Democratic and Popular Republic of Algeria

Ministry of Higher Education

and Scientific Research

Abderrahmane Mira University Bejaia

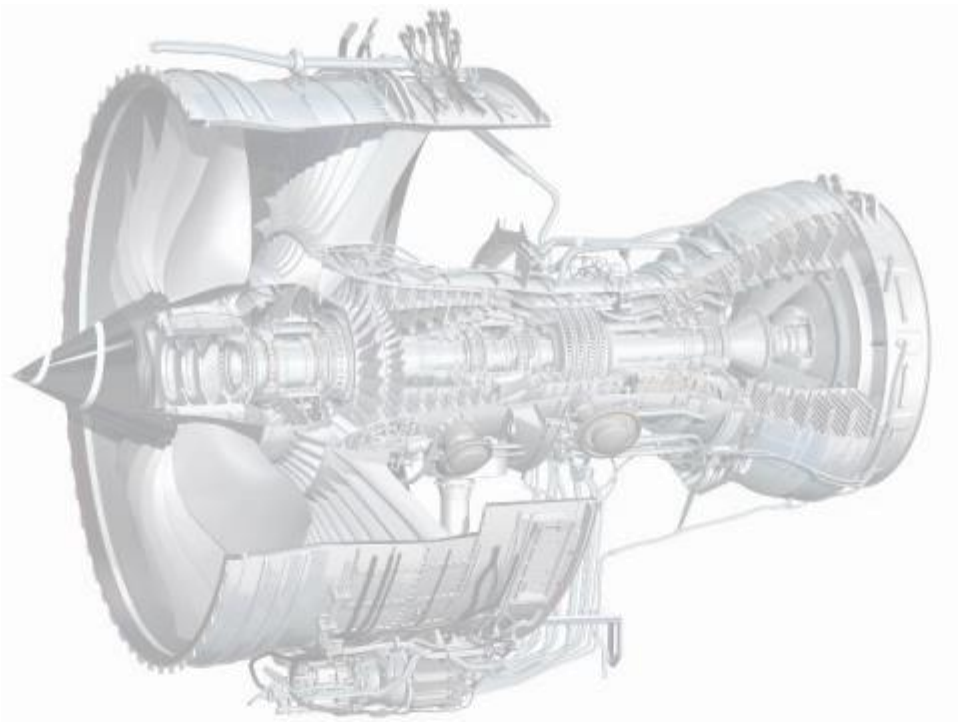
Faculty of Technology

Department of Mechanical Engineering

TURBOMACHINES 1

COURSES AND EXERCISES

For students enrolled in the third year of an Academic License (LMD) in Mechanical Engineering, specializing in Energetics.



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جامعة بجاية
Tasdawit n Bgayet
Université de Béjaïa

Module Presentation

1. Course Description :

Semestre : 5

Unit of Teaching : UTF 3.1.2

SHV : 45h00 (Course : 01h30, Tutorials :1h30)

Credits : 4

Coefficient : 2

This course, complemented by practical exercises, serves as an indispensable educational resource aimed at facilitating the acquisition of key concepts in the "Turbomachines 1" module. It is specially designed for students enrolled in the third year of an Academic License (LMD) in Mechanical Engineering, specializing in Energetics. The objective of this learning program is to provide students with the fundamental knowledge needed to understand the operation of turbomachines, a crucial area of mechanical engineering.

The practical exercises integrated into this course offer students the opportunity to apply the theoretical knowledge they have acquired, thereby strengthening their understanding of the fundamental principles of turbomachines. Through this approach, they will develop essential skills that will help them succeed in their academic journey and apply this knowledge in practical contexts.

2. Teaching objectives :

- Apply the principles of fluid mechanics to analyze and solve complex problems within the context of technical systems specific to generative and expansion turbomachines.
- Master the skills required for designing pumps, understanding parameters such as flow rate, pressure, power, and selecting the appropriate components for specific applications.
- Acquire in-depth knowledge of potential causes of pump failure, primarily focusing on understanding cavitation phenomena.
- Be capable of conducting advanced calculations to evaluate the performance of pumps and hydraulic turbines, considering factors such as efficiency and pressure losses.

- Know how to select and install different types of hydraulic turbines based on specific project requirements, understanding the advantages and disadvantages of both impulse and reaction turbines.

3. Recommended prior knowledge :

- Fluid Mechanics 1
- Thermodynamics

4. Evaluation mode :

Continuous Assessment: 40%, Examination: 60%.

5. Contents of the handout :

Nomenclature

Symbols	Definition	SI Unit
D	: Turbomachine wheel diameter	m
r	: Turbomachine wheel radius	m
S	: Flow cross-section	m
t	: time	s
C	: Flow velocity	$m.s^{-1}$
C_r	: Radial flow velocity	$m.s^{-1}$
C_a	: Axial flow velocity	$m.s^{-1}$
C_U	: Peripheral flow velocity	$m.s^{-1}$
U	: Peripheral wheel velocity	$m.s^{-1}$
W	: Tangential blade velocity	$m.s^{-1}$
ω	: Angular velocity.	$rad.\ s^{-1}$
M	: Machine torque	N.m
N	: Rotation speed	s^{-1}
N_s	: Specific speed	-
b	: Wheel thickness, Wheel width	m
P_a	: Absolute power, Absorbed power (power absorbed by the pump), Shaft power (motor shaft power)	W
P_f	: Fluid power	W
p	: Pressure	Pa
q_V	: Volumetric flow rate	$m^3.s^{-1}$
q_m	: Mass flow rate	$kg.\ s^{-1}$
H	: Height	m
H_{thZ}	: Height at number of blades Z	m
$H_{th\infty}$	: Height at infinite number of blades	m
g	: Gravitational acceleration	$m\ s^{-2}$
R	: Degree of reaction	-
F	: Resultant of external forces	N
F_a	: Axial thrust	N
F_i	: Inertial force	N
F_p	: Pressure force	N

F_g	: Gravity force	N
F_γ	: Surface tension force	N
τ	: Specific work	J.kg
ρ	: Density	$kg.m^{-3}$
ν	: Kinematic viscosity	$m^2.s^{-1}$
γ	: Surface tension	$J.m^{-2}$
ε	: Utilization factor	-
μ	: Slip Coefficient	-
α	: Flow inclination	°
β	: Blade inclination	°
η_H	: Hydraulic efficiency, manometric efficiency	-
η_V	: Volumetric efficiency	-
η_m	: Mechanical efficiency	-
η_g	: Overall efficiency	-

Indices

1	: Inlet
2	: Outlet
m	: Meridional flow direction
th	: Theoretical
thZ	: Theoretical at number of blades Z
th∞	: Theoretical at infinite number of blades

Chapter I

Definitions and General Theory of Turbomachines

Table of contents

1. Definitions

a. Turbomachines

Turbomachines are devices designed to facilitate the transfer of mechanical energy between a continuously flowing fluid and a rotor rotating around a central axis at a constant speed. These rotors, also known as wheels or impellers, are equipped with blades (in the case of pumps and compressors), vanes (in gas or steam turbines), or buckets (in the case of Pelton hydraulic turbines).

b. Classifications des turbomachines

There is a diversity of turbomachines, and to conduct systematic studies on them, it is essential to develop a specific terminology for turbomachines. It is also necessary to become acquainted with their specific components as a preliminary step. Among the various classifications for turbomachines, some notable ones include: :

i. Classification of turbomachines based on the direction of energy exchange

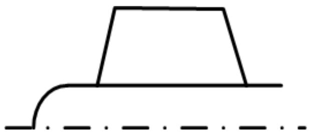
When considering the direction of energy flow in machines, an input of energy into the fluid results in an increase in its pressure, and conversely, an extraction of energy is associated with a decrease in pressure. This is commonly referred to as :

- Generating or compression turbomachines (Pumps and compressors)
- Receiving or expansion turbomachines (Turbines)

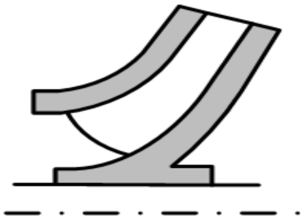
> <u>Note:</u> Since it consumes or collects mechanical energy on its shaft, it must necessarily be coupled with another machine acting as a motor in the first case (electric motor, Diesel engine, receiving turbomachine) or as a driven machine in the second case (generator, alternator, generating turbomachine).

ii. Classification of turbomachines based on the flow path

The shape of the fluid path in the rotor of a turbomachine also provides a basis for classifying the types of turbomachines.

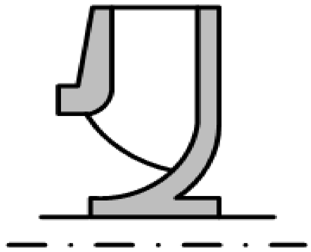


Axial Turbomachines



Semi-Axial Turbomachines

They are also referred to as helico-centrifugal or helical machines.



Radial Turbomachines

Centrifugal machines are characterized by flow moving away from the axis, whereas centripetal machines exhibit flow moving toward the axis..

iii. Classification of turbomachines based on the nature of the fluid

Depending on the liquid or gaseous nature of the conveyed fluid and its compressibility, turbomachines can be classified into two categories :

- Compressible fluid turbomachine
- Incompressible fluid turbomachine

Note: When categorizing various types of turbomachines, **marine and aerial propellers**, primarily used for energy generation, occupy a distinctive position as they operate by taking in and discharging the fluid within the same unlimited medium.

iv. Classification of turbomachines based on the number of stages

Depending on whether a turbomachine has one or multiple rotors, it is referred to as single-stage or multi-stage.

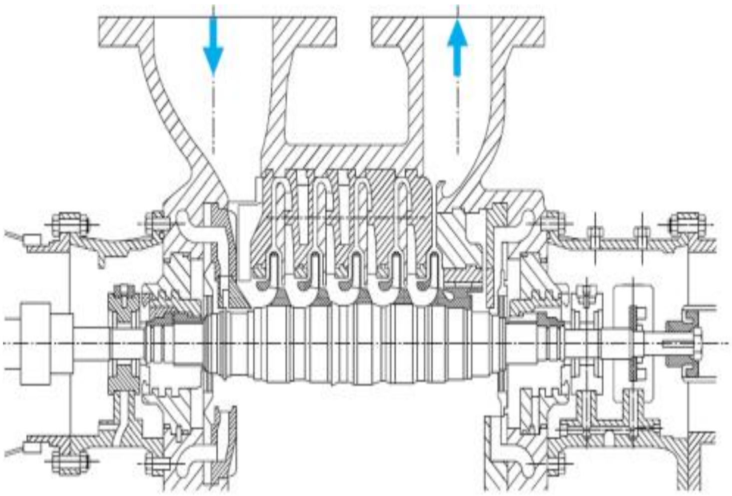


Figure 1. Example of a multi-stage turbomachine

2. General Theory of Turbomachines

The most critical elements derived from the design process of a turbomachine are found in the detailed characteristics of the rotor blades or vanes. This encompasses their profiles, the necessary angles at the inlet and outlet, as well as the radius at the entry and exit point.

a. Flow within a turbomachine rotor

The Euler equation is the fundamental equation on which the study of turbomachines is based. Therefore, the constancy of fluid states separately at the inlet and outlet sections, mass conservation, and the exclusion of acceleration at the start and deceleration at the stop are assumed when applying this equation.

In accordance with Newton's second law :

$$F = m\,a = m\dot{C} = m\frac{dC}{dt} = \frac{d(mC)}{dt} \qquad (I.1)$$

Recall of Newton's second law: "In the case of a closed system, the resultant force **F** exerted on a material point, with a given mass m, is equal to the product of the point's mass and its acceleration **a**."

This equation is known as the linear momentum equation, which can also be written as:

$$Fdt = d(mC) \qquad (I.2)$$

The latter is known as the impulse equation (Fdt is the impulse of force F). Following what was previously mentioned, the equation applicable to turbomachines is as follows:

$$F \cdot r = r \cdot m\frac{d(C)}{dt} \qquad (I.3)$$

Here, r is the radial distance of the force., $(F \cdot r)$ is the moment of the force and is referred to as the machine's torque. $(r \cdot m(dC/dt))$ or $(r \cdot (mdC/dt))$ is the rate of change of angular momentum. In other words, the equation above means that "the applied torque is equal to the rate of change of angular momentum.".

By considering (r) as the generalized radius vector $(\bar{r})$ and (C) as the generalized velocity vector, each having components in the axial, radial, and tangential directions, we obtain the equation of angular momentum :

$$M = \frac{d\left(m\bar{r} \times \bar{C}\right)}{dt} \tag{I.4}$$

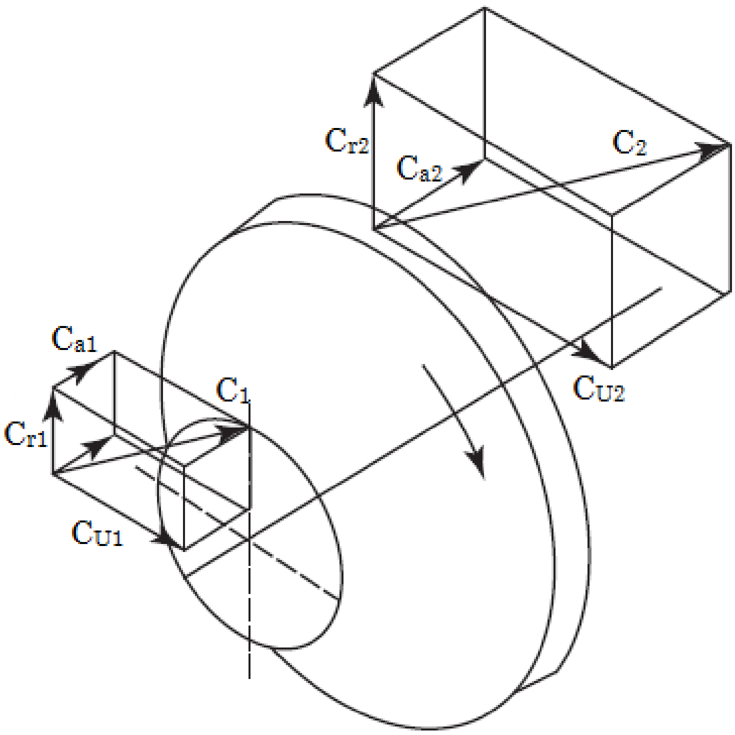


Figure 2. Velocities in a rotor

The torque and power developed in a turbomachine are solely due to the tangential components, while the axial thrust developed is due to the axial components.

Figure 2 depicts a generalized rotor of a turbomachine with a typical fluid flow, represented by the absolute fluid velocities : C_1 t the inlet and C_2 at the outlet. It also illustrates all the components of fluid velocities : C_{U1}, C_{U2}, C_{a1}, C_{a2}, C_{r1} et C_{r2}.

(C_U) is the component of the absolute fluid velocity inclined at an angle (*a*) with respect to the wheel's periphery (U), it's also the "whirl" component that contributes to the torque (M). (C_a) is the component of the absolute fluid velocity in the axial direction ; this causes axial thrust. (C_r) is the radial component of the absolute fluid velocity.

Important : For a purely axial flow machine, the radial components C_{r1} and C_{r2} are reduced to zero, making the axial components C_{a1} and , C_{a2} the flow components. Similarly, for a purely radial flow machine, C_{r1} and C_{r1} become the flow components, and in this case, C_{a1} and , C_{a2} are reduced to zero.

