>0	<0
D	<0	<0
E	<0	=0
F	<0	>0
G	=0	=0

Question 1.3.2: (2p) Is the magnitude of the velocity greater in region B than it is in E ? Why ?

The magnitude of the velocity at a given time is the magnitude of the slope of the tangent line in the above graph at that given time. The slope at time range B is about +2 squares/2 squares, with a magnitude of +1. The slope at time range E is about -4 squares/2 squares with a magnitude of -2. Hence, the answer is NO; the magnitude of the velocity at region B is smaller than that at region E.

Part 2: Tennis shot.

Problem 2.1: (20p) You are at a distance of $\boldsymbol{d}$ meters from your friend's car which is driving away from you with a velocity of $\boldsymbol{V_c}$ <u>and some acceleration</u>. In an effort to pass a tennis ball to your friend before he is too far away you throw the tennis ball at an angle of ***45*** degrees up with a horizontal component of $\boldsymbol{V_{xb}}$. Gravity is present and neglect the size of the car as well as yours

Question 2.1.1: (4p) In the space below draw a schematic diagram of the problem and write any associated times. In addition choose and draw a coordinate system and clearly indicate its origin.

Question 2.1.2: (6p) Write the equations of motion of the accelerating car and tennis ball according to your coordinate system. Indicate which known parameters are zero.

$$X_c(t) = d + V_c t + \frac{a_c}{2}t^2$$
$$X_b(t) = V_{xb} t$$
$$Y_b(t) = V_{yb} t - \frac{g}{2}t^2 = V_{xb} t - \frac{g}{2}t^2$$

Question 2.1.3: (4p) Find the time the tennis ball will be in the air.

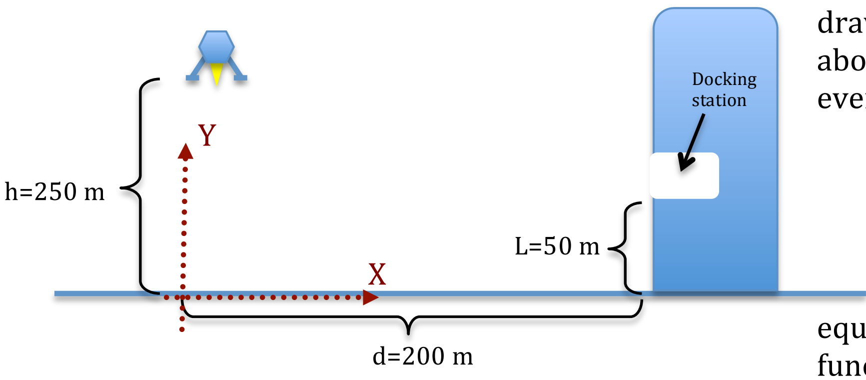
$$Y_b(t_L) = V_{xb} t_L - \frac{g}{2}t_L^2 = 0 \Rightarrow t_L = \frac{2V_{xb}}{g}$$

Question 2.1.4: (6p) Find the acceleration the car needs to have such that the tennis ball will land on the car.

$$X_c(t_L) = X_b(t_L) \Rightarrow d + V_c t_L + \frac{a_c}{2}t_L^2 = V_{xb} t_L \Rightarrow$$
$$\frac{a_c}{2}t_L^2 = V_{xb} t_L - d - V_c t_L \Rightarrow a_c = \frac{2}{t_L^2}\left(V_{xb} t_L - d - V_c t_L\right)$$

Part 3: Parking the rocket.

Problem 3.1: (20p) A spaceship is approaching its docking station located on the side of a building and ***50m*** above ground. When the spaceship is at a horizontal distance ***d=200m*** from the docking station its horizontal velocity is ***5m/s*** towards the docking station and ***10m/s*** towards the ground. The engine of the spaceship is ignited producing acceleration in the vertical direction. <u>The following questions must be answered in the form of a number with proper units.</u>



Question 3.1.1: (3p) Choose and draw your coordinate system on the figure above and associate times to the relevant events.

Question 3.1.2: (5p) Write the equation of motion of the spaceship as a function of time

$$X(t) = 5\frac{m}{s}t$$

$$Y(t) = h - 10\frac{m}{s}t + \frac{1}{2}a_y t^2$$

Question 3.1.3: (5p) Find the time at which the spacecraft reaches the building.

$$X(t_L) = 5\frac{m}{s}t_L = 200m \quad \Rightarrow \quad t_L = \frac{200}{5}s = 40s$$

Question 3.1.4: (7p) Find the acceleration of the spacecraft in the vertical direction such that it lands exactly at the docking station.

