

Physics 218 Exam 2

Spring 2010, Sections 521-525

Do not fill out the information below until instructed to do so!

Name : _____
Signature : _____
Student ID : _____
E-mail : _____
Section # : _____

Rules of the exam:

1. You have the full class period to complete the exam.
2. When calculating numerical values, be sure to keep track of units.
3. You may use this exam or come up front for scratch paper.
4. Be sure to **put a box around your final answers** and clearly indicate your work to your grader.
5. Clearly erase any unwanted marks. No credit will be given if we can't figure out which answer you are choosing, or which answer you want us to consider.
6. Partial credit can be given only if your work is clearly explained and labeled.
7. **All work must be shown to get credit for the answer marked. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer.**

Put your initials here after reading the above instructions: _____

Table to be filled by the graders	
Part	Score
Part 1 (25)	
Part 2 (20)	
Part 3 (25)	
Part 3 (30)	
Bonus (5)	
Exam Total	

Part 1: Basic Concepts

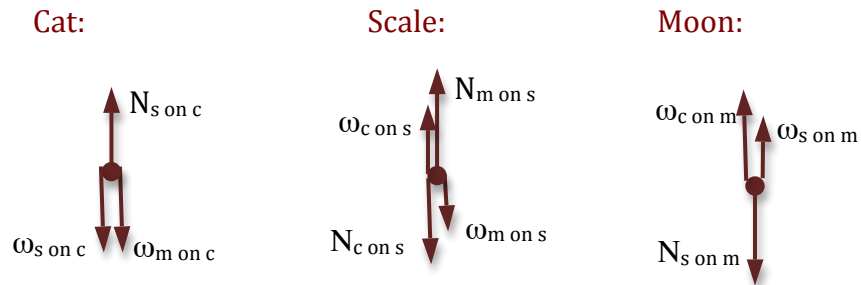
Problem 1.1: (4p) Fill the following table

Quantity	Units	Is it a vector ?, or a scalar ?
Force	Netwon	Vector
Work	Joule	Scalar
Power	Watts	Scalar
Energy	Joule	Scalar

Problem 1.2: A cat is standing over a scale standing on the surface of the moon.



Question 1.2.1: (5p) The cat, the scale, and the moon have masses. Neglect friction forces between them. Below draw the free-body diagrams of all three objects giving the forces distinct names.



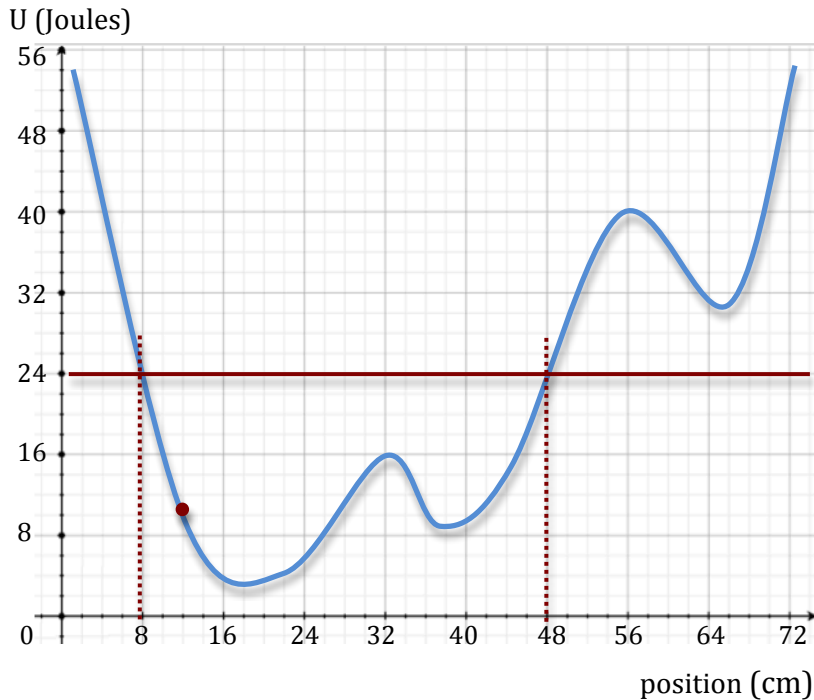
Question 1.2.2: (5p) From the drawn forces, fill the table below with the names of the forces that are action/reaction pairs.

Action	Reaction
$W_m \text{ on } s$	$W_s \text{ on } m$
$W_m \text{ on } c$	$W_c \text{ on } m$
$W_s \text{ on } c$	$W_c \text{ on } s$
$N_s \text{ on } c$	$N_c \text{ on } s$
$N_m \text{ on } s$	$N_s \text{ on } m$

Question 1.2.3: (2p) The weight of the cat in the moon is $|w| = m_{cat} g_{moon}$, which depends on the mass of the cat. Does the mass of the moon enters into play anywhere ? If it does, where ?