Equation (I.4) can be written in a general relation between the inlet and outlet sections :

$$M = q_m \left(\bar{r} \times \bar{C}\right) \tag{I.5}$$

$$M = q_m \left(r_2 C_2 \cos\alpha_2 - r_1 C_1 \cos\alpha_1 \right) = \rho \cdot q_v \left(r_2 C_2 \cos\alpha_2 - r_1 C_1 \cos\alpha_1 \right) = \rho \cdot q_v \left(r_2 C_{U2} - r_1 C_{U1} \right) \tag{I.6}$$

The power supplied to the liquid by the pump impeller is given by the formulas :

$$P = M\omega = \rho q_v \left(C_{u_2} r_2 \omega - C_{u_1} r_1 \omega \right) \tag{I.7}$$

where (ω) is the angular velocity.

$$P = \rho q_v \left(U_2 C_{u_2} - U_1 C_{u_1} \right) \tag{I.8}$$

$$P = \rho q_v \left(U_2 C_2 \cos\alpha_2 - U_1 C_1 \cos\alpha_1 \right) \tag{I.9}$$

This equation is known as the "general form of the Euler equation" and can be adjusted according to the nature of the turbomachine (expansion "turbine," compression "pump/compressor"):

- Euler's turbine equation is as follows:

$$P = \rho q_v \left(U_2 C_{u_2} - U_1 C_{u_1} \right) < 0$$
$$P = \rho q_v \left(U_1 C_{u_1} - U_2 C_{u_2} \right) > 0 \tag{I.10}$$

- Euler's pump equation is as follows::

$$P = \rho q_v \left(U_2 C_{u_2} - U_1 C_{u_1} \right) > 0 \tag{I.11}$$

It can also be deduced from the equation(I.9), the specific work (τ) :

$$\tau = \frac{P}{q_m} = \frac{P}{\rho q_v} = U_2 C_2 \cos\alpha_2 - U_1 C_1 \cos\alpha_1 \tag{I.12}$$

Just as the tangential components of fluid velocities generate tangential force (and subsequently torque, as mentioned above), the axial components of fluid velocities generate axial thrust to be supported by the bearings. Referring to the thrust as (F_a), we have :

$$F_a = \frac{d}{dt}\left(m \cdot \overline{C_a} \right) = q_m \left(C_{a_1} - C_{a_2} \right) \tag{I.13}$$

When the obtained thrust $\mathbf{F_a}$ is positive, this thrust on the bearings is in the direction of the inlet flow (whether the machine is a pump or a turbine), and a negative value of $\mathbf{F_a}$ represents thrust in the opposite direction of the incoming flow.

b. Alternative Form to Euler's Equation

Another form of Euler's equation in terms of six velocities (three from each velocity triangle) can be derived. Let's consider a general velocity triangle without "1" and "2" indices corresponding to the inlet and outlet of a turbomachine. (see Figure 3).

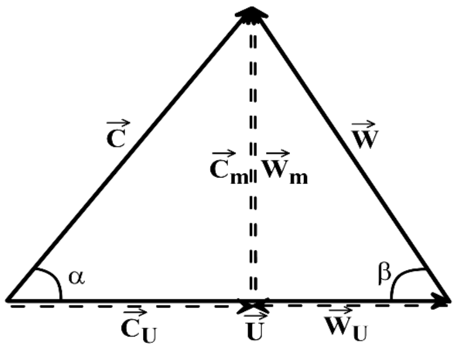


Figure 3. General Velocity Triangle

Note: The meridional component is radial for a centrifugal turbomachine and axial for an axial turbomachine.

Thus, according to this reasoning:

$$\begin{cases} \vec{C} = \vec{U} + \vec{W} \\ \vec{W} = \vec{C} - \vec{U} \\ W^2 = \left(\vec{W}\right)^2 = C^2 + U^2 - 2\vec{C}\cdot\vec{U} \\ W^2 = C^2 + U^2 - 2CU\cos\alpha \\ 2CU\cos\alpha = C^2 + U^2 - W^2 \\ CU\cos\alpha = \frac{1}{2}\left(C^2 + U^2 - W^2\right) \end{cases} \longrightarrow \begin{cases} C_1U_1\cos\alpha_1 = \frac{1}{2}\left(C_1^2 + U_1^2 - W_1^2\right) \\ C_2U_2\cos\alpha_2 = \frac{1}{2}\left(C_2^2 + U_2^2 - W_2^2\right) \end{cases} \tag{I.14}$$

Therefore, the second form of the Euler expression can be written as:

$$P = \rho q_v\left(\frac{C_2^2 - C_1^2}{2} + \frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2}\right)$$
$$\tau = \frac{C_2^2 - C_1^2}{2} + \frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2} \tag{I.15}$$

3. Universal Parameters of Turbomachines

a. Degree of Reaction R

In expression (I.15) for the specific work (τ), the first term in the expression $\left(\frac{C_2^2 - C_1^2}{2}\right)$ represents the change in kinetic energy of the fluid between the inlet and outlet of the rotor, and this component is transferred directly between the fluid and the rotor, without requiring the presence of nozzles or diffusers. The remaining terms $\left(\frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2}\right)$ are the components that require transformation within the rotor itself, simultaneously with the energy transfer process. These components are referred to as "reaction components."

Hence, the ratio of the components of energy transferred due to the change in fluid pressure between the inlet and outlet of the rotor to the total energy transferred is called the degree of reaction, denoted as R.

$$R = \frac{\frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2}}{\tau} = \frac{\left(U_2^2 - U_1^2\right) + \left(W_1^2 - W_2^2\right)}{\left(C_2^2 - C_1^2\right) + \left(U_2^2 - U_1^2\right) + \left(W_1^2 - W_2^2\right)} \tag{I.16}$$

b. Turbine Utilization Factor

The utilization factor of a turbine is defined as the ratio of the ideal work of the turbine to the available energy for conversion into work at the fluid's inlet flow:

$$\varepsilon = \frac{\tau}{\tau_a} \tag{I.17}$$

(τ) is the specific work defined by the Euler equation associated with turbines. The available energy in the fluid at the inlet, (τ_a), consists of two parts : its kinetic energy, $\left(C_1^2/2\right)$, and the energy that can be obtained due to the pressure drop (referred to as the reaction component).

$$\begin{cases} \tau = \dfrac{C_1^2 - C_2^2}{2} + \dfrac{U_1^2 - U_2^2}{2} + \dfrac{W_2^2 - W_1^2}{2} \\ \tau_a = \dfrac{C_1^2}{2} + \dfrac{U_1^2 - U_2^2}{2} + \dfrac{W_2^2 - W_1^2}{2} \end{cases} \tag{I.18}$$

Thus,

$$\varepsilon = \frac{\left(C_1^2 - C_2^2\right) + \left(U_1^2 - U_2^2\right) + \left(W_2^2 - W_1^2\right)}{C_1^2 + \left(U_1^2 - U_2^2\right) + \left(W_2^2 - W_1^2\right)} \tag{I.19}$$

The previous expression can also be written by introducing the degree of reaction :

$$R = \frac{\left(U_2^2 - U_1^2\right) + \left(W_1^2 - W_2^2\right)}{\left(C_2^2 - C_1^2\right) + \left(U_2^2 - U_1^2\right) + \left(W_1^2 - W_2^2\right)}$$

$$\Rightarrow \frac{1}{R} = 1 + \frac{\left(C_2^2 - C_1^2\right)}{\left(U_2^2 - U_1^2\right) + \left(W_1^2 - W_2^2\right)}$$

$$\Rightarrow \frac{1}{R} - 1 = \frac{1-R}{R} = \frac{\left(C_2^2 - C_1^2\right)}{\left(U_2^2 - U_1^2\right) + \left(W_1^2 - W_2^2\right)}$$

$$\Rightarrow \frac{R}{1-R} = \frac{\left(U_2^2 - U_1^2\right) + \left(W_1^2 - W_2^2\right)}{\left(C_2^2 - C_1^2\right)}$$

$$\begin{aligned} \left(U_2^2 - U_1^2\right) + \left(W_1^2 - W_2^2\right) &= \frac{R\left(C_2^2 - C_1^2\right)}{1-R} \\ \left(U_1^2 - U_2^2\right) + \left(W_2^2 - W_1^2\right) &= \frac{R\left(C_1^2 - C_2^2\right)}{1-R} \end{aligned} \tag{I.20}$$

By substituting (I.20) into (I.19), we obtain :

$$\varepsilon = \frac{C_1^2 - C_2^2}{C_1^2 - RC_2^2} \tag{I.21}$$

c. Component of transferred energy

The energy transfer process between a fluid and a rotor, as expressed by the specific work in the alternative form of the Euler equation (I.15), can also be formulated differently using the first law of thermodynamics. This principle is manifested as follows for a generating turbomachine :

$$\tau = \left(h_2 - h_1\right) + \frac{C_2^2 - C_1^2}{2} + \left(z_2 - z_1\right)g \tag{I.22}$$

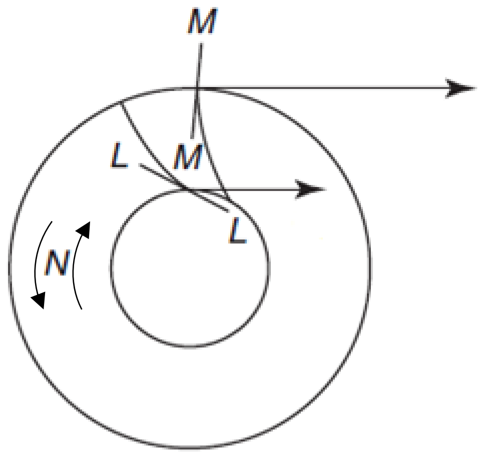
A common component in equations (I.15)and (I.22) is the kinetic component of energy transfer $\left(\left(C_2^2 - C_1^2\right)/2\right)$. The other components are as follows:

- The elevation component, $\left(z_2 - z_1\right)g$.
- The enthalpy drop component, $\left(h_2 - h_1\right)$.
- The centrifugal energy component, $\left(\left(U_2^2 - U_1^2\right)/2\right)$.
- The relative velocity component, $\left(\left(W_1^2 - W_2^2\right)/2\right)$.

The kinetic component is considered the primary component, and all the other components can be seen as another combined component. The kinetic component can interact directly between the fluid and the rotor, while the other components, when appropriately grouped, may have a somewhat indirect interaction.

4. Exercises

- **Exercise n°1 :** Centrifugal/Centripetal Turbomachines



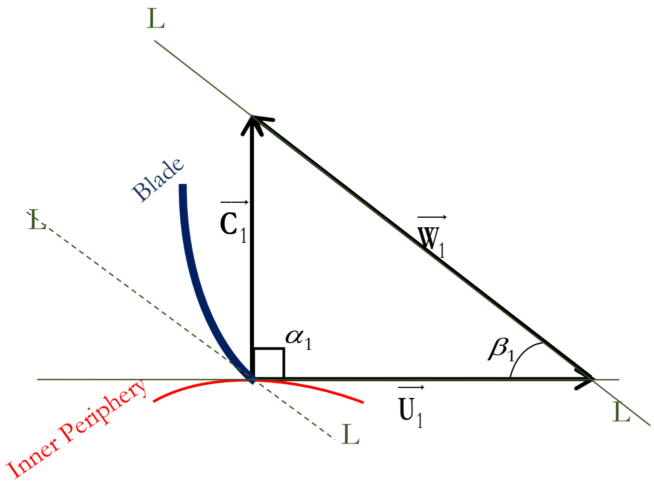
In the context of the rotor illustrated in the figure above, it is worth noting that LL and MM refer to the tangents along the leading and trailing edges of the blades, respectively.

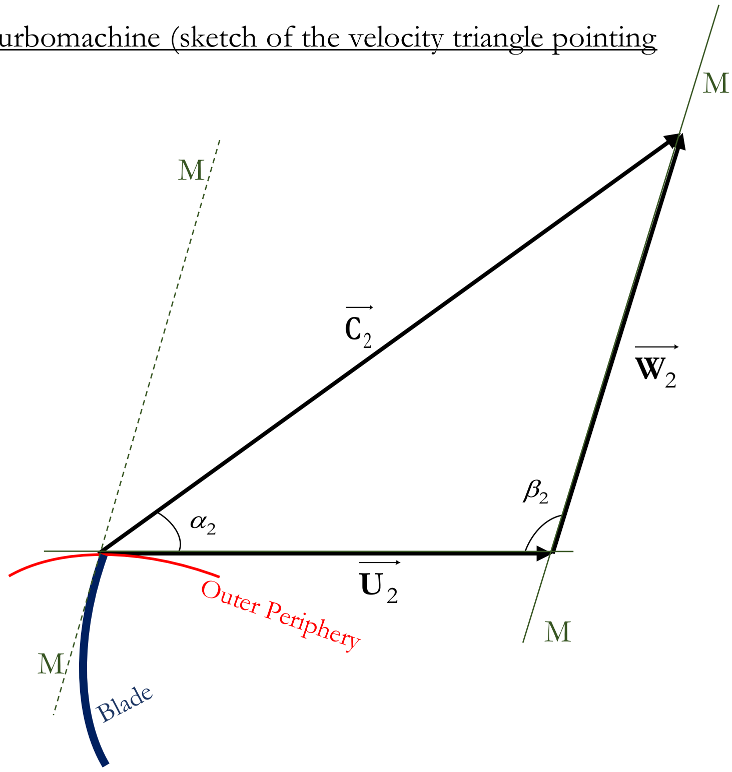
Draw the velocity triangles for the following cases :

- -Case 1 : Fluid inlet is radial, and the flow is outward. The blades are forward-curved at the outlet.
- Case 2 : Fluid inlet is radial, and the flow is outward. The blades are forward-curved and radial at the outlet.
- Case 3: Fluid inlet is inclined, and the flow is directed inward. The blades are inclined at the outer diameter, and the flow is radial at the inner diameter (the blades are backward-curved).
- Case 4: Fluid inlet is inclined, and the flow is directed inward. The blades are inclined at a 90° angle at the inlet with respect to the periphery, and the flow is radial at the inner diameter (the blades are backward-curved)

Solution :

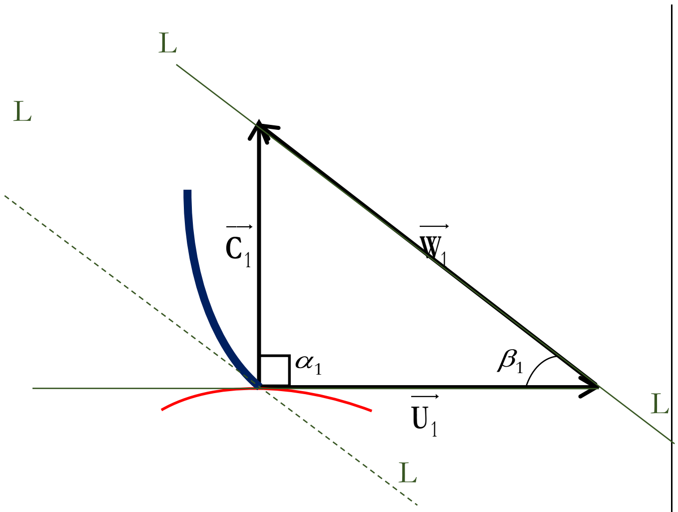
- Case 1 : Outward Flow → Centrifugal Turbomachine (sketch of the velocity triangle pointing upward)

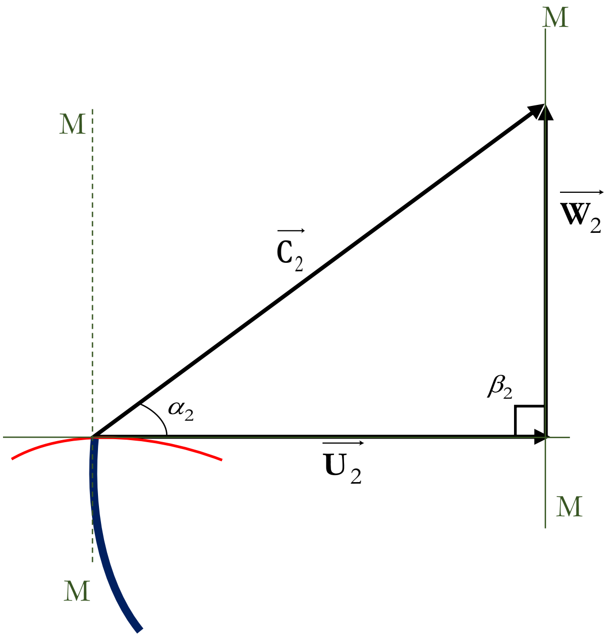




- Radial Inlet: The flow characterized by velocity C_1 is in the radial direction
- Blades are forward-curved: Based on the shape of the blade associated with the direction of the rotor's rotation (see the direction of $\mathbf{U_2}$)

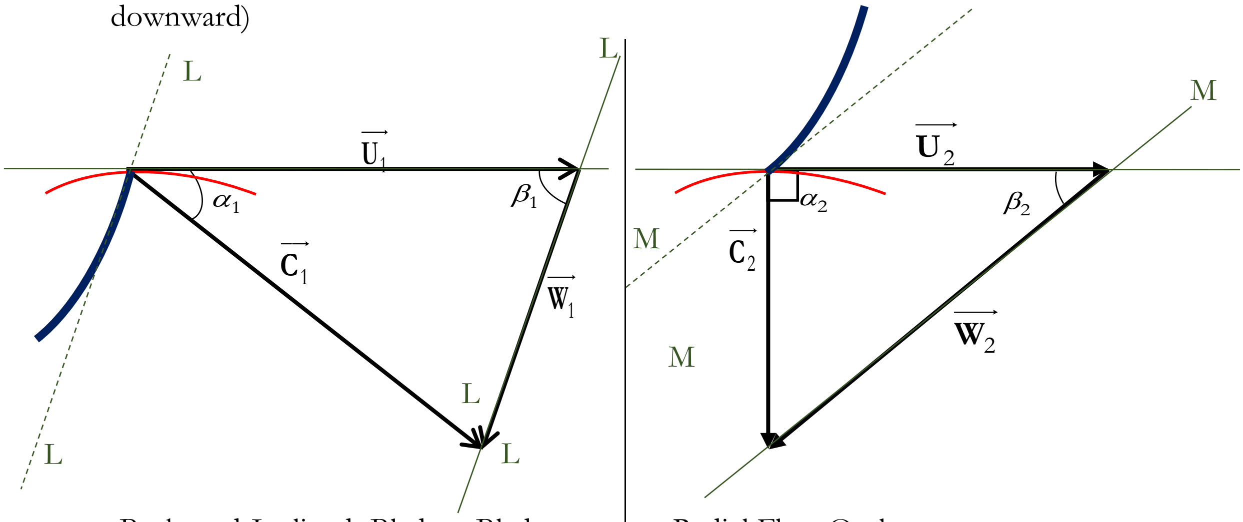
- Case 2 : Outward Flow → Centrifugal Turbomachine : Blades are forward-curved and radial at the outlet.





