Auto + ELN01

Jeudi 11/01/24

Intero N° 2

1] Soit $D=\{(x,y)\in\mathbb{R}^2 \,/\, y\geq 0,\ x-y+1\geq 0,\ x+2y-4\leq 0\}$

a) Tracer D (1 pt)

b) Calculer $\iint_D x\,dx\,dy$ (2 pt)

2] Soit $V=\{(x,y,z)\in\mathbb{R}^3 \,/\, x^2+y^2\leq 4,\ 0\leq z\leq 1\}$ (2 pt)

Calculer $I_2=\iiint_V \frac{z}{\sqrt{x^2+y^2}}\,dx\,dy\,dz$

3] Etudier la nature des intégrales impropres suivantes :

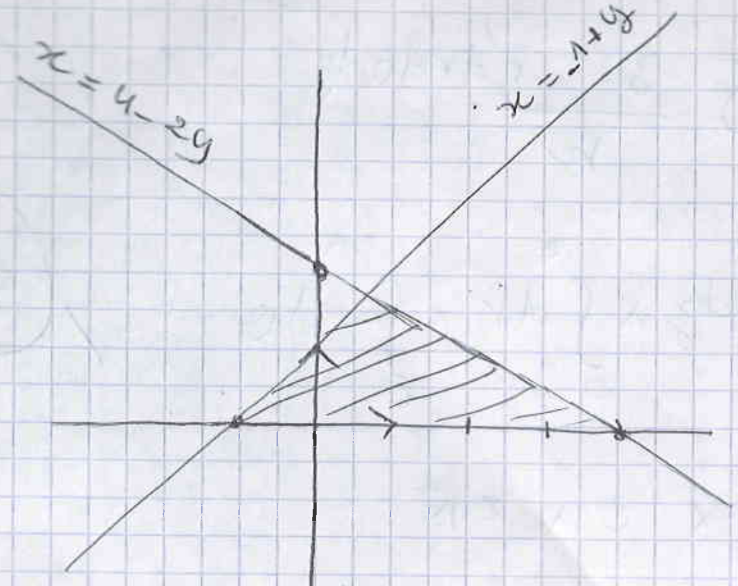
a) $\int_1^3 \frac{2x-3}{x^2-1}\,dx$ (1,5 pt)

b) $\int_0^1 \cos^2\left(\frac{1}{x}\right)dx$ (1,5 pt)

Bonne chance

Auto + EL N°1

Q1)



1pt

$$D = \{(x,y) \in \mathbb{R}^2 \ / \ 0 \leq y \leq \tfrac{5}{3} \ ; -1+y \leq x \leq 4-2y\}$$

$$I = \int_0^{\frac{5}{3}} \left[\int_{-1+y}^{4-2y} x \, dx \right] dy$$

$$= \int_0^{\frac{5}{3}} \left[\frac{1}{2} x^2 \right]_{-1+y}^{4-2y} dy$$

$$= \int_0^{\frac{5}{3}} (8 + 2y^2 - 8y) - \left(\frac{1}{2} + \frac{1}{2} y^2 - y\right) \Big] dy$$

$$= \int_0^{\frac{5}{3}} \left(\frac{15}{2} + \frac{3}{2} y^2 - 7y \right) dy$$

$$= \frac{15}{2} y + \frac{1}{2} y^3 - \frac{7}{2} y^2 \Big]_0^{\frac{5}{3}} = \frac{275}{54}$$

2 pt

2)

$x = r \cos\theta \qquad 0 \leq r \leq 2$

$y = r \sin\theta \qquad 0 \leq \theta \leq 2\pi$

$z = z \qquad 0 \leq z \leq 1$

$$|J| = r$$

1

$$I_2 = \iiint_{D_3} \frac{z \cdot \cancel{r}\, dr\, d\theta\, dz}{\cancel{r}}$$

$$= \int_0^1 z\, dz \times \int_0^2 dr \times \int_0^{2\pi} d\theta$$

2 pts

$$= \frac{1}{\cancel{4}} \times \cancel{2} \times 2\pi$$

$$= 2\pi$$

3/

$$\int_1^3 \frac{2x-3}{x^2-1}\, dx \quad (\text{PS} : 1)$$

1,5 pts

$$\frac{2x-3}{x^2-1} = \frac{2x-3}{x+1} \times \frac{1}{x-1} \sim_1 -\frac{1}{2} \frac{1}{x-1}$$

$$\int_1^3 -\frac{1}{2} \frac{1}{x-1}\, dx = -\frac{1}{2} \ln(x-1) \Big]_t^3$$

$$\lim_{t \to 1} = -\frac{1}{2} \ln 2 + \frac{1}{2} \ln(t-1)$$

$$= -\infty \quad (\text{Diverge})$$

Alors $\int_1^3 \frac{2x-3}{x^2-1}\, dx$ diverge

4/

$$\int_0^1 \cos^2\left(\frac{1}{x}\right) dx$$

1,5 pts

$$\cos^2\left(\frac{1}{x}\right) < 1$$

$$\int_0^1 1\, dx = 1 \quad (\text{converge})$$

Alors $\int_0^1 \cos^2\left(\frac{1}{x}\right) dx$ converge

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