Jeudi 11/01/24

Tel + ELN02

# Intero N° 2

1) Soit $D = \{(x,y) \in \mathbb{R}^2 / 1 \leq x \leq 3 ; y \geq 2 ;$
$x + y \leq 5\}$

1 a) Tracer D

2 b) Calculer $I_1 = \iint_D \frac{1}{(x+y)^3} \, dx\,dy$

2) Soit $V = \{(x,y,z) \in \mathbb{R}^3 / x \geq 0, y \geq 0, z \geq 0,$
et $x + y + z \leq 1\}$.

2 - Calculer $I_2 = \iiint_V x \, dx\,dy\,dz$.

3) Etudier la nature des intégrales impropres suivantes :

1,5 a) $\int_0^1 \frac{dx}{e^x - 1}$

1,5 b) $\int_1^{+\infty} \frac{\sqrt{x^2 + x + 1}}{x^3} \, dx$

Bonne chance

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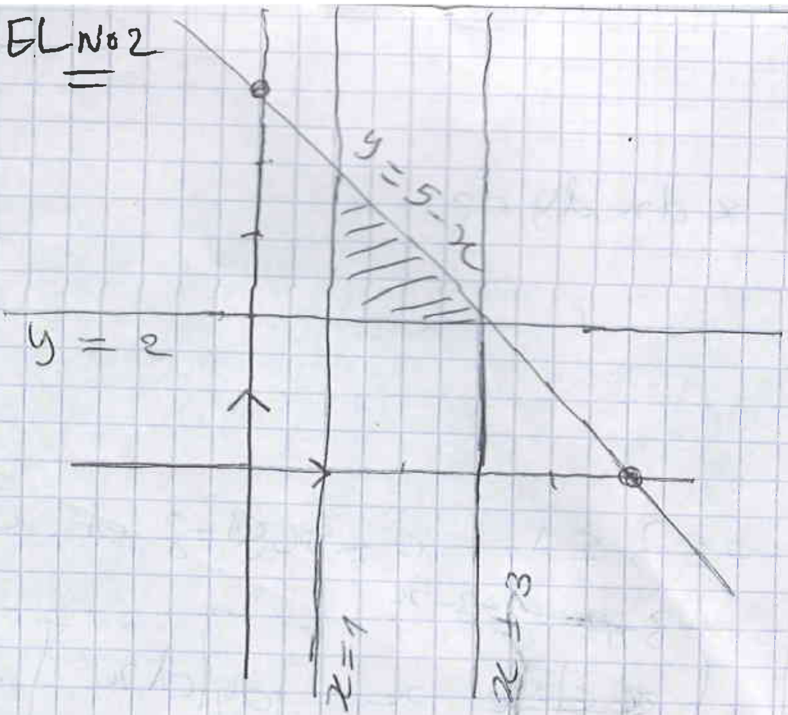
1)




1pt5

$$D = \{ (x, y) \in \mathbb{R}^2 \mid 1 \leq x \leq 3 \,;\, 2 \leq y \leq 5 - x \}.$$

$$I = \iint_D \frac{1}{(x+y)^3}\, dx\, dy$$

$$= \int_1^3 \left[ \int_2^{5-x} \frac{1}{(x+y)^3}\, dy \right] dx$$

$$= \int_1^3 \left[ \frac{-1}{2(x+y)^2} \right]_2^{5-x} dx$$

02pt5

$$= -\frac{1}{2} \int_1^3 \left( \frac{+1}{25} - \frac{1}{(x+2)^2} \right] dx$$

$$= -\frac{1}{2} \left[ \frac{1}{25} x + \frac{1}{x+2} \right]_1^3$$

$$= -\frac{1}{2} \left[ \frac{3}{25} + \frac{1}{5} - \frac{1}{25} - \frac{1}{3} \right] = -\frac{1}{2} \left[ \frac{2}{25} - \frac{2}{15} \right] = \frac{2}{75}$$

2)

$$I_2 = \iiint_V x \, dx \, dy \, dz$$

$$V = \{ 0 \leq z \leq 1 \; ; \; 0 \leq x \leq 1-z \text{ et } 0 \leq y \leq 1-z-x \}$$

$$I_2 = \int_0^1 \left[ \int_0^{1-z} \left[ \int_0^{1-z-x} x \right] dy \right] dx \, \Big] dz$$

2pts

$$= \int_0^1 \left[ \int_0^{1-z} \left[ x y \right]_0^{1-z-x} dx \right] dz$$

$$= \int_0^1 \left[ \int_0^{1-z} (x - zx - x^2) \, dx \right] dz$$

$$= \int_0^1 \left[ \frac{1}{2} x^2 - \frac{z}{2} x^2 - \frac{1}{3} x^3 \right]_0^{1-z} dz$$

$$= \int_0^1 \frac{1}{2}(1-z)^2 - \frac{z}{2}(1-z)^2 - \frac{1}{3}(1-z)^3 \, dz$$

$$= \frac{1}{24}$$

3)

$$\int_0^1 \frac{1}{e^x - 1}\,dx \qquad (PS = 0)$$

$$e^x - 1 \underset{0}{\sim} x$$

$$\frac{1}{e^x - 1} \sim \frac{1}{x}$$

1,5 pt

$$\int_0^1 \frac{1}{x}\,dx = \lim_{t \to 0} \int_t^1 \frac{1}{x}\,dx$$

$$= \lim_{t \to 0} \ln t$$

$$= -\infty \quad (\text{Diverge})$$

Alors $\int_0^1 \frac{1}{e^x - 1}\,dx$ Diverge.

b) $$\frac{\sqrt{x^2+x+1}}{x^3} \underset{+\infty}{\sim} \frac{x}{x^3} \sim \frac{1}{x^2}$$

1,5 pt

$$\int_1^{+\infty} \frac{1}{x^2}\,dx \quad CV \quad (\text{Riemann } \alpha = 2)$$

Alors $\int_1^{+\infty} \frac{\sqrt{x^2+x+1}}{x^3}\,dx$ CV

$\frac{2}{15}$