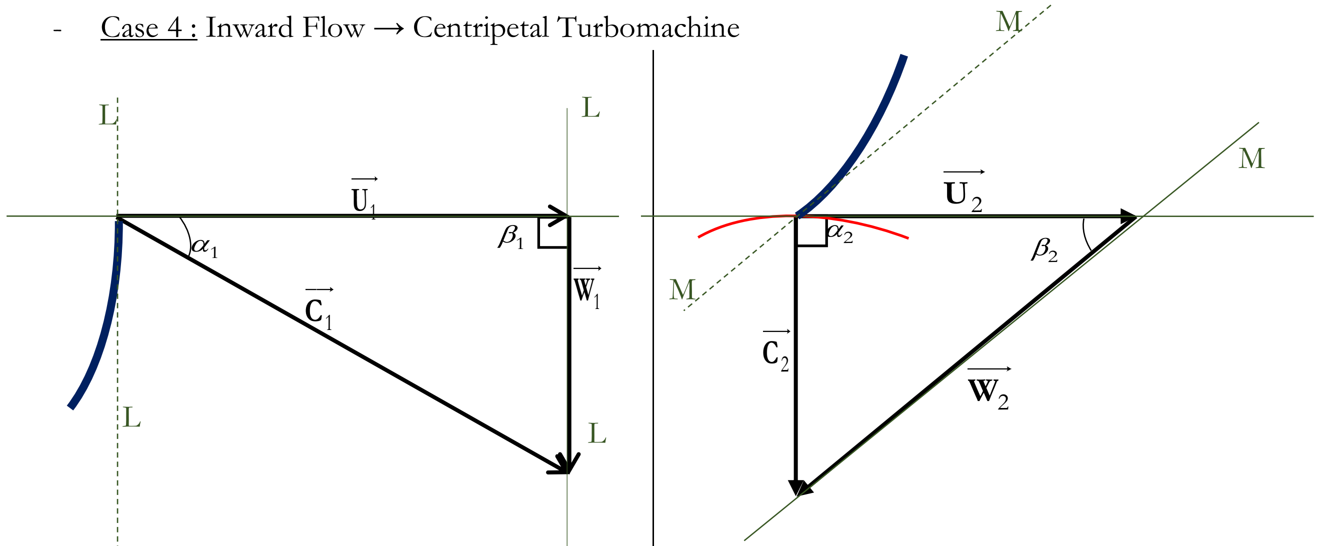
- Radial Flow Inlet
- Radial Blades: Blades are perpendicular to the periphery.

- <u>Case 3 :</u> Inward Flow → Centripetal Turbomachine (sketch of the velocity triangle pointing downward)



- Backward-Inclined Blades: Blades are inclined backward from the intrados to the extrados (according to the direction of peripheral velocity U).
- Radial Flow Outlet

- <u>Case 4 :</u> Inward Flow → Centripetal Turbomachine



- Radial Blades at Inlet
- Radial Flow at Inner Diameter

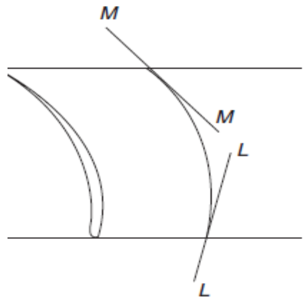
- **Exercise n°2 :** Axial Turbomachines

For the cases below:

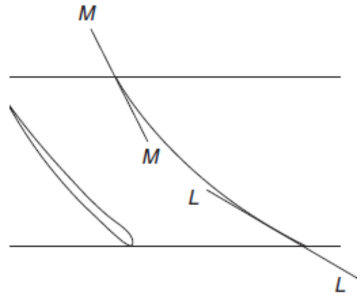
- - Indicate the flow direction, whether it's a turbine or a compressor, and the direction of rotation.
- - Draw the velocity triangles

Note: MM at the outlet and LL at the inlet.

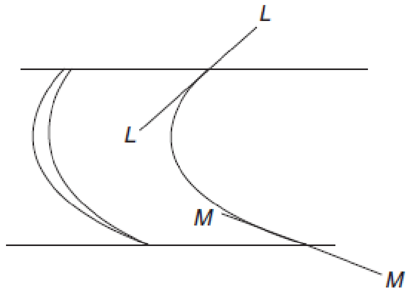
Case 1 :



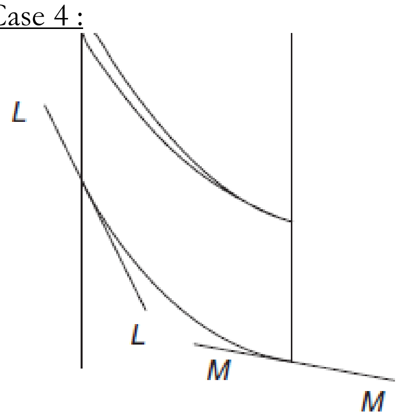
Case 2 :



Case 3 :



Case 4 :

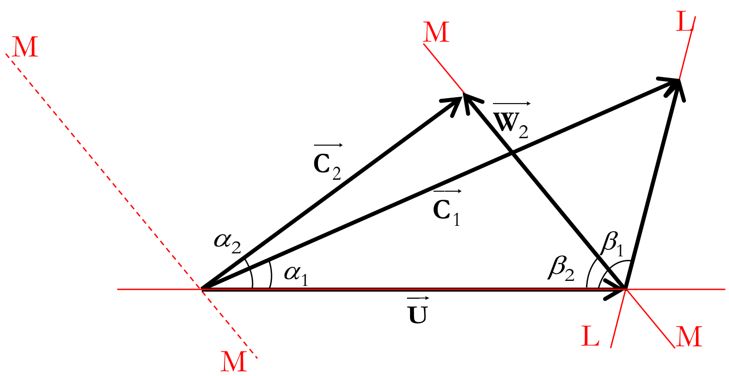


Solution :

Based on the sketches provided in the question, where the direction of the blades is illustrated by the tangents (MM) and (LL), we can determine the following :

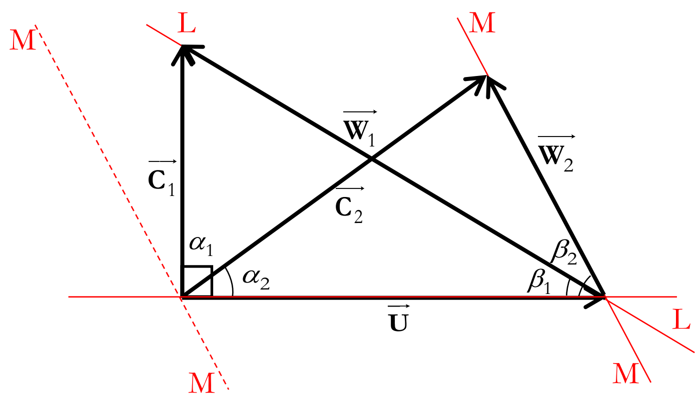
Case 1 :

- Based on the designations of the tangents at the leading edges (LL → Inlet; MM → Outlet) →Flow is directed upwards.
- The turbomachine is a Turbine since the blades are profiled.
- Turbines receive on the intrados, so the rotation is clockwise (to the right).



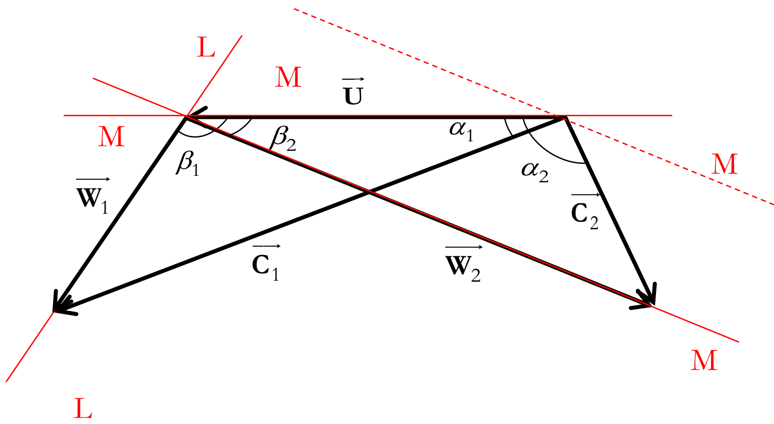
Case 2 :

- Flow is directed upwards.
- The device is a Compressor.
- The rotation is clockwise (to the right).



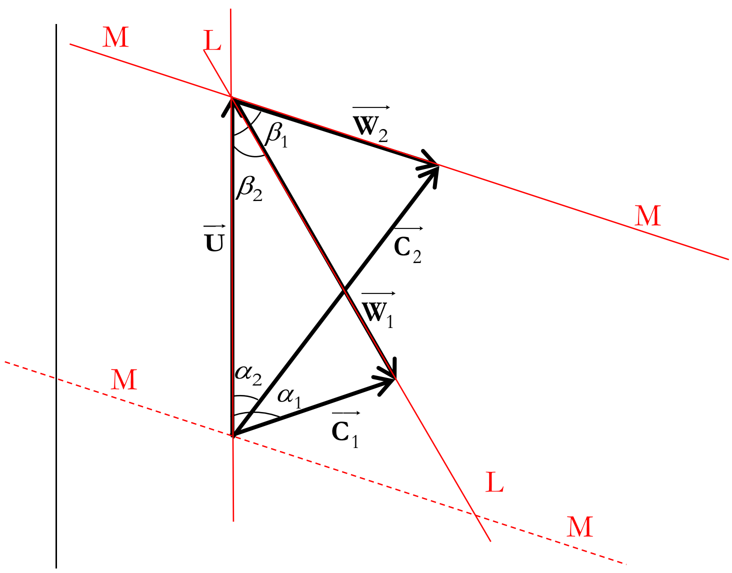
Case 3 :

- Flow is directed downwards.
- It's a Turbine.
- The movement is counterclockwise (to the left).



Case 4 :

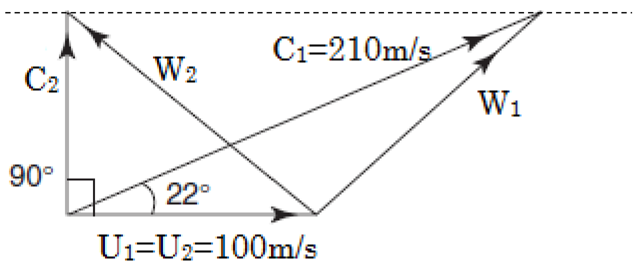
- Flow is directed to the right.
- It's a Compressor.
- Movement is upwards.



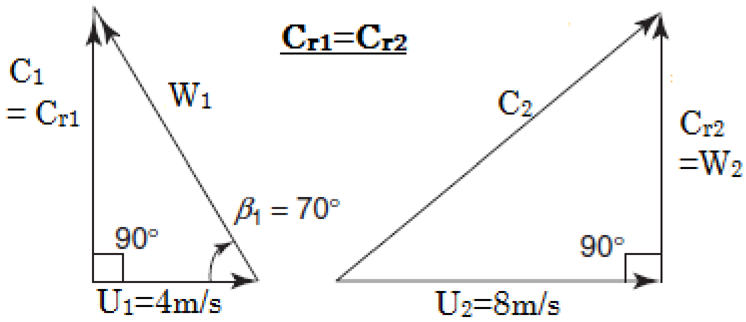
- **Exercise n° 3 :**

Consider the diagrams below, representing the velocity triangles at the inlet and outlet of each turbomachine.

Case 1 :



Case 2 :

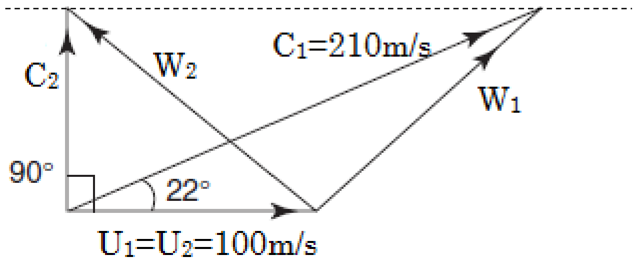


For each case, answer the following questions :

a. The machine is of the radial flow type or axial flow type ?
b. The turbomachine is a generator (compressor) or a receiver (expander)
c. Calculate all the elements of each velocity triangle and then deduce the specific work (τ).
d. Deduct power per unit of flow.
e. Calculate
    1. Degree of reaction R.
    2. Axial thrust.
    3. The utilization factor, if applicable.

Solution :

<u>Case 1 :</u>



a. Type of machine

$U_1 = U_2 \rightarrow$ The machine is of axial flow

b. Nature of the turbomachine

$C_1 > C_2 \rightarrow$ the machine is receiving (for expansion), (Turbine)

c. 1. Specific work **τ**

Expansion turbomachin, then: $\tau = U_1 C_{U_1} - U_2 C_{U_2} = U\left(C_{U_1} - C_{U_2}\right)$

$$C_{U_2} = 0 \text{ car } C_2 \perp U_2$$

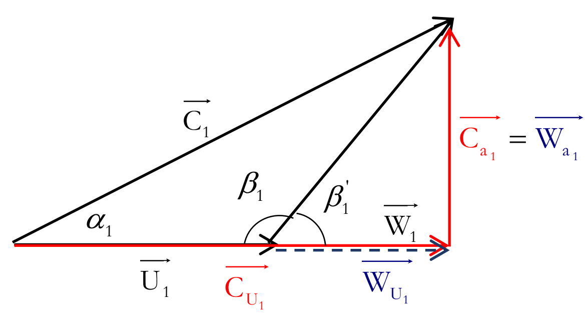
$$C_{U_1} = C_1 \cos\alpha_1 = 194{,}71\,\text{m/s}$$

$$\tau = U_1 C_{U_1} = 100 \times 194{,}71 = 19{,}471\,\text{kJ/kg}$$

2. Elements of velocity triangles

<u>Inlet</u>

$U_1 = 210\ \text{m/s}, \quad U_1 = 100\ \text{m/s}, \quad \alpha_1 = 22°$



<u>Outlet</u>

$U_2 = U_1 = 100\ \text{m/s}, \quad \alpha_2 = 90°$

$$C_2 = C_{a_2} = C_{a_1} = 78{,}66\,\text{m/s}$$

$$\beta_2 = \arctan\left(\frac{C_2}{U_2}\right) = 38{,}19°$$

$$\cos\beta_2 = \frac{U_2}{W_2} \rightarrow W_2 = \frac{U_2}{\cos\beta_2} = 127{,}23\,\text{m/s}$$

$C_{U_1} = 194,71\,m/s$

$W_{U_1} = C_{U_1} - U_1 = 94,71\,m/s$

$C_{a_1} = C_1 \sin\alpha_1 = W_{a_1} = 78,66\,m/s$

$W_1 = \sqrt{W_{U_1}^2 + W_{a_1}^2} = 123,12\,m/s$

$\beta_1^{'} = \arctan\left(\frac{W_{a_1}}{W_{U_1}}\right) = 39,71°,$

$\beta_1 = 180° - \beta_1^{'} = 140,29°$

d. Power per unit of flow rate

$$P = q_m \cdot \tau \rightarrow \tilde{P} = \frac{P}{q_m} = \tau = 19,471\,kW/(kg/s)$$

e. 1. Degree of reaction

$$R = \frac{(U_1^2 - U_2^2) + (W_2^2 - W_1^2)}{(C_1^2 - C_2^2) + (U_1^2 - U_2^2) + (W_2^2 - W_1^2)} = \frac{\tau - \frac{C_1^2 - C_2^2}{2}}{\tau} \rightarrow R = 0,024$$

2. Axial thrust

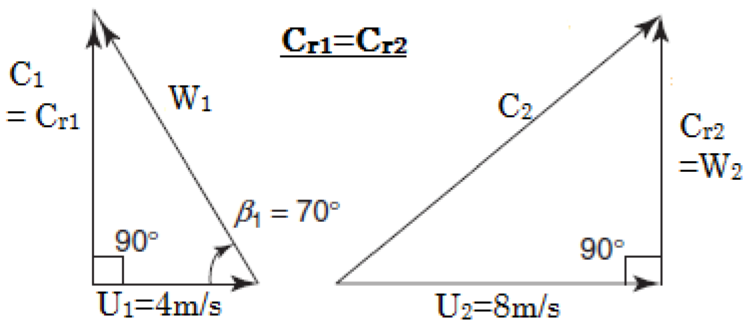
$$F_a = q_m \cdot (C_{a_1} - C_{a_2})$$

$$(C_{a_1} = C_{a_2}) \rightarrow F_a = 0$$

3. Utilization factor

$$\varepsilon = \frac{\tau}{\tau + \frac{C_2^2}{2}} = \frac{19,471 \times 10^3}{19,471 \times 10^3 + \frac{78,66^2}{2}} = 0,863$$

Case 2 :



a. Type of machine

$U_2 > U_1 \rightarrow$ The machine is of outward radial flow

f. Nature of the turbomachine

$C_2 > C_1 \rightarrow$ The machine is a generator (pump or compressor)

b. 1. Specific work **τ**

Generator turbomachine : $\tau = U_2 C_{U_2} - U_1 C_{U_1}$

According to the velocity triangles :

$$U_2 = 8 \text{ m/s}$$
$$U_1 \perp C_1 \rightarrow C_{U_1} = 0 \text{ m/s}$$

$$\tau = U_2 C_{U_2} = 8 \times 8 = 64 \text{ J/kg}$$

2. Elements of velocity triangles

Inlet

$U_1 = 4 \text{ m/s}, \quad \alpha_1 = 90°, \quad \beta_1 = 70°$

$$\tan\beta_1 = \frac{C_1}{U_1} \rightarrow C_1 = U_1 \tan\beta_1 = 10,99 \text{ m/s}$$

$$C_1 = C_{r_1} = 10,99 \text{ m/s}$$

$$\sin\beta_1 = \frac{C_1}{W_1} \rightarrow W_1 = \frac{C_1}{\sin\beta_1} = 11,70 \text{ m/s}$$

Outlet

$U_2 = 8 \text{ m/s}, \quad \beta_2 = 90°$

$$C_{r_1} = C_{r_2} = 10,99 \text{ m/s} \rightarrow W_2 = C_{r_2} = 10,99 \text{ m/s}$$

$$C_2 = \sqrt{U_2^2 + W_2^2} = 13,6 \text{ m/s},$$

$$\alpha_2 = \arctan\left(\frac{C_{r_2}}{U_2}\right) = 53,94 \text{ m/s}$$

c. Power per unit of flow rate

$$P = q_m \cdot \tau \rightarrow \tilde{P} = \frac{P}{q_m} = \tau = 64 \text{ W/(kg/s)}$$

d. 1. Degree of reaction

$$R = \frac{\left(U_2^2 - U_1^2\right) + \left(W_1^2 - W_2^2\right)}{\left(C_2^2 - C_1^2\right) + \left(U_2^2 - U_1^2\right) + \left(W_1^2 - W_2^2\right)} = \frac{\frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2}}{\tau}$$

As :

$$\tau = \frac{C_2^2 - C_1^2}{2} + \frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2}$$

$$\frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2} = \tau - \frac{C_2^2 - C_1^2}{2}$$

then,

$$R = \frac{\tau - \frac{C_2^2 - C_1^2}{2}}{\tau} = \frac{64 - \frac{13{,}6^2 - 10{,}99^2}{2}}{64} \rightarrow R = 0{,}5$$

2. Axial thrust

The machine is of radial flow type ; therefore, axial thrust is negligible (non-existent).

3. Utilization factor

The utilization factor is applicable only to turbines.

Chapter II

Similarities in Turbomachines

Table des matières

1. Introduction

The resolution of a fluid mechanics challenge typically begins with a theoretical study. However, there are cases where this theoretical approach may prove to be complex or insufficient to provide the expected answers. In such situations, it becomes necessary to resort to experimental simulation.

Conducting experiments can often be an insurmountable challenge, especially when exploring the behavior of turbomachines. In these situations, it becomes both more practical and faster to opt for tests conducted on a scale model. However, to ensure that the main characteristics of the scale model are accurately extrapolated to the full-scale model, the laws of similarity play a crucial role. These laws, formulated through dimensional analysis, allow for the establishment of crucial relationships.

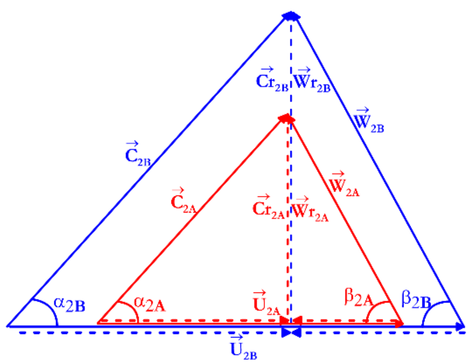
Similarity is defined as follows : "Two flows are considered similar if they occur within comparable geometric bounds, and the trajectories of corresponding particles are also geometrically comparable." This approach ensures an adequate transition between the scale model and the full-scale model.

2. General relationships

Two turbomachines are considered to be of the same type if they exhibit geometric similarity, implying that one can be morphed into the other by uniformly scaling all linear dimensions using a common factor known as the geometric similarity coefficient.

Furthermore, two turbomachines of the same type operate in similarity when, for all corresponding pairs of points taken in these machines, the velocity triangles are similar. When there is operational similarity between two turbomachines of the same type, A and B, we have the following for all corresponding pairs of points :

$$\begin{cases} \left(\dfrac{C}{U}\right)_A = \left(\dfrac{C}{U}\right)_B \\ \left(\dfrac{W}{U}\right)_A = \left(\dfrac{W}{U}\right)_B \\ \alpha_A = \alpha_B \quad \text{et} \quad \beta_A = \beta_B \end{cases}$$



3. Rateau's invariants

The Rateau coefficients, often referred to as non-dimensional Rateau coefficients, serve a pivotal role in the analysis and modeling of turbomachinery. These coefficients are dimensionless parameters that amalgamate similarity properties, effectively simplifying the intricacies of operational variables.

The adimensional expression of Rateau coefficients is of particular importance because it highlights the essential dimensionless numbers that must be preserved to ensure similarity when extrapolating results. For example, when conducting experiments on a small-scale model in the laboratory with the intention of applying these results to a full-scale prototype, it is imperative that all relevant dimensionless numbers remain identical for both configurations.

This dimensionless methodology is fundamental to ensure that the performance, flow characteristics, and other critical properties of turbomachines remain consistent between the model and the prototype. In summary, the Rateau coefficients and their dimensionless form serve as an essential guide to maintain the validity and accuracy of extrapolations in the complex field of turbomachines..

To define these dimensionless variables in the context of turbomachines, it is imperative to start by listing the essential quantities, which are :

- Wheel diameter **D**
- Wheel rotation speed **N**
- The power supplied by the machine **P**
- The discharge flow rate q_v
- Manometric energy (energy head) **Hg**
- A fluid property
  - Density ρ

Then illustrate their dimensions, using the following table :

	D	**N**	ρ	**Hg**	q_v	**P**
[L]	1	0	-3	2	3	2
[M]	0	0	1	0	0	1
[t]	0	-1	0	-2	-1	-3
[T]	0	0	0	0	0	0

[L] : Length [M] : Mass [t] : time [T] : temperature

Among these 6 variables, 3 fundamental units (M, L, T) are involved. The application of the **Vaschy-Buckingham** theorem reduces the number of dimensionless characteristic variables governing the machine's operation to 6 - 3 = 3.

By choosing the fundamental quantities **D**, **N** and ρ (with a nonzero determinant), we obtain the following dimensionless coefficients of Rateau :

a. Pressure Coefficient (Manometric Power) ψ

$$\psi = \frac{Hg}{D^a N^b \rho^c}$$

$$\begin{cases} a = 2 \\ b = 2 \\ c = 0 \end{cases} \Rightarrow \psi = \frac{Hg}{N^2 \cdot D^2}$$

D	**N**	ρ	**Hg**
a	0	-3c	2
0	0	c	0
0	-b	0	-2

b. Flow Coefficient (Flowing Power) δ

$$\delta = \frac{q_V}{D^d N^e \rho^f}$$

$$\begin{cases} d = 3 \\ e = 1 \\ f = 0 \end{cases} \Rightarrow \delta = \frac{q_V}{N \cdot D^3}$$

D	**N**	ρ	**qv**
d	0	-3f	3
0	0	f	0
0	-e	0	-1

c. Power Coefficient τ_P

$$\tau_P = \frac{P}{D^g N^h \rho^i}$$

$$\begin{cases} g = 5 \\ h = 3 \\ i = 1 \end{cases} \Rightarrow \tau_P = \frac{P}{\rho \cdot N^3 \cdot D^5}$$

D	**N**	ρ	**P**
g	0	-3i	2
0	0	i	1
0	-h	0	-3

d. Efficiency η

We note that the efficiency η is also an invariant, as by definition :

$$\eta = \frac{\rho \cdot g \cdot H \cdot q_V}{P} = \frac{\rho \cdot g \cdot \left(\frac{\psi \cdot N^2 \cdot D^2}{g}\right) \cdot \left(\delta \cdot N \cdot D^2\right)}{\tau \cdot \rho \cdot N^3 \cdot D^5}$$

$$\Rightarrow \eta = \frac{\psi \cdot \delta}{\tau_P}$$

4. Similar operation of turbomachines

The similarity properties of turbomachines form a foundational set of principles that establish crucial relationships for hydraulic turbomachines of the same type operating under similarity conditions. These principles can be succinctly summarized as follows: when turbomachines are designed and operated in accordance with the relevant similarity laws, several key parameters, including the pressure coefficient (ψ), flow coefficient (δ), internal power coefficient (τ), and internal efficiencies (η), remain constant.

This statement implies several crucial elements:

- The Invariant Nature of Parameters: When turbomachines of the same type are brought into similarity, the mentioned parameters (ψ, δ, τ, η) remain constant, regardless of the scales or operating conditions. This means that how these machines behave under different sizes or conditions remains consistent and predictable.
- Pressure Coefficient (ψ): The pressure coefficient, which measures the pressure variation across the turbomachine, is kept constant during similarity. This ensures that the pressure distribution on the blades or vanes remains consistent.
- Flow Coefficient (δ) : The flow coefficient, which characterizes the volumetric fluid displacement through the turbomachine, is also maintained in similarity. This ensures that the mass and volumetric flow rates remain proportional.
- Internal Power Coefficient (τ) : The internal power coefficient, which evaluates the turbomachine's ability to transfer energy to the fluid, remains unchanged. This means that the machine's energy performance remains consistent regardless of scale or conditions.
- Internal Efficiencies (η) : The internal efficiencies of the turbomachine, which indicate its efficiency in energy conversion, remain constant. This maintains the predictability of overall performance.

By applying these principles of similarity, engineers are able to design smaller-scale models of turbomachines for laboratory testing while confidently extrapolating the expected performance to the full-scale prototype. This saves time and money while ensuring the reliability and consistency of turbomachines in various industrial applications, from power generation to hydraulic engineering.

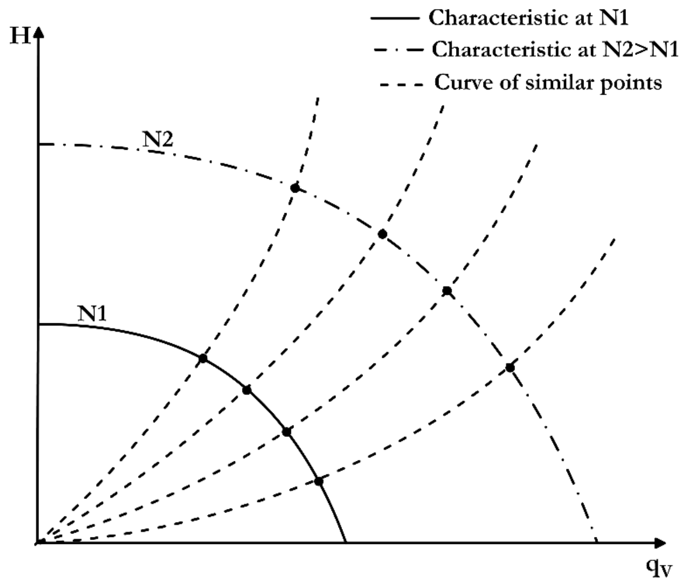


Figure 1. Illustration of similarity through variation of rotational speed (N) for the head-flow characteristic of a turbopump.

5. Specific speed

The specific speed, represented by the symbol N_s, is a crucial parameter in the field of turbomachinery. Unlike other parameters like wheel or impeller diameter, N_s does not explicitly depend on them. It plays a vital role in comparing various types of turbomachines because each category of these devices has its unique specific speed, revealing its intrinsic characteristics.

- Pump specific speed :

The specific speed of a centrifugal pump is a fundamental characteristic that quantifies the performance of the pump. It is defined as the speed at which the pump is capable of delivering one cubic meter of liquid per second while overcoming a one-meter head. It is expressed in its dimensionless form. :

$$N_s = \frac{N\sqrt{q_V}}{gH^{3/4}} \tag{II.1}$$

- Turbine specific speed :

The specific speed of a turbine is a crucial parameter that reflects its performance. It is defined as the speed at which the turbine is capable of producing unit power while operating under standardized thermal load. This value is essential for evaluating and comparing turbine performance, allowing for the characterization of a turbine's ability to efficiently convert thermal energy into mechanical energy. It is expressed in its dimensionless form :

$$N_s = \frac{N\sqrt{P}}{\sqrt{\rho}H^{5/4}} \quad (II.2)$$

Turbomachines	N_s
Pelton turbines	0,02−0,39
Francis turbines	0,14−0,39
Kaplan turbines	2,7−5,4
Centrifugal pumps	0,24−1,8
Axial pumps	3,2−5,7
Radial compressors	0,4−1,4
Axial compressors/blowers	1,4−20
Axial-flow steam/gas turbines	0,35−1,9

Table 1. Specific speed ranges of turbomachinery.

Table 1 highlights that each category of turbomachine operates within a specific range of specific speeds. This specific speed is the essential parameter that reflects the variation of all variables, namely N and Q, or P and H, resulting in comparable flows within turbomachines that share geometric similarities.

6. Generalization

Before manufacturing full-scale large machines, closely resembling models, also known as prototypes, are created. Tests are conducted on these models, and the performance of the prototypes is predicted. Complete similarity between the actual machine and the model can only be achieved if the Râteau coefficients (ψ, δ, τ, η) and the rotation speed are meticulously adhered to.

7. Other coefficients

In addition to the coefficients mentioned throughout this chapter, there are also other important dimensionless numbers that can be used as criteria for dynamic similarity. These include, but are not limited to:

- Reynolds Number (Re) : It is defined as the ratio of the fluid's inertial force in motion to the fluid's viscous force. It is expressed as :

$$Re = \frac{C \cdot D}{\nu} \quad (II.3)$$

- Euler Number (Eu): It is defined as the square root of the ratio between the fluid's inertial force in motion and the pressure force. It is expressed as:Nombre d'Euler (Eu) :

$$Eu = \sqrt{\frac{\text{Inertia force } (F_i)}{\text{pressure force } (F_p)}} = \sqrt{\frac{\rho \cdot S \cdot C^2}{p \cdot S}} = \frac{C}{\sqrt{p/\rho}} \quad (II.4)$$

- Froude Number (Fe): It is defined as the square root of the ratio between the fluid's inertial force in motion and gravitational force. Its expression is as follows :

$$Fe = \sqrt{\frac{\text{Inertia force}(F_i)}{\text{Gravity force}(F_g)}} = \sqrt{\frac{\rho \cdot S \cdot C^2}{\rho \cdot S \cdot D \cdot g}} = \frac{C}{\sqrt{D \cdot g}} \quad (II.5)$$

- Weber Number (We): It is defined as the square root of the ratio between the fluid's inertial force in motion and the surface tension force. It is given as follows :

$$We = \sqrt{\frac{\text{Inertia force}(F_i)}{\text{surface tension force}(F_\gamma)}} = \sqrt{\frac{\rho \cdot S \cdot C^2}{\gamma \cdot D}} \quad (II.6)$$

8. Exercises

- **Exercise n°1 :**

The impeller of a purely radial centrifugal pump with a diameter D = 0.25 m and thickness b = 12 mm (outer dimensions), rotating at 1350 rpm has the following characteristics:

q_V (m^3/s)	0,0614	0,0735	0,0859	0,0984	0,1107	0,1228
H (m)	21,1	19,9	19,3	18,1	16,6	13,3
η net	0,72	0,77	0,84	0,88	0,86	0,78

We have geometrically similar pumps with diameters of 0.5 m, 0.45 m, 0.42 m, and 0.39 m that can rotate at speeds of 1750 rpm, 1475 rpm, and 1125 rpm.

rpm= rounds per minute=tr/min

1. What diameter and rotational speed should be chosen to achieve a flow rate of 0,478 m^3/s and a net head of 40,725 m
2. Calculate the power absorbed by the selected pump at the operating point from the previous question (1).
3. Plot in the same diagram the velocity triangles corresponding to the similar points obtained, then deduce the outer thickness of the selected pump wheel.

Hypotheses :

- The slip coefficient μ=0,9 and the density $\rho = 1000\,kg/m^3$
- The mechanical and volumetric losses in the pump are negligible

Solution :

Initial pump : $D_i = 0,25m$ et $N_i = 1350rpm$

1. Selection of the pump to obtain $\left(q_{V_j} = 0,478\,m^3/s \quad et \quad H_j = 40,725m\right)$

Method 1 :

$$\begin{cases} \delta_i = \delta_j \Rightarrow \dfrac{q_{V_i}}{N_i D_i^3} = \dfrac{q_{V_j}}{N_j D_j^3} \Rightarrow q_{V_j} = q_{V_i} \dfrac{N_j}{N_i}\left(\dfrac{D_j}{D_i}\right)^3 \\ \psi_i = \psi_j \Rightarrow \dfrac{H_i}{N_i^2 D_i^2} = \dfrac{H_j}{N_j^2 D_j^2} \Rightarrow H_j = H_i \left(\dfrac{N_j}{N_i}\right)^2 \left(\dfrac{D_j}{D_i}\right)^2 \end{cases}$$

q_{Vi} (m^3/s)	0,0614	0,0735	0,0859	0,0983	0,1107	0,1228
H_i (m)	21,1	19,9	19,3	18,1	16,6	13,3
η_i net	0,72	0,77	0,84	0,88	0,86	0,78

D_i(m)	0,25
N_i(rpm)	1350

See the tables below for the location of the requested point

N_j=1750rpm

D_j=0,5m		D_j=0,45m		D_j=0,42m		D_j=0,39m	
0,637	141,824	0,464	114,878	0,377	100,071	0,302	86,286
0,762	133,759	0,556	108,344	0,452	94,380	0,362	81,379
0,891	129,726	0,649	105,078	0,528	91,534	0,423	78,925
1,020	121,660	0,744	98,544	0,605	85,843	0,484	74,018
1,148	111,578	0,837	90,378	0,680	78,729	0,545	67,884
1,273	89,396	0,928	72,411	0,755	63,078	0,604	54,389
q_{Vi}(m^3/s)	H_j(m)	q_{Vi}(m^3/s)	H_j(m)	q_{Vi}(m^3/s)	H_j(m)	q_{Vi}(m^3/s)	H_j(m)

N_j=1475rpm

D_j=0,5m		D_j=0,45m		D_j=0,42m		D_j=0,39m	
0,537	100,753	0,391	81,610	0,318	71,091	0,255	61,298
0,642	95,023	0,468	76,969	0,381	67,048	0,305	57,812
0,751	92,158	0,547	74,648	0,445	65,027	0,356	56,069
0,860	86,428	0,627	70,007	0,510	60,984	0,408	52,583
0,968	79,266	0,705	64,205	0,574	55,930	0,459	48,225
1,073	63,508	0,782	51,441	0,636	44,811	0,509	38,638
q_{Vi}(m^3/s)	H_j(m)	q_{Vi}(m^3/s)	H_j(m)	q_{Vi}(m^3/s)	H_j(m)	q_{Vi}(m^3/s)	H_j(m)

N_j=1125rpm

D_j=0,5m		D_j=0,45m		D_j=0,42m		D_j=0,39m	
0,409	58,611	0,298	47,475	0,243	41,356	0,194	35,659
0,490	55,278	0,357	44,775	0,290	39,004	0,233	33,631
0,573	53,611	0,417	43,425	0,339	37,828	0,272	32,617
0,656	50,278	0,478	40,725	0,389	35,476	0,311	30,589
0,738	46,111	0,538	37,350	0,437	32,536	0,350	28,054
0,819	36,944	0,597	29,925	0,485	26,068	0,388	22,477
$q_{Vi}(m^3/s)$	$H_j(m)$	$q_{Vi}(m^3/s)$	$H_j(m)$	$q_{Vi}(m^3/s)$	$H_j(m)$	$q_{Vi}(m^3/s)$	$H_j(m)$

Method 2 :

Search for $\left(q_{V_i}\right)$ and $\left(H_i\right)$ located in the data table of the initial pump;

$$\begin{cases} \delta_i = \delta_j \Rightarrow \dfrac{q_{V_i}}{N_i D_i^3} = \dfrac{q_{V_j}}{N_j D_j^3} \Rightarrow q_{V_i} = q_{V_j} \dfrac{N_i}{N_j}\left(\dfrac{D_i}{D_j}\right)^3 \\ \psi_i = \psi_j \Rightarrow \dfrac{H_i\, g}{N_i^2 D_i^2} = \dfrac{H_j\, g}{N_j^2 D_j^2} \Rightarrow H_i = H_j \left(\dfrac{N_i}{N_j}\right)^2 \left(\dfrac{D_i}{D_j}\right)^2 \end{cases}$$

Refer to the table below for the location of the initial point.

		$H_j(m)$	40,725			$D_i(m)$	0,25
		$q_{Vj}(m^3/s)$	0,478			$N_i(rpm)$	1350
		$N_j(rpm)$			$N_j(rpm)$		
		1125	1475	1750	1125	1475	1750
$D_j(m)$	0,5	0,0717	0,0547	0,0461	14,66	8,53	6,06
	0,45	0,0984	0,0750	0,0632	18,10	10,53	7,48
	0,42	0,1210	0,0923	0,0778	20,78	12,09	8,59
	0,39	0,1511	0,1152	0,0971	24,10	14,02	9,96
		$q_{Vi}(m^3/s)$			$H_i(m)$		

2. Power absorbed by the selected pump at the operating point

The operating point:

$$\left| \begin{array}{l} q_{Vj} = q_{V_F} = 0,478\,m^3/s \\ H_j = H_F = 40,725m \end{array} \right. \xrightarrow{\text{Similarity condition}} \left| \begin{array}{l} q_{V_i} = 0,0983\,m^3/s \\ H_i = 18,1m \end{array} \right.$$

$$\eta_j = \eta_F \quad \overset{\text{Equal}}{\longleftrightarrow} \quad \eta_i = 88\%$$

Finally,

$$P_a = \frac{P_f}{\eta_F} = \frac{\rho g H_F q_{V_F}}{\eta_F} = \frac{1000 \cdot 9{,}81 \cdot 0{,}478 \cdot 40{,}725}{0{,}88} \rightarrow \boxed{P_a = 217{,}008\text{kW}}$$

3. Diagram of velocity triangles at similar points

$$\left| \begin{array}{l} N_j = 1125\text{rpm} \\ Q_{Vj} = 0{,}478\,\text{m}^3/\text{s} \\ H_j = 40{,}725\text{m} \\ D_{2j} = 0{,}45\text{m} \\ b_{2j} = ? \end{array} \right. \quad \xleftrightarrow{\text{Similar points}} \quad \left| \begin{array}{l} N_j = 1125\text{rpm} \\ Q_{Vj} = 0{,}478\,\text{m}^3/\text{s} \\ H_j = 40{,}725\text{m} \\ D_{2j} = 0{,}45\text{m} \\ b_{2j} = ? \end{array} \right.$$

$\mu = 0{,}90$

Negligible mechanical and volumetric losses $\Rightarrow \eta_M = 1$ et $\eta_V = 1 \Rightarrow \eta_{net}\left(\eta_g\right) = \eta_H$

$\mu = H_{thZ}/H_{th\infty} = 0{,}90$ and $\eta_H = H/H_{thZ} = 88\% \Rightarrow H_{th\infty} = H/\mu\eta_H$

Point (i)

$U_{2i} = D_{2i}\pi N_i/60$

$$H_{th\infty} = \frac{U_2 C_{U2} - U_1 C_{U1}}{g}$$

$\rightarrow C_{U1} = 0$ (Radial inlet)

$$C_{U2_i} = \frac{gH_{th\infty_i}}{U_2}$$

$Q_V = C_{r2i}\pi D_{2i} b_{2i} \Rightarrow C_{r2i} = Q_V/\pi D_{2i} b_{2i} = W_{r2i}$

$\alpha_{2i} = \arctan\left(C_{r2i}/C_{U2i}\right)$

$C_{2i} = \sqrt{C_{U2i}^2 + C_{r2i}^2}$

$U_{2i} > C_{U2_i} \Rightarrow W_{U2_i} = U_{2i} - C_{U2_i}$

$\beta_{2i} = \arctan\left(W_{r2i}/W_{U2i}\right)$

$W_{2i} = \sqrt{W_{U2i}^2 + W_{r2i}^2}$

Point (j)

$U_{2j} = D_{2j}\pi N_j/60$

$$H_{th\infty} = \frac{U_2 C_{U2} - U_1 C_{U1}}{g}$$

$\rightarrow C_{U1} = 0$ (Radial inlet)

$$C_{U2_j} = \frac{gH_{th\infty_j}}{U_2}$$

Similar flow for points (i) and (j)

$\Rightarrow \alpha_{2j} = \alpha_{2i}$

$C_{2j} = C_{U2_j}/\cos\alpha_{2j}$

$C_{r2j} = C_{2j}\cos\alpha_{2j} = W_{r2j}$

$U_{2j} > C_{U2_j} \Rightarrow W_{U2_j} = U_{2j} - C_{U2_j}$

$\beta_{2j} = \arctan\left(W_{r2j}/W_{U2j}\right) = \beta_{2i}$ (Similarity)

$W_{2j} = \sqrt{W_{U2j}^2 + W_{r2j}^2}$

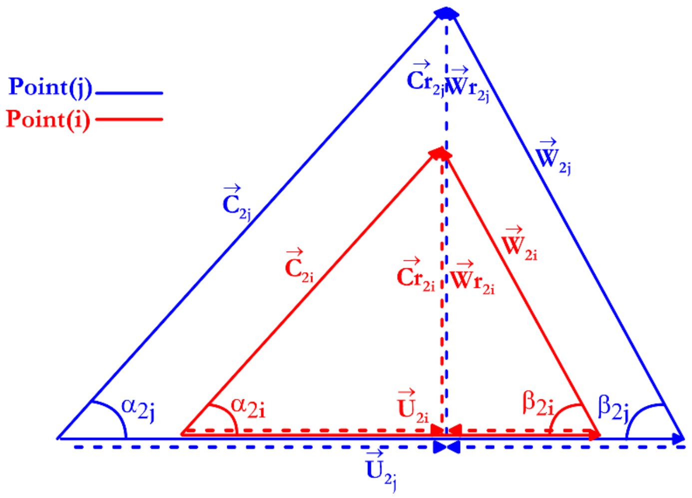
Which results in, ultimately:

	U_2 [m/s]	$H_{th\infty}$ [m]	C_{U2} [m/s]	W_{U2} [m/s]	$C_{r2} = W_{r2}$ [m/s]	α_2 [°]	C_2 [m/s]	β_2 [°]	W_2 [m/s]
Point(i)	17,671	22,854	12,687	4,984	10,436	39,44	16,428	64,47	11,565
Point(j)	26,507	51,420	19,030	7,477	15,654	39,44	24,641	64,47	17,348

- Impeller thickness of the pump (j)

$$Q_{Vj} = C_{r2j} D_{2j} \pi b_{2j} \Rightarrow b_{2j} = Q_{Vj} / C_{r2j} D_{2j} \pi = 0{,}022\text{m} = 22\text{mm}$$

- Diagram of velocity triangles



- **Exercise n°2 :**

The head H, net efficiency η, and the hydraulic system characteristic H_r associated with a centrifugal pump are given by the equations :

$$H = 52{,}169 + 1{,}783 q_V - 3{,}276 q_V^2$$
$$\eta = 29{,}643 q_V - 3{,}2143 q_V^2$$
$$H_r = 4{,}325 + 2{,}761 q_V^2$$

he pump's rotation speed is 1800 rpm. The distance is given in meters, the flow rate is expressed in ℓ/s and the efficiency is in (%), rpm= rounds per minute

1. If the rotor's rotation speed in the same system is increased to 3600 rpm, what will be the flow rate, and what will be the power at the operating point ?
2. What is the integer value of the rotation speed (in rpm) required to increase the flow rate to 1.7 times the value obtained at 1800 rpm?

With : $\rho = 1000\,\text{kg/m}^3$

Solution :

$$N = 1800\,rpm \quad \begin{vmatrix} H = 52,169 + 1,783q_V - 3,276q_V^2 \\ \eta = 29,643q_V - 3,2143q_V^2 \\ H_r = 4,325 + 2,761q_V^2 \end{vmatrix}$$

1. Rotation speed increases to $N = 3600\,rpm$, Determination of flow rate and power at the operating point.
   a. Operating point for $N = 1800\,rpm$

This is equivalent to solving the $(H = H_r)$,

$$H = H_r \Leftrightarrow 52,169 + 1,783q_V - 3,276q_V^2 = 4,325 + 2,761q_V^2$$
$$\Rightarrow 47,844 + 1,783q_V - 6,037q_V^2 = 0$$

Reminder :

$$ax^2 + bx + c = 0 \rightarrow \Delta = b^2 - 4ac \Rightarrow \begin{cases} x_1 = \dfrac{-b-\sqrt{\Delta}}{2a} \\ x_2 = \dfrac{-b+\sqrt{\Delta}}{2a} \end{cases}$$

then,

$$\Delta = 1158,516001 \Rightarrow \begin{cases} q_V' = 2,9667\,\ell/s \\ q_V'' = -2,6714\,\ell/s < 0 \end{cases}$$

The negative value is excluded.

Then the operating point,

$$\begin{cases} q_{V_F} = 2,9667\,\ell/s \\ H_F = H\left(q_{V_F}\right) = H_r\left(q_{V_F}\right) = 28,63m \end{cases}$$

   b. Determination of the flow rate at the operating point at $N = 3600\,rpm$

The coefficients for the Râteau (similarity law for turbomachinery) :

$$\psi = \frac{H \cdot g}{N^2D^2}, \quad \delta = \frac{Q_V}{ND^3}, \quad \tau = \frac{P}{\rho N^3 D^5}$$

To determine the new operating point at the rotation speed $N = 3600\,rpm$, we need to determine the new head curve at this rotation speed N.

For two similar operating points at different speeds with $\left(g = 9,81\,m^2/s = C^{ste}, D = C^{ste}\right)$.