Yes, the mass of the moon enters implicitly into g_{moon} .

Problem 1.3: A particle is subjected to a single force, whose potential energy is drawn in the table below as a function of position. The particle's total energy is 24 Joules.



Question 1.3.1: (3p) If the particle is initially at position 12cm, what is the range of positions this particle can ever reach ?

The particle must be between 8 and 48 cm. Any position outside this range would have negative kinetic energy.

Question 1.3.2: (3p) At what position is the kinetic energy maximum ? Why ?

The kinetic energy is maximum at about 18 cm, where the potential energy U is minimum, and therefore $K=24\text{Joules}-U$ is maximum.

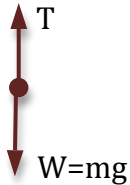
Question 1.3.3: (3p) Is the point at position 32cm a stable or unstable point ? Explain why.

It is a point of unstable equilibrium, as small departments from that point tend to move the particle further away from the point.

Part 2: (20p) Forces and Acceleration

Problem 2.1: A crate of mass m is accelerated upward by a chain of negligible mass whose breaking strength is T_{max} .

Question 2.1.1: (4p) Draw the free-body diagram of the crate.



Question 2.1.2: (8p) What is the maximum acceleration a_{max} that can be given to the crate without breaking the chain.

$$T+W=T-mg=ma \rightarrow a_{max} = T_{max}/m - g$$

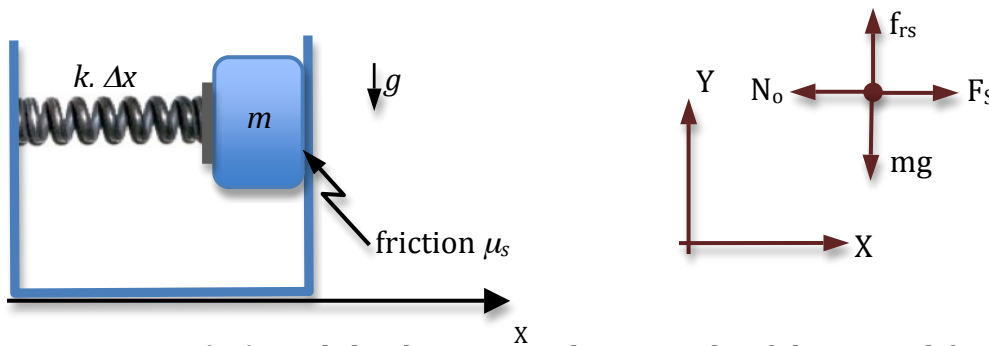
Question 2.1.3: (8p) If the acceleration is a (less than a_{max}) what is the tension T in the chain ?

From 2.1.2 we get $T=m(g+a)$

Part 3: (25p) Static Friction

Problem 3.1: A box contains a spring with constant $k=40$ N/m. The string is compressed $\Delta X=0.5$ m and pushes against a mass $m=1$ Kg as shown in the picture below. The mass is pushed by the spring towards the internal wall of the box. The coefficient of static friction between the mass and the wall is 0.8. **A number with proper units is expected for all these questions.**

Question 3.1.1: (5p) Draw the free-body diagram of the mass, as well the chosen coordinate system.



Question 3.1.2: (4p) Find the direction and magnitude of the normal force the wall of the box exerts on the mass.

$$F_s = k\Delta X \text{ (positive as it points in the + x direction)} = 40 \text{ N/m} \cdot 0.5 \text{ m} = 20 \text{ N}$$

$$N_0 + F_s = 0 \rightarrow N_0 = -F_s = -20 \text{ N. So Magnitude } 20 \text{ N, direction in the negative x axis.}$$

Question 3.1.3: (4p) What is the magnitude and direction of the static friction of force? Is it enough to keep the mass in its position?, or will the mass fall down to the bottom of the box? EXPLAIN.

For the block to stay in place the static friction force need to counteract the weight of $mg=9.8$ N. The static friction force can exert up to a maximum of $f_{rs} \leq \mu_s |N_0| = 16$ N, which is clearly more than the 9.8N and so the block will stay in place.

However, the actual friction force holding the block is just 9.8N, exactly what is needed to counteract the weight. (If the friction force would be bigger than 9.8N then the block would be moving upwards!)

