Corrigé Série n°2

Suite

Exo 2 :

1/ (STP)

d/ Écart type corrigé de la population

Selon l'équation ANOVA ⇒

$$S_Y^2 = \frac{1}{N}\sum_{h=1}^{4} N_h S_h^2 + \frac{1}{N}\sum_{h=1}^{4} N_h\left(\hat{\bar{Y}}_{st} - \bar{Y}_h\right)^2$$

$$S_Y^2 = \frac{1}{770}\left[310\,(9,5)^2 + 220\,(6,1)^2 + 130\,(3,5)^2 + 110\,(2,1)^2\right] + \frac{1}{770}\left[2482,719 + 1035,958 + 6683,157 + 2566,129\right]$$

$$S_Y^2 = \frac{1}{770}\left(3824,3\right) + \frac{1}{770}\left(12768,053\right)$$

$$S_Y^2 = 49,664 + 16,581 = 66,245$$

$$\boxed{S_Y^2 = 66,245} \Rightarrow S_Y = \sqrt{66,245} = 8,139$$

Écart type de la population

1

3/ on considère un plan de sondage aléatoire Stratifié de type optimale (STO)

a/ Les tailles des échantillons pour chacune des Strates :

$$n = 77 \Rightarrow n_h = \frac{N_h S_h}{\sum_{h=1}^{4} N_h S_h} \cdot n \quad \text{pour } h = 1, 2, 3, 4$$

Donc

$$\begin{cases} n_1 = \dfrac{(310)(9,5)}{4974,007} \cdot (77) = 45,5 \\[2ex] n_2 = \dfrac{(220)(6,1)}{4974,007} \cdot (77) = 20,779 \\[2ex] n_3 = \dfrac{(130)(3,5)}{4974,007} \cdot (77) = 7,045 \end{cases}$$

$$n_4 = \frac{(110)(2,1)}{4974,007} \cdot (77) = 3,576$$

Comme $\sum_{h=1}^{4} n_h > 77 \Rightarrow$ donc on va ajuster les tailles des échantillon de sorte que $\sum_{h=1}^{4} n_h = 77$

Alors
$$\begin{cases} n_1 = 46 \\ n_2 = 21 \\ n_3 = 7 \\ n_4 = 3 \end{cases}$$

b/ $\hat{\bar{Y}}_{st} = \sum_{h=1}^{4} \frac{N_h}{N} \bar{y}_h = \underline{7,83}$

c/ $V(\hat{\bar{Y}}_{st})_{op} = \sum_{h=1}^{4} \frac{N_h^2}{N^2} \frac{S_h^2}{n_h}$ $\left(\begin{array}{l} n \geq 30 \\ \Rightarrow TAR \simeq TSR \end{array} \right)$

$$V(\hat{\bar{Y}}_{st})_{op} = \frac{1}{770^2} \left[310^2 \left(\frac{9,5}{46} \right)^2 + 220 \left(\frac{6,1^2}{21} \right) + 130^2 \left(\frac{3,5^2}{7} \right) + 110 \cdot \left(\frac{2,1^2}{3} \right) \right]$$

3]

$$V(\hat{\bar{y}}_{st})_{op} = \frac{1}{592900}\left[96100(1,961) + 48400(1,771) + 16900(1,75) + 12100(1,45)\right]$$

$$= \frac{1}{592900}(321530,5)$$

$$\boxed{V(\hat{\bar{y}}_{st})_{op} = 0,542}$$

5/ $\begin{cases} V(\hat{\bar{y}}_{st})_{prop.} = 0,644 \\ V(\hat{\bar{y}}_{st})_{opt} = 0,542 \end{cases}$

Conclusion

$$V(\hat{\bar{y}}_{st})_{prop.} > V(\hat{\bar{y}}_{st})_{opt}.$$

4/