We'll have:

$$\begin{cases} \delta_1 = \delta_2 \Rightarrow \dfrac{q_{V_1}}{N_1 D_1^3} = \dfrac{q_{V_2}}{N_2 D_2^3} \Rightarrow \dfrac{q_{V_1}}{N_1} = \dfrac{q_{V_2}}{N_2} \Rightarrow q_{V_1} = q_{V_2}\dfrac{N_1}{N_2} \\ \psi_1 = \psi_2 \Rightarrow \dfrac{H_1 g}{N_1^2 D_1^2} = \dfrac{H_2 g}{N_2^2 D_2^2} \Rightarrow \dfrac{H_1}{N_1^2} = \dfrac{H_2}{N_2^2} \Rightarrow H_1 = H_2\left(\dfrac{N_1}{N_2}\right)^2 \end{cases}$$

Then,

$$N_1 = 1800\,rpm \;\rightarrow H_1 = 52,169 + 1,783 q_{V1} - 3,276 q_{V1}^2$$

$$\text{So ; } N_2 = 3600\,rpm \;\rightarrow H_2\left(\frac{N_1}{N_2}\right)^2 = 52,169 + 1,783 q_{V_2}\frac{N_1}{N_2} - 3,276\left(q_{V_2}\frac{N_1}{N_2}\right)^2$$

$$\rightarrow \boxed{H_2 = 208,676 + 3,566 q_{V_2} - 3,276 q_{V_2}^2}$$

The new operating point:

$$H_2 = H_r \Leftrightarrow 208,676 + 3,566 q_V - 3,276 q_V^2 = 4,325 + 2,761 q_V^2$$

$$\Delta = 4947,384304 \Rightarrow \begin{cases} q_V' = 6,1209\,\ell/s \\ q_V'' = -5,5302\,\ell/s < 0 \quad (\text{Exclu}) \end{cases}$$

Finally, the coordinates of the new operating point

$$\begin{cases} q_{V_{F2}} = 6,1209\,\ell/s \\ H_{F2} = H_2\left(q_{V_{F2}}\right) = H_r\left(q_{V_{F2}}\right) = 107,77\,m \end{cases}$$

- Power at point of operation

Deduction of the efficiency curve

$$N_1 = 1800\,rpm \;\rightarrow \eta_1 = 29,643 q_{V_1} - 3,2143 q_{V_1}^2$$

$$N_2 = 3600\,rpm \;\rightarrow \eta_2 = 29,643 q_{V_2}\frac{N_1}{N_2} - 3,2143\left(q_{V_2}\frac{N_1}{N_2}\right)^2 \Rightarrow \boxed{\eta_2 = 14,8215 q_{V_2} - 0,8036 q_{V_2}^2}$$

The performance of the new operating point : $\eta_2\left(q_{V_{F2}}\right) = 60,61\%$

Finally, absolute power :

$$P_a = \frac{P_f}{\eta_f} = \frac{\rho g H_F q_{V_F}}{\eta_F} = \frac{6471,1605}{0,6061} \rightarrow \boxed{P_a = 10676,721W}$$

2. Rotation speed required to increase the flow rate by a factor of 1.7 from the initial flow rate.

<u>Method 1 :</u>

The intersection of the pump characteristic curve and the hydraulic system's curve yields a flow rate and a head (previously calculated) :

$$\left\{q_{V_{F1}} = 2,9667\,\ell/s,\ H_{F1} = 28,63m\right\}$$

The required flow rate :

$$q_{V_d} = 1,7 \times q_{V_{F1}} = 5,0434\,\ell/s$$

The head associated with this flow rate :

$$H_d = H_r\left(q_{V_d}\right) = 4,325 + 2,761 q_{V_d}^2 = 74,55m$$

The operating point $\left(q_{V_{Fd}} = 5,0434\,\ell/s,\ H_{Fd} = 74,55m\right)$, and the point $\left(Q_{V_{F1}} = 2,9667\,\ell/s,\ H_{F1} = 28,63m\right)$ are not similar because the connection is made only through the system curve. To find a similar condition, the similarity laws are used.

$$g = C^{ste}, D = C^{ste} \rightarrow \left\{\left(\frac{q_V}{N}\right)_i = \left(\frac{q_V}{N}\right)_j \text{ et } \left(\frac{H}{N^2}\right)_i = \left(\frac{H}{N^2}\right)_j\right\}$$

With that, j is associated with the requested point, so:

$$\left(\frac{q_V}{N}\right)_i = \left(\frac{q_V}{N}\right)_j \Rightarrow \frac{q_{V_i}}{q_{V_j}} = \frac{N_i}{N_j}$$

$$\left(\frac{H}{N^2}\right)_i = \left(\frac{H}{N^2}\right)_j \Rightarrow \frac{H_i}{H_j} = \left(\frac{N_i}{N_j}\right)^2 = \left(\frac{q_{V_i}}{q_{V_j}}\right)^2$$

So,

$$\frac{H_i}{H_j} = \left(\frac{q_{V_i}}{q_{V_j}}\right)^2 \Rightarrow \frac{H_i}{q_{V_i}^2} = \frac{H_j}{q_{V_j}^2} = \frac{H_d}{q_{V_d}^2} = \frac{74,55}{5,0434^2} = 2,9309$$

$$\rightarrow \boxed{H_i = 2,9309 \cdot q_{V_i}^2}$$

This expression represents the set of points similar to the requested point $\left(q_{V_{Fd}} = 5{,}0434\,\ell/s,\ H_{Fd} = 74{,}55m\right)$ located at different rotation speeds.

The intersection of this curve with the curve for the pump at $N = 1800\text{rpm}$ $\left(H = 52{,}169 + 1{,}783q_V - 3{,}276q_V^2\right)$, gives:

$$H = H_i \Rightarrow 52{,}169 + 1{,}783q_V - 3{,}276q_V^2 = 2{,}9309q_V^2 \Rightarrow 52{,}169 + 1{,}783q_V - 6{,}2069q_V^2 = 0$$

$$\Delta = 1298{,}410153 \Rightarrow \begin{cases} q_{V_1} = 3{,}0463\,\ell/s \\ q_{V_2} = -2{,}7591\,\ell/s < 0 \quad (\text{Exclu}) \end{cases}$$

The point associated with the requested point, which is similar, has the following coordinates :

$$\begin{cases} q_{V_{Fi}} = 3{,}0463\,\ell/s \\ H_{Fi} = H_i\left(q_{V_{Fi}}\right) = H\left(q_{V_{Fi}}\right) = 27{,}20m \end{cases}$$

Finally, using the laws of similitude:

$$N_j = N_d = N_i \frac{q_{V_d}}{q_{V_i}} = 2980\text{rpm}$$

Or

$$N_j = N_d = N_i \sqrt{\frac{H_d}{H_i}} = 2980\text{rpm}$$

Method 2 :

We know that :

$$\begin{cases} N_i = 1800\text{rpm} \\ q_{Vj} = q_{V_d} = 5{,}0435\,\ell/s \\ H_j = H_d = 74{,}55m \end{cases}$$

The laws of similitude give :

$$\left\{\frac{q_{V_i}}{q_{V_j}} = \frac{N_i}{N_j}\ ;\ \frac{H_i}{H_j} = \left(\frac{N_i}{N_j}\right)^2\right\} \rightarrow \left\{q_{V_i} = \frac{N_i}{N_j} q_{V_j}\ ;\ H_i = H_j\left(\frac{N_i}{N_j}\right)^2\right\}$$

We know that : $H_i = 52{,}169 + 1{,}783q_{V_i} - 3{,}276q_{V_i}^2$

Substituting the values of (H_i) and $\left(q_{V_i}\right)$, we obtain :

$$H_j\left(\frac{N_i}{N_j}\right)^2 = 52{,}169 + 1{,}783\frac{N_i}{N_j}q_{V_j} - 3{,}276\left(\frac{N_i}{N_j}q_{V_j}\right)^2$$

By replacing (H_j) and (q_{V_j}) with their associated values :

$$52{,}169+1{,}783\frac{N_i}{N_j}q_{V_i}-3{,}276\left(\frac{N_i}{N_j}q_{V_i}\right)^2-H_j\left(\frac{N_i}{N_j}\right)^2=0$$

We then have :

$$\frac{52{,}169N_j^2+16186{,}288N_j-511524573}{N_i}=0\rightarrow 52{,}169N_j^2+16186{,}288N_j-511524573=0$$

$$\Delta=1{,}07005\times10^{11}\Rightarrow\begin{cases}N_1=2980\text{rpm}\\N_2=-3290\text{rpm}<0\ (\text{Exclu})\end{cases}$$

Rotation speed $\rightarrow$ $\boxed{N=2980\text{rpm}}$

- Efficiency of the two similar points

<u>Point 1 :</u> $N_1=2980\text{rpm}$

$$\begin{cases}q_{V_1}=5{,}0434\,\ell/s\\H_1=74{,}55m\end{cases}$$

<u>Point 2 :</u> $N_2=1800\text{rpm}$

$$\begin{cases}q_{V_2}=3{,}0463\,\ell/s\\H_2=27{,}20m\end{cases}$$

For point 2 : $N_2=1800\text{rpm}$

$$\eta(q_{V_2})=29{,}643q_{V_2}-3{,}2143q_{V_2}^2=29{,}643(3{,}0463)-3{,}2143(3{,}0463)^2$$

$$\rightarrow\boxed{\eta_2=\eta(q_{V_2})=60{,}47\%=\eta_1}$$

For point 1 : $N_1=2980\text{rpm}$

$$\begin{cases}q_{V_2}=\dfrac{N_2}{N_1}q_{V_1}\\N_2=1800\text{rpm}\rightarrow\eta_2=29{,}643q_{V_2}-3{,}2143q_{V_2}^2\end{cases}$$

$$\eta_1=29{,}643\frac{N_2}{N_1}q_{V_1}-3{,}2143\left(\frac{N_2}{N_1}q_{V_1}\right)^2\Rightarrow\eta_1=17{,}9052q_{V_1}-1{,}1727q_{V_1}^2$$

$$\eta_1(q_{V_1})=17{,}9052(5{,}0434)-1{,}1727(5{,}0434)^2\rightarrow\boxed{\eta_1=60{,}47\%}$$

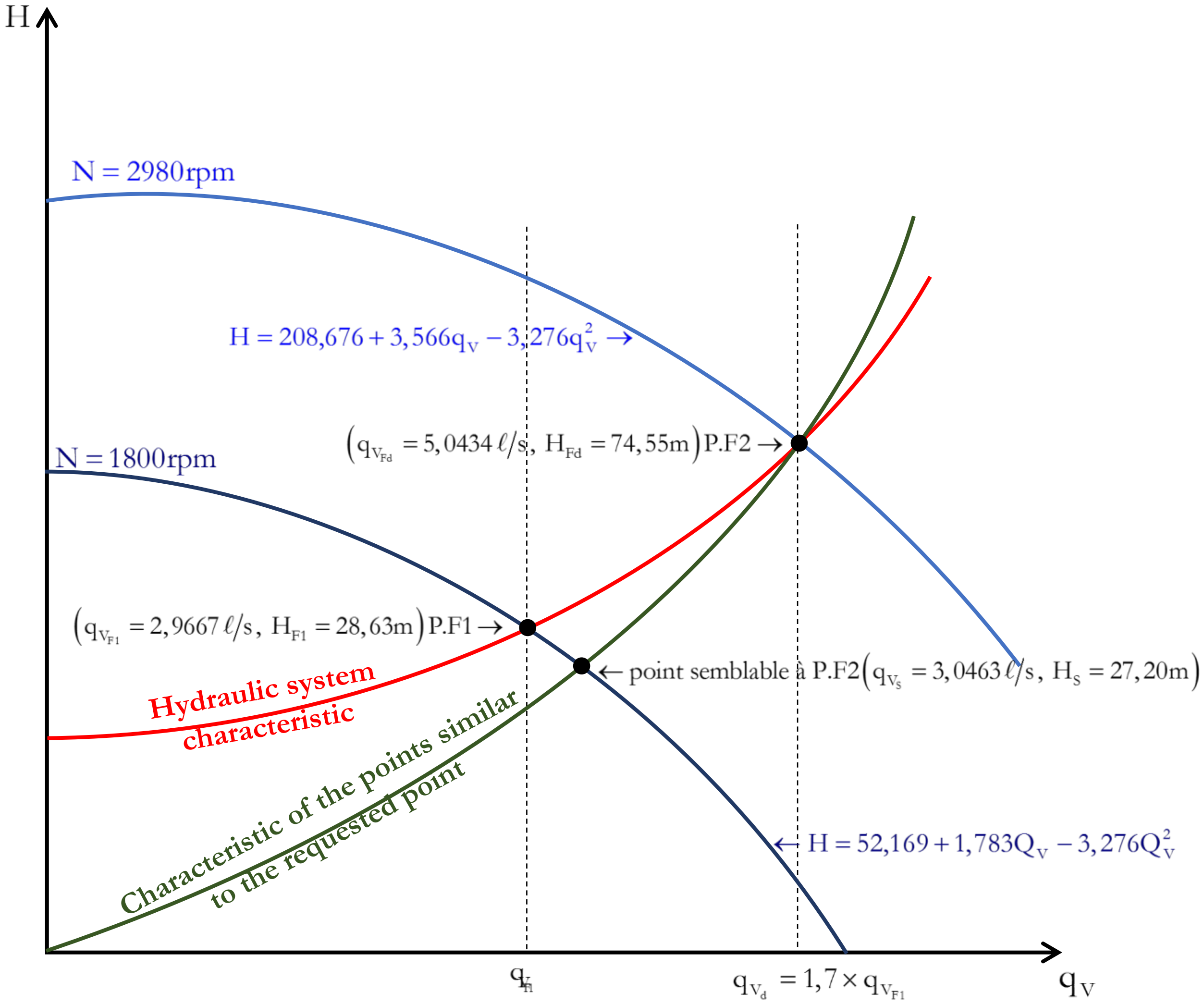
- Power absorbed at the requested point:

$$\begin{cases}q_{V_d}=5{,}0434\,\ell/s\\H_d=74{,}55m\end{cases}\rightarrow P_{a_d}=\frac{\rho gH_dq_{V_d}}{\eta}=6099{,}582W$$

- Power absorbed at the point similar to the requested point

$$\begin{cases} q_{V_s} = 3,0463\,\ell/s \\ H_s = 27,20\text{m} \end{cases} \rightarrow P_{a_s} = \frac{\rho g H_s q_{V_s}}{\eta} = 1344,221\text{W}$$

The results are quite logical, as an increase in the rotation speed of $N = 1800\text{rpm} \rightarrow N = 2980\text{rpm}$ leads to an increase in the absorbed power.



Chapter III

Pumps

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1. Introduction

Exploring the characteristics of liquid flow within a pump impeller primarily relies on an experimental approach. This approach confirms that the trajectories of particles in a real liquid are similar to those of an ideal liquid. To analyze the flows within the channels of pump impellers and guide vanes analytically, we assume an infinite number of thin and identical vanes. As a result, each liquid particle acquires the same amount of energy. This methodology enhances our comprehension of fluid dynamics within these intricate systems.

2. Pompes centrifuges

a. Construction d'une pompe centrifuge

Figure 1 illustrates a schematic layout of a centrifugal pump installation. The central element of the centrifugal pump is its impeller, complete with blades, enclosed within a casing and mounted on the shaft. The impeller and the shaft are set in motion by the motor.

When the motor drives the shaft and, in turn, the pump's impeller, the impeller imparts a centrifugal force to the water held between its blades. Consequently, the water is compelled to move radially outward, creating a vacuum, suction, or "pull" at the center, known as the eye of the impeller. This action results in the drawing of water from the suction pipe into the impeller, thus maintaining the pumping process.

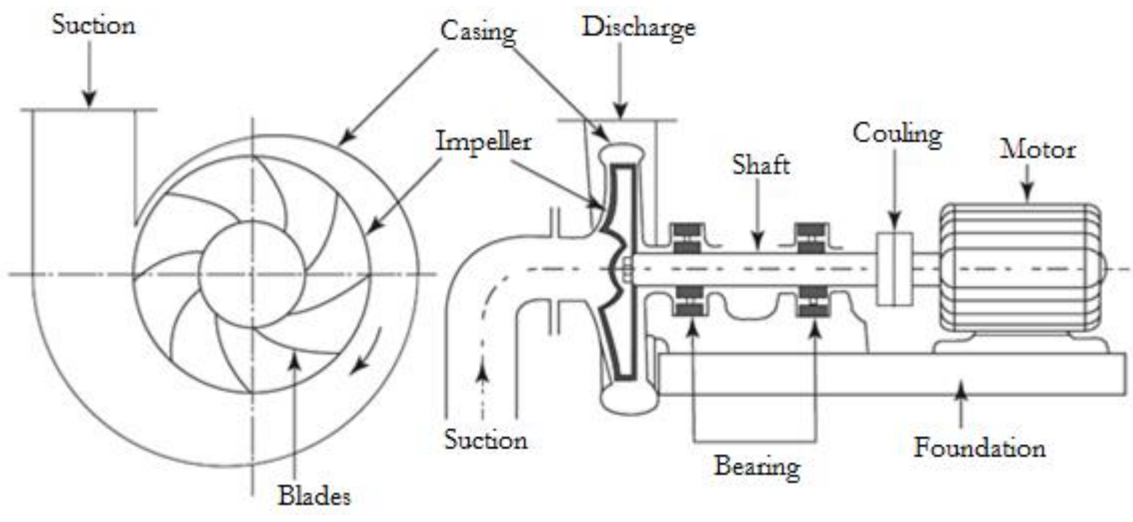


Figure 1. Schéma de principe d'une pompe centrifuge

As the pumping process continues, water enters the spiral casing surrounding the periphery of the impeller. The spiral casing has a continuously increasing cross-sectional area. This increase in area can serve the dual purpose of accommodating the incremental amount of water coming from the impeller's periphery and further diffusing the velocity of the water, thereby increasing its pressure.

b. Centrifugal pump operation and energy transfer mechanism

In Figure 2, the fluid is drawn in through an inlet conduit, which is typically convergent to ensure a uniform distribution of velocities at the entrance to the mobile channels, also known as the pump's intake. The area between points B_1 and C exhibits a steady flow with respect to a fixed reference, known as the suction side.