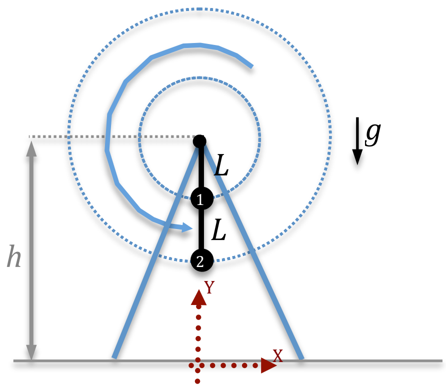
$$Y(t_L) = 50m$$

$$250 - 10\frac{m}{s}t_L + \frac{1}{2}a_y t_L^2 = 50m \Rightarrow \frac{1}{2}a_y t_L^2 = -200m + 10\frac{m}{s}40s = +200m$$

$$\Rightarrow a_y = \frac{400m}{1600s^2} = 0.25\frac{m}{s^2}$$

Part 4: A more complex problem.

Problem 4.1: (20p) Two balls are tied up to a rod of length ***2L*** and connected to a motor that makes it spin in the vertical plane with a circular uniform motion. Ball #1 is at a radius ***L*** and moving with velocity V_1 and ball #2 is at a radius ***2L***. The center of rotation is located at height ***h*** above the ground as shown in the picture below. All answers must be expressed in terms of known parameters.



Question 4.1.1: (2p) Which ball moves faster the one farther to the ground or the closer one ? why ?

$$V_1 = \frac{2\pi L}{T}, V_2 = \frac{2\pi 2L}{T} \Rightarrow \frac{V_2}{V_1} = 2$$

The ball closer to the ground moves faster as it has larger radius.

Question 4.1.2: (5p) When the rod is at the minimum position (as shown in the diagram) both balls break loose with a horizontal velocity and eventually fall to the ground due to gravity. Indicate a coordinate system and write the equations of motion of each ball in the vertical and horizontal components.

$X_1(t) = V_1 t$	$X_2(t) = V_2 t = 2V_1 t$
$Y_1(t) = h - L - \frac{g}{2}t^2$	$Y_2(t) = h - 2L - \frac{g}{2}t^2$

Question 4.1.3: (5p)Find the time it would take each ball to fall to the ground. Which ball touches the ground first ?

$$Y_1(t_{1L}) = h - L - \frac{g}{2}t_{1L}^2 = 0 \Rightarrow t_{1L} = \sqrt{\frac{2(h-L)}{g}}$$

$$Y_2(t_{2L}) = h - 2L - \frac{g}{2}t_{2L}^2 = 0 \Rightarrow t_{2L} = \sqrt{\frac{2(h-2L)}{g}}$$

Ball #2 touches the ground faster.

Question 4.1.4: (8p) Assuming that the height ***h*** is exactly ***3L***, which ball has a larger range? (***Hint:*** calculate the range of each and take the ratio)

$$X_1(t_{1L}) = V_1 t_{1L} = V_1\sqrt{\frac{2(h-L)}{g}} = V_1\sqrt{\frac{4}{g}}$$

$$X_2(t_{2L}) = V_2 t_{2L} = 2V_1 t_{2L} = 2V_1\sqrt{\frac{2}{g}}$$

$$\Rightarrow \frac{X_2(t_{2L})}{X_1(t_{1L})} = \frac{2V_1}{V_1}\sqrt{\frac{2}{g}}\sqrt{\frac{g}{4}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Ball #2 has a larger range.

Formula sheet:

Vectors:

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\tan(\varphi) = \frac{A_y}{A_x}$, where φ = angle between $\hat{x}$ axis and projection of vector $\vec{A}$ to the $(\hat{x}\ \hat{y})$ plane.

In 2-D, φ = angle between $\hat{x}$ axis and vector $\vec{A}$.

Mathematical Formulae:

$$at^2 + bt + c = 0 \quad \Rightarrow \quad t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $x = at^n \Rightarrow \frac{dx}{dt} = nat^{n-1}$.

If $x = at^n \Rightarrow \int_{t_2}^{t_2} x(t)\,dt = \frac{a}{n+1}\left(t_2^{n+1} - t_1^{n+1}\right)$

The following equations are always true:

$$\vec{v} = \frac{d\vec{r}}{dt} \quad \Rightarrow \quad v_x = \frac{dx}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \Rightarrow \quad a_x = \frac{dv_x}{dt}$$

$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t)\,dt \quad \Rightarrow \quad x(t) = x_0 + \int_0^t v_x(t)\,dt$$

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t)\,dt \quad \Rightarrow \quad v_x(t) = v_{0x} + \int_0^t a_x(t)\,dt$$

$$v_{av\text{-}x} = \frac{(x_2 - x_1)}{(t_2 - t_1)}, \qquad a_{av\text{-}x} = \frac{(v_{2x} - v_{1x})}{(t_2 - t_1)}$$

The following apply for constant acceleration:

$$\vec{r} = \vec{r}_0 + \vec{v}_o t + \frac{1}{2}\vec{a}t^2 \quad \Rightarrow \quad x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$\vec{v} = \vec{v}_0 + \vec{a}t \quad \Rightarrow \quad v_x(t) = v_{0x} + a_x t$$

$$v_x^2(t) = v_{0x}^2 + 2a_x\Delta x$$

$$\Delta x = \frac{v_{0x} + v_x(t)}{2}t$$

Other Equations:

$$a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$$

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$