Question 3.1.4: (6p) Now a motor start pushing the whole box with an acceleration of 4 m/s^2 in the positive X direction. Find the magnitude and direction of the normal force of the wall on the mass in this condition.

$$N_0 + F_s = ma \rightarrow N_0 = ma - F_s = 1 \text{ Kg} \cdot 4 \text{ m/s}^2 - 20 \text{ N} = 4 \text{ N} - 20 \text{ N} = -16 \text{ N}$$

Magnitude 16N, direction in the negative x axis.

Question 3.1.5: (6p) With the box accelerating, determine the maximum acceleration of the box for which the mass does not fall.

Block does not fall as long as the max friction force supports the weight $f_{rsMAX} - mg = 0$. With acceleration the max static friction force is $f_{rsMAX} = \mu_s |N_0| = \mu_s (F_s - ma)$. And so:

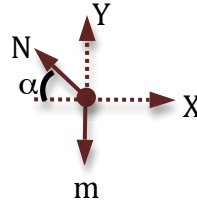
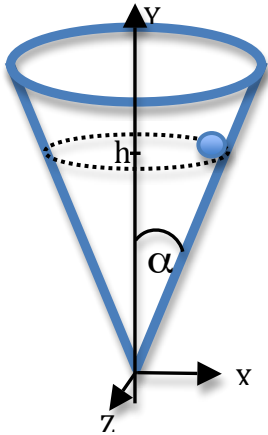
$$f_{rsMAX} - mg = 0$$

$$\mu_s (F_s - ma) - mg = 0 \rightarrow a = (F_s/m - g/\mu_s) = (20 - 12.25) \text{ m/s}^2 = 7.75 \text{ m/s}^2$$

Part 4: (30p) Forces and Energy Conservation

Problem 4.1: A ball of mass m is moving in a uniform circular motion inside a cone with angular aperture α at a height h . There is negligible friction between the ball and the cone. All the question must be answered in terms of the variables of the problem.

Question 4.1.1: (2p) Draw the free body diagram of the ball with the given axes.



Question 4.1.2: (5p) Find the acceleration of the ball in its movement around the circle. In general, what is the direction of the acceleration?

The ball is moving in a uniform circular motion, so the acceleration of the ball lies in the plane of rotation and points towards the axis of the cone. Notice that the radius of rotation is $R = h \tan(\alpha)$.

$$N \sin(\alpha) - mg = 0 \quad \Rightarrow \quad N = \frac{mg}{\sin(\alpha)}$$

$$-N \cos(\alpha) = ma_x \quad \Rightarrow \quad a_x = \frac{-N \cos(\alpha)}{m} = \frac{-mg}{m \tan(\alpha)} = \frac{-g}{\tan(\alpha)}$$

acceleration comes out negative (points in the negative x direction) for the position in which we draw the free-body diagram.

Question 4.1.3: (5p) Find the speed of the ball in its movement around the circle.

$$a = \frac{v^2}{R} = \frac{v^2}{h \tan(\alpha)} \Rightarrow v = \sqrt{ah \tan(\alpha)} = \sqrt{\frac{g}{\tan(\alpha)} h \tan(\alpha)} = \sqrt{gh}$$

Question 4.1.4: (6p) Find the time it takes the ball to complete a full circle.

$$v = \frac{2\pi R}{T} = \frac{2\pi h \tan(\alpha)}{T} \Rightarrow T = \frac{2\pi h \tan(\alpha)}{v} = \frac{2\pi h \tan(\alpha)}{\sqrt{hg}} = 2\pi \tan(\alpha) \sqrt{\frac{h}{g}}$$

Problem 4.2: Over a brief period of time the system loses energy. After the energy is lost the ball is now found rotating in circular motion at a height $h/4$.

Question 4.2.1: (7p) Find the energy lost.

$$E_h = \frac{mv_h^2}{2} + mgh = \frac{1}{2} mhg + mgh = \frac{3}{2} mgh$$

$$E_{h/4} = \frac{mv_{h/4}^2}{2} + mgh/4 = \frac{3}{2} mg \frac{h}{4} \quad \Rightarrow \quad \Delta E = E_{h/4} - E_h = \frac{3}{2} mgh \left(\frac{1}{4} - 1 \right) = -\frac{9}{8} mgh$$

Question 4.2.2: (5p) What is the period of rotation now compared to what it was when the ball was at a height of h ? Is it making a full revolution faster, or slower?

$$\frac{T_{h/4}}{T_h} = \frac{2\pi \tan(\alpha) \sqrt{\frac{h}{4g}}}{2\pi \tan(\alpha) \sqrt{\frac{h}{g}}} = \frac{1}{2} \Rightarrow T_{h/4} = \frac{T_h}{2}$$

That means that period of rotation at $h/4$ is smaller than that at h .

Therefore at $h/4$ it makes a full revolution faster.