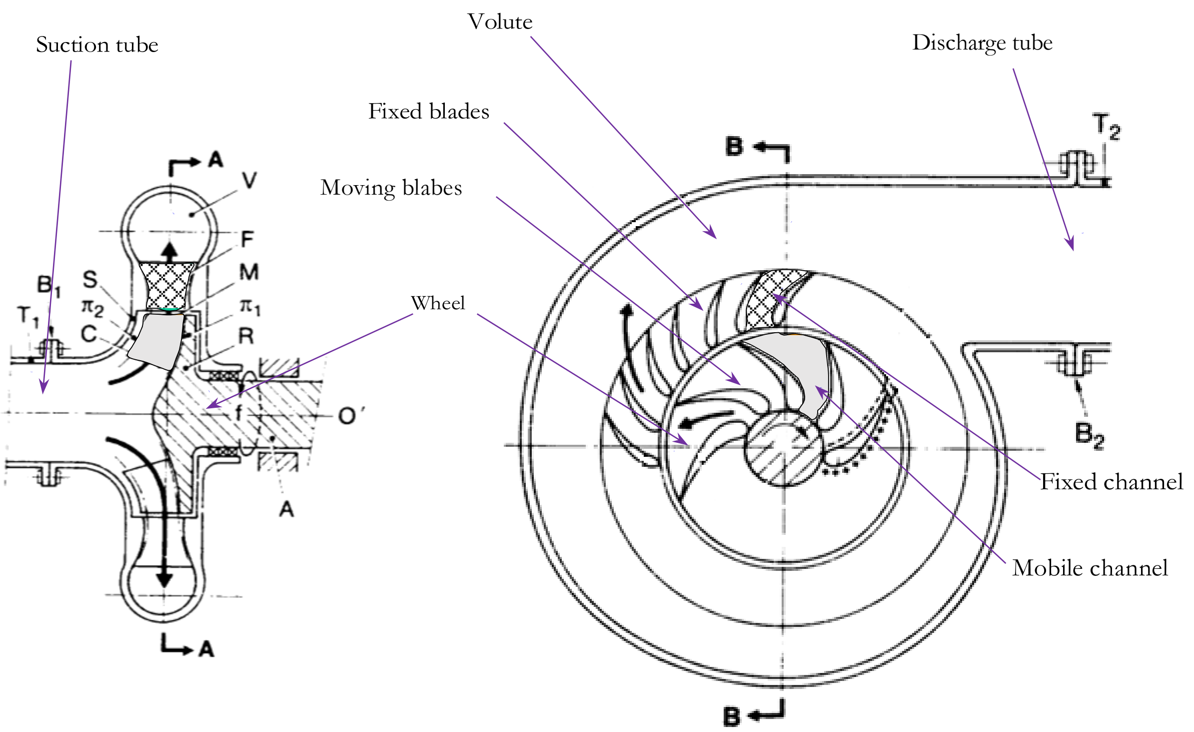


Figure 2. Internal components of a centrifugal turbopump

Due to the motion imposed by the drive shaft, with a constant angular velocity, the blades exert pressure forces on the fluid, creating an overpressure along their extrados and an underpressure on their intrados (Figure 3). To sustain this motion, continuous mechanical energy must be supplied by the shaft A, which is driven by a motor. This is the fundamental principle of operation of a generating turbomachine, where there is a transfer of energy between the shaft and the fluid. The mechanical energy provided by the shaft to the fluid as it passes through the mobile channels results in an increase in fluid pressure on one hand and its kinetic energy on the other hand.

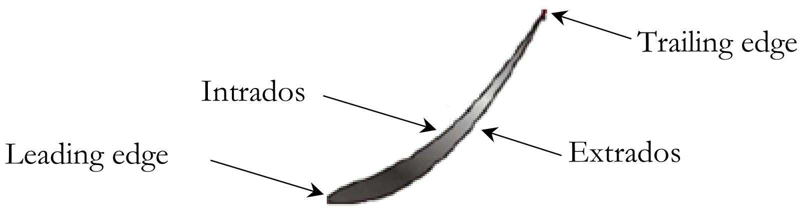


Figure 3. Representation of a blade

At the exit of the fixed blades F, the fluid must be collected and directed towards the pipeline T_2. This function is performed by the volute V, and to some extent, this space is used to further convert the fluid's kinetic energy into piezometric energy.

c. Idealized flow concept : Infinite number of blades

An analysis of the midline is carried out in a plane perpendicular to the rotor shaft, initially taking into account an infinite number of infinitely slender blades.

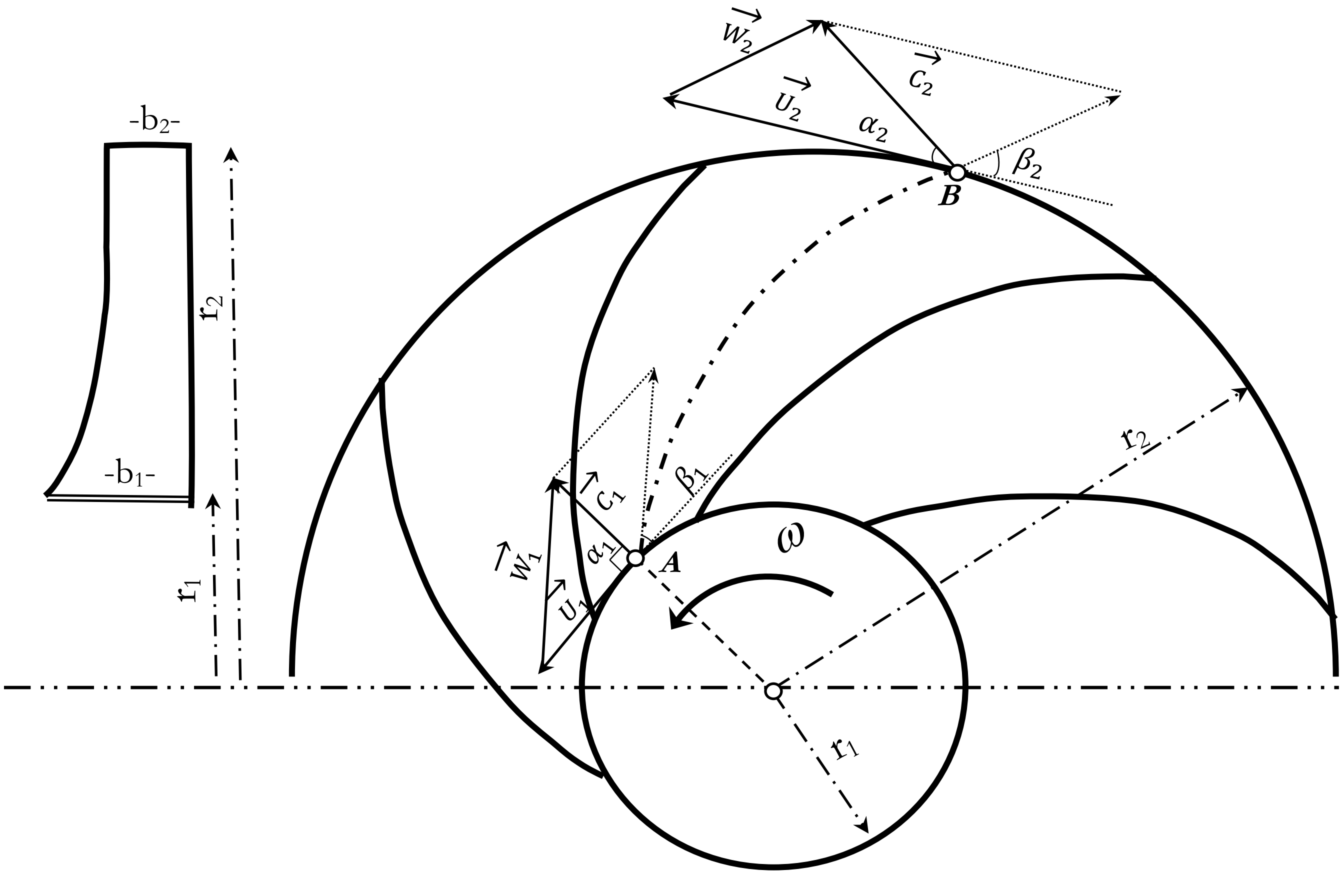


Figure 4. Flow through a centrifugal pump impeller

Figure 4 represents a meridional section and an orthogonal section of a rotor. The term "infinite number of blades" implies that the flow is expected to perfectly follow the blade's geometry. This would indeed be achieved with an infinite number of infinitely thin blades. The relative velocity would then be tangent to the blades at every point of the flow. There is no incident flow at the rotor's inlet, and there is no friction between the fluid and the blades. An incident-free inlet means that the flow direction coincides with the blade orientation just upstream of the inlet. In a general case, the absolute flow direction at the inlet is not perfectly perpendicular to the inlet circle and may exhibit a certain level of pre-swirl velocity $\left(C_{U1}\right)$. Nevertheless, in the case of most machines, pre-swirl is negligible or non-existent. Much like our assumptions for the rotor, we consider an incidence-free inlet for the volute and assume a frictionless environment within the volute. The flow-related quantities, as described above, are indicated with subscripts "th" (theoretical flow, representing ideal conditions with no losses) and "∞" (representing conditions assuming an infinite number of blades).

d. Performance characteristics with idealized flow : General relationship

Let's examine the impeller, as illustrated in Figure 4, with an inlet radius of $\left(r_1\right)$ and an outlet radius of $\left(r_2\right)$, along with respective widths (thicknesses) $\left(b_1\right)$ and $\left(b_2\right)$. This impeller rotates at a constant angular velocity. According to the idealized flow model, the movement of the liquid inside the impeller's channel can be conceptualized as the displacement of liquid particles, with flow concentrated along the midline (AB) of the channel. This theoretical approach to flow is known as the one-dimensional theory of turbopumps. In this scenario, liquid particles enter the impeller through a cylindrical surface $\left(S_1 = 2\pi r_1 b_1\right)$ with a radius of $\left(r_1\right)$ and exit through a cylindrical surface $\left(S_2 = 2\pi r_2 b_2\right)$ with a radius of $\left(r_2\right)$.

The angles $\left(\alpha_1\right)$ and $\left(\alpha_2\right)$ are referred to as the angles of absolute velocities, while the angles $\left(\beta_1\right)$ and $\left(\beta_2\right)$, which are respectively formed by the vectors $\left(\overrightarrow{W_1}\right)$ and $\left(-\overrightarrow{U_1}\right)$, and $\left(\overrightarrow{W_2}\right)$ and $\left(-\overrightarrow{U_2}\right)$, are known as the angles of relative velocities at the inlet and outlet of the impeller. These relative angles serve as essential parameters in characterizing the properties of the blades.

The Euler equation adapted to the impeller of a generating turbomachine allows us to write:

$$P = M\omega = \gamma q_v H_{th_\infty} = \rho g q_v H_{th_\infty} = Q_m \tau \qquad (I.1)$$

$$H_{th_\infty} = \frac{U_2 C_{U_2} - U_1 C_{U_1}}{g}$$
$$H_{th_\infty} = \frac{C_2^2 - C_1^2}{2g} + \frac{U_2^2 - U_1^2}{2g} + \frac{W_1^2 - W_2^2}{2g} \quad (I.2)$$

This equation is known as the "Euler's Theoretical Head with an Infinite Number of Blades."

The velocity triangles for a radial flow impeller at the inlet of a centrifugal pump wheel are illustrated in Figure 5. The absolute velocity $\left(\overrightarrow{C_1}\right)$ at the inlet is inclined at an angle $(\alpha_1 = 90°)$ with respect to the peripheral velocity $\left(\overrightarrow{U_1}\right)$. This result is ensured by the presence of guide vanes that direct the fluid into the impeller at a specific angle, thereby ensuring that no pre-swirl velocity (C_{U1}) is created in the turbopump.

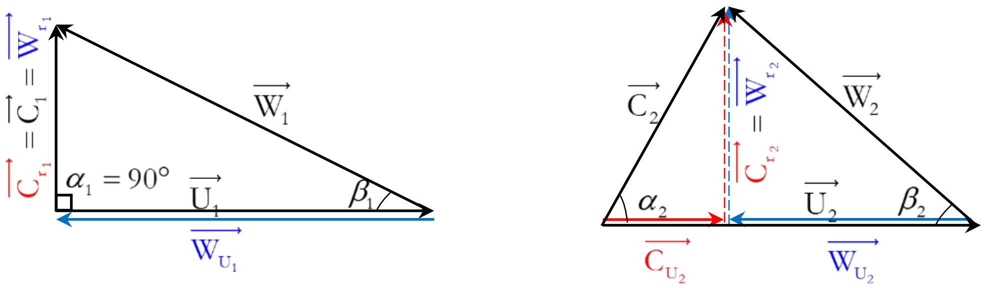


Figure 5. Velocity triangles in a purely radial inlet centrifugal pump impeller

Under such conditions, the theoretical Euler head developed by the pump with an infinite number of blades can be expressed as follows :

$$H_{th_\infty} = \frac{U_2 C_{U_2}}{g} \quad (I.3)$$

$$\left(C_{U_2} = U_2 - W_{U_2}\right), \left(\tan\beta_2 = C_{r_2} / W_{U_2}\right) \text{ et } \left(C_{r_2} = q_v / 2\pi r_2 b_2\right)$$

So, the expression for the theoretical Euler head for an infinite number of blades is as follows:

$$H_{th_\infty} = \frac{U_2^2}{g} - \frac{U_2}{g} \frac{q_v}{2\pi r_2 b_2 \tan\beta_2} \quad (I.4)$$

The previous expression represents a linear relationship and can take on three different orientations depending on the value of the angle (β_2), as illustrated in Figure 6 :

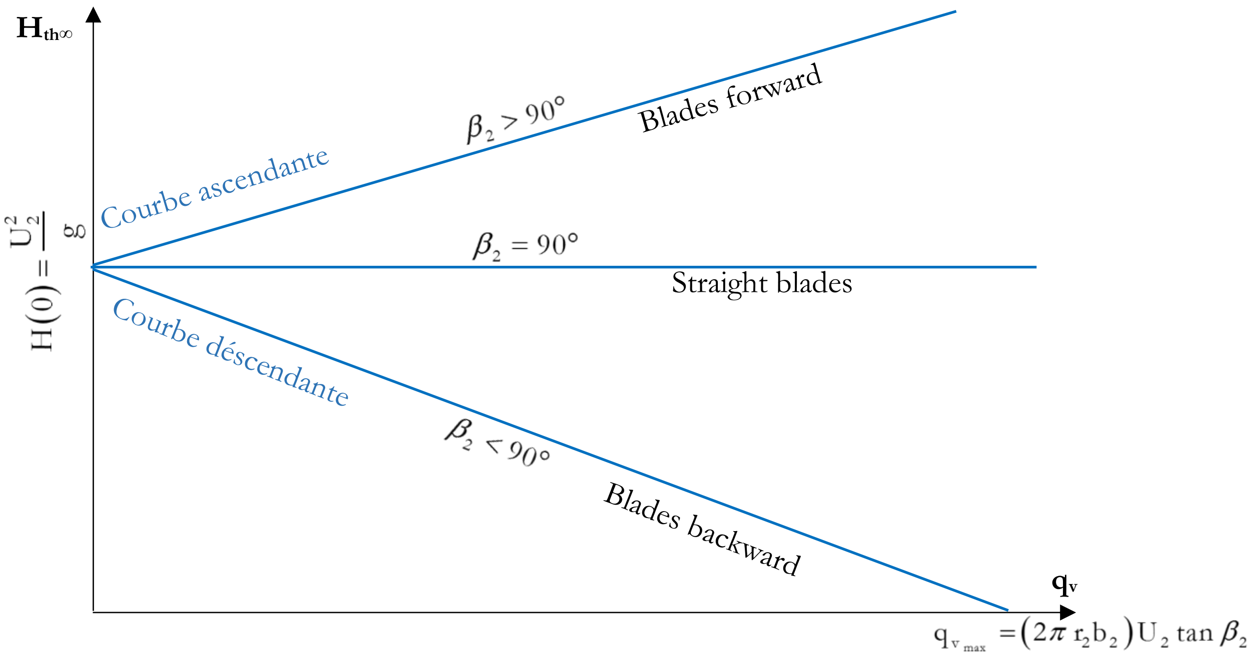
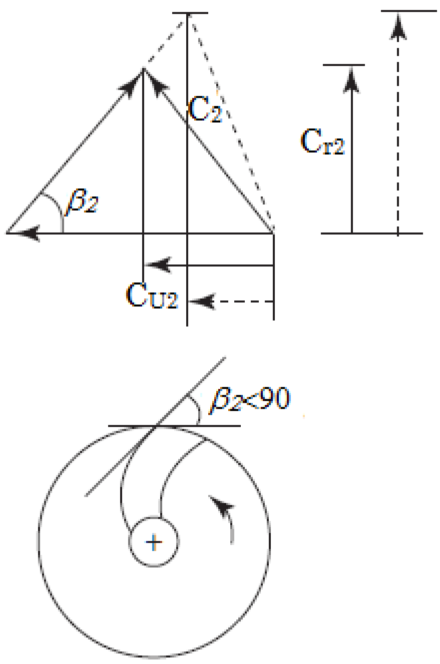
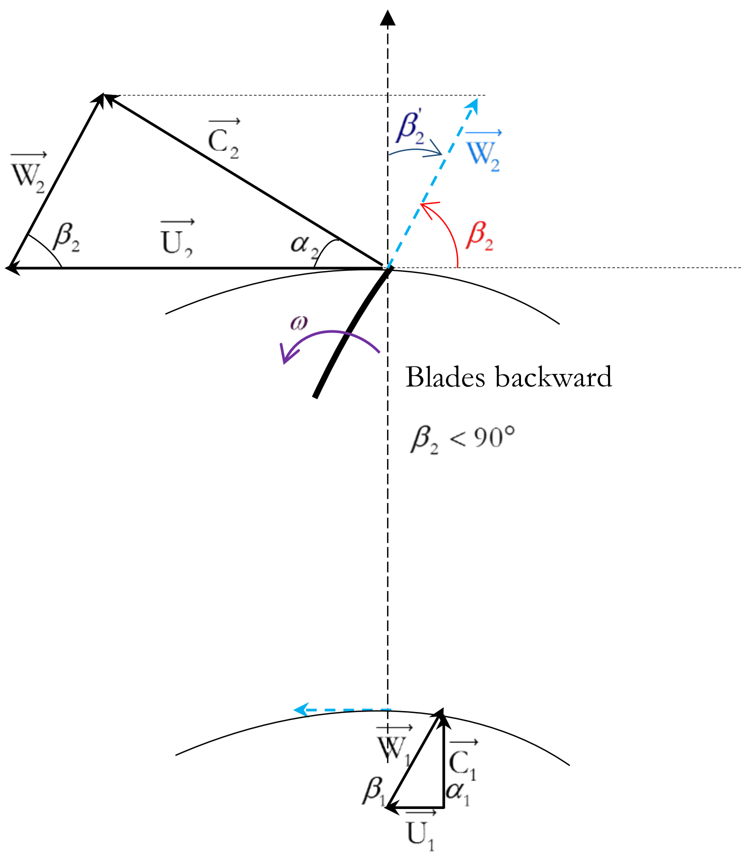


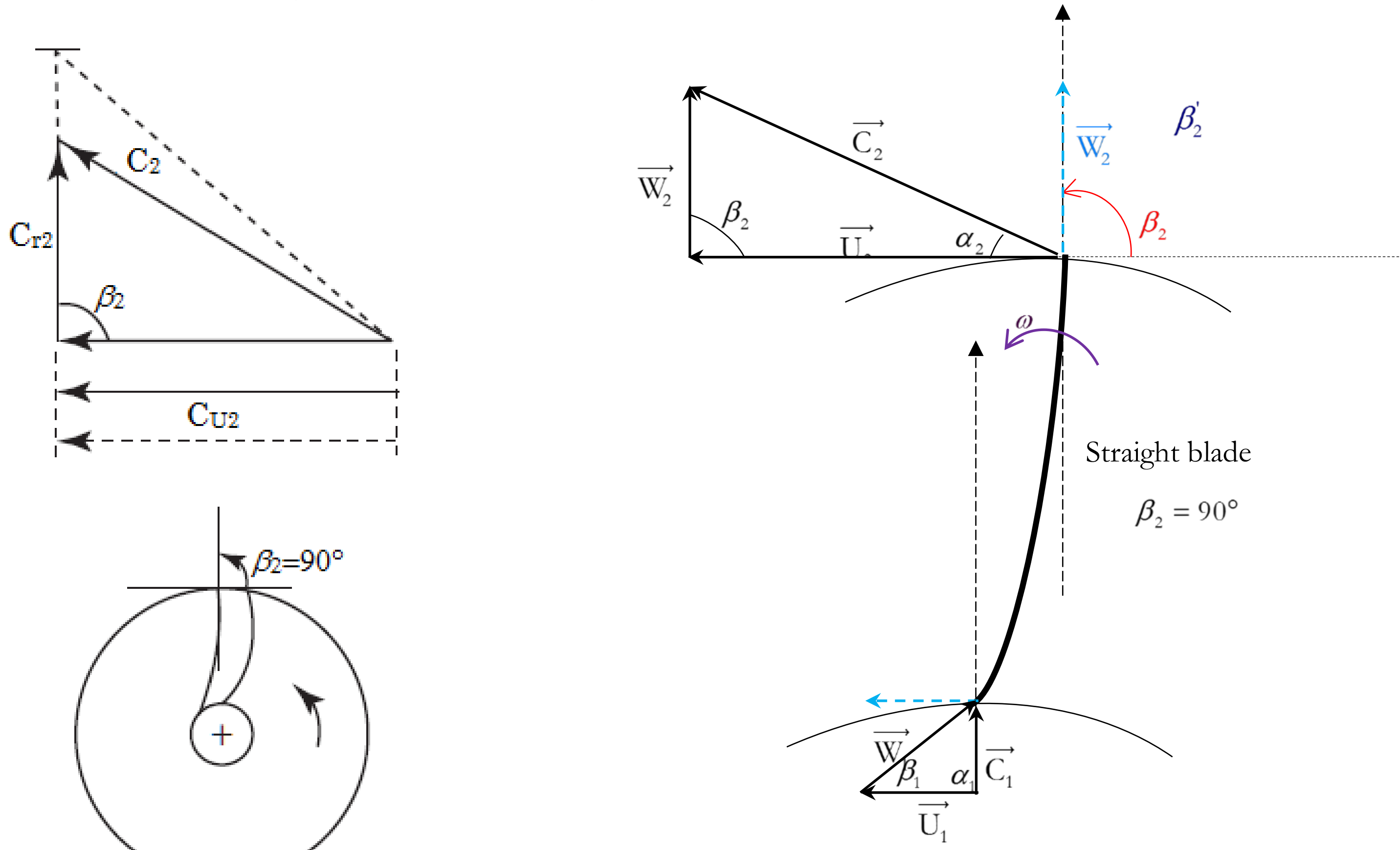
Figure 6. Variation of the theoretical Euler head with flow rate as a function of the angle (β_2)

Case 1 : $\beta_2 < 90° \rightarrow \left(q_v \nearrow \Rightarrow C_{U_2} \searrow \Rightarrow H_{th_\infty} \searrow\right)$

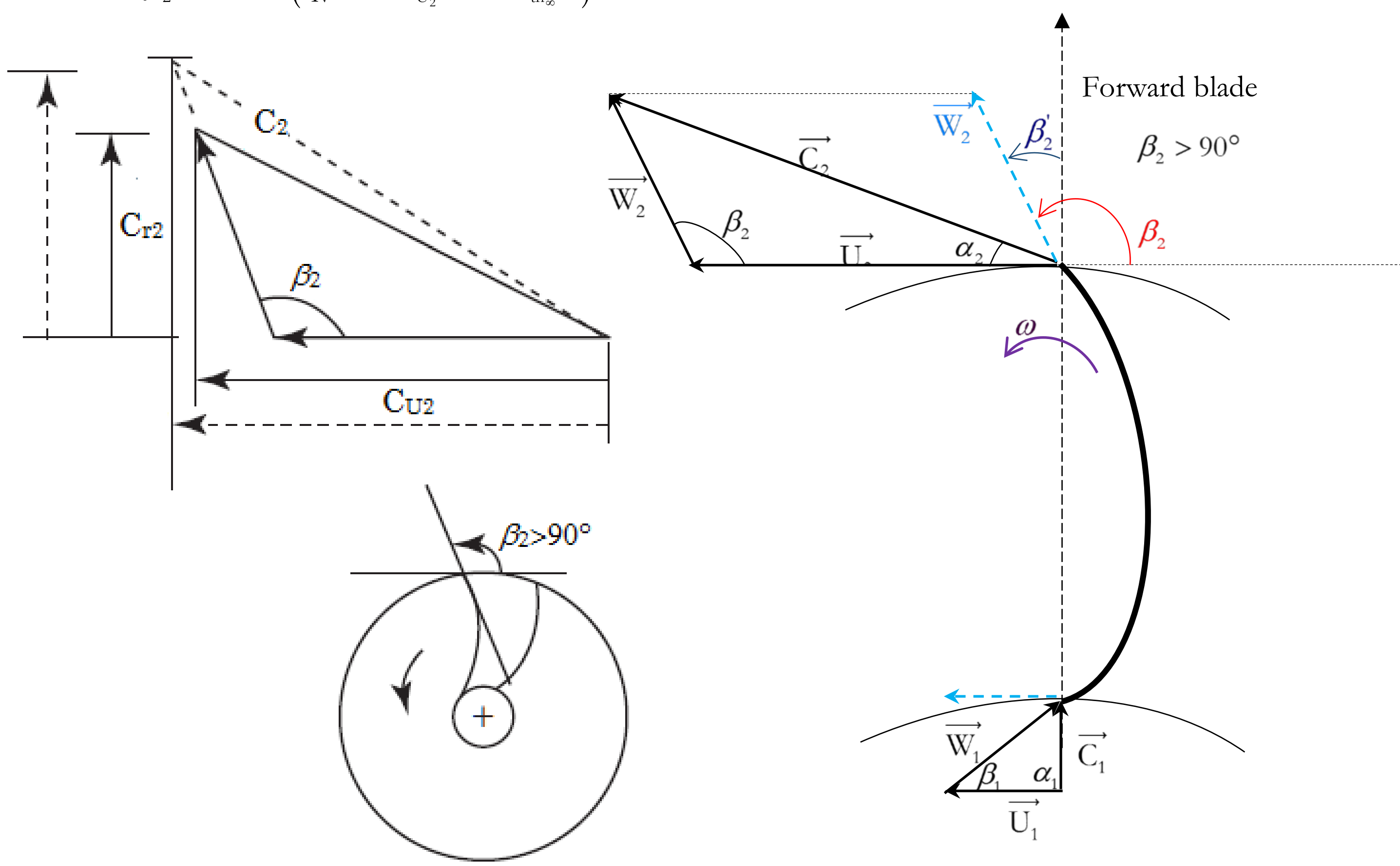




<u>Case 2 :</u> $\beta_2 = 90° \rightarrow \left(q_v{}^{\nearrow} \Rightarrow C_{U_2}{}^{Cst} \Rightarrow H_{th_\infty}{}^{Cst}\right)$



<u>Case 3 :</u> $\beta_2 > 90° \rightarrow \left(q_v{}^{\nearrow} \Rightarrow C_{U_2}{}^{\nearrow} \Rightarrow H_{th_\infty}{}^{\nearrow}\right)$



e. Transition to a finite number of blades: Slip and slip factor or slip coefficient.

Considering that the real rotor of a centrifugal pump is equipped with a specific number of blades, the flow between these blades deviates from our original idealized assumption. Consequently, this deviation causes an uneven distribution of velocities in the cross-sectional areas of the flow. A closer examination reveals distinct differences: on the extrados of the blades, the pressure is noticeably higher, while the velocity is comparatively lower than on the intrados, where the pressure is lower, and the velocity is higher. These variations in pressure and velocity manifest in the velocity distribution depicted in Figure 7.

Due to these combined effects, with a constant discharge flow rate, the swirl component of the outlet velocity (C_{U2}) is reduced, resulting in a modification of the velocity triangles at the rotor outlet (Figure 8). As a result, the magnitude of the energy transfer (Euler head) is reduced.

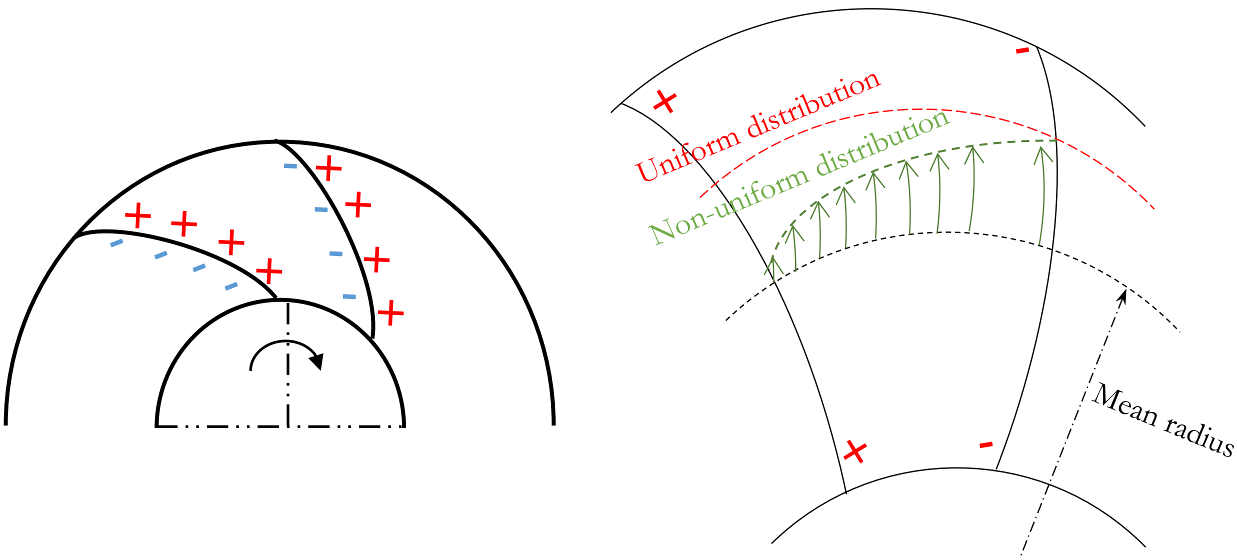


Figure 7. Flow in the inter-blade channel

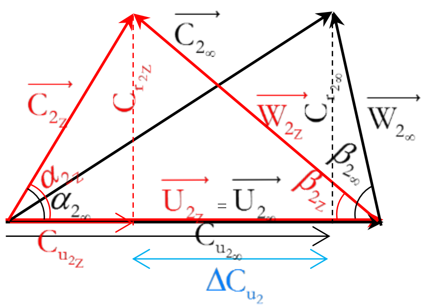


Figure 8. Modification of the velocity triangles due to the reduction in swirl velocity. (C_{U2})

This velocity triangle plot is obtained using the same velocity (U_2) and the same discharge velocity (C_{r_2}), which means at the same rotation speed (N) and the same flow rate (q_v).

This overall reduction in the head calculated by the Euler equation is called slip. This slip is explained by the "slip factor" or "slip coefficient" (represented as μ), which can be expressed as the ratio of $\left(\frac{H_{th_Z}}{H_{th_\infty}}\right)$, where :

$H_{th_\infty} = \frac{U_2 C_{U_{2\infty}}}{g} \rightarrow$ Theoretical head with an infinite number of blades

$H_{th_Z} = \frac{U_2 C_{u_{2Z}}}{g} \rightarrow$ Theoretical head with a finite number of blades Z

Slip is not a loss; it simply reflects the actual performance of an impeller based on its geometric characteristics. Therefore, its value is influenced by several factors specific to the design of the turbomachine's impeller.

In the absence of the slip factor (μ) provided by the manufacturer, there are semi-empirical formulas available in the literature to calculate it. As an example, we can mention the STODOLA formula :

$$\mu = 1 - \frac{\pi \sin(\beta_2)}{Z\left(1 - \frac{Cr_2}{U_2}\cot(\beta_2)\right)} = 1 - \frac{\pi \sin(\beta_2)}{Z\left(1 - \frac{Cr_2}{U_2}\frac{\cos(\beta_2)}{\sin(\beta_2)}\right)} \quad \text{(I.5)}$$

f. Losses in the blade passages of the impeller

The losses that occur when the fluid flows through the passages between the rotor blades are analogous to those observed in fluid mechanics. They result from various phenomena, including wall friction, turbulence, vortices, viscous resistance, and so on, all contributing to the reduction in pump head. These losses can be categorized into two categories :

1. The first category is attributed to friction on the blades due to the viscosity of the fluid. These frictions may, in terms of their characteristics, resemble linear head losses within the impeller $\left(\Delta h_{fric} = k_{fric} \cdot q_v^2\right)$. Consequently, by their nature, these losses are proportional to the square of the flow velocity, the surface roughness, and the passage length.

2. The second category of losses is due to the deviation of the flow from the ideal flow around the blades. This occurs when the flow rate varies (increases or decreases) from the design conditions. Off-design flow rates result in non-conforming flow velocities or relative velocities, leading to the occurrence of fluid shocks on the blades. These losses are referred to as "rotational losses" and are akin to singular head losses; they are proportional to the square of the flow rates and become zero at the design flow rate $\left(q_v = q_{v_a}\right)$, where the shock loss takes the form of a step function $\Delta h_{shock} = k_{shock} \cdot \left(q_v - q_{v_a}\right)^2$.

Figure 9 provides a visual representation of these two loss categories, whose cumulative effect is commonly referred to as "overall hydraulic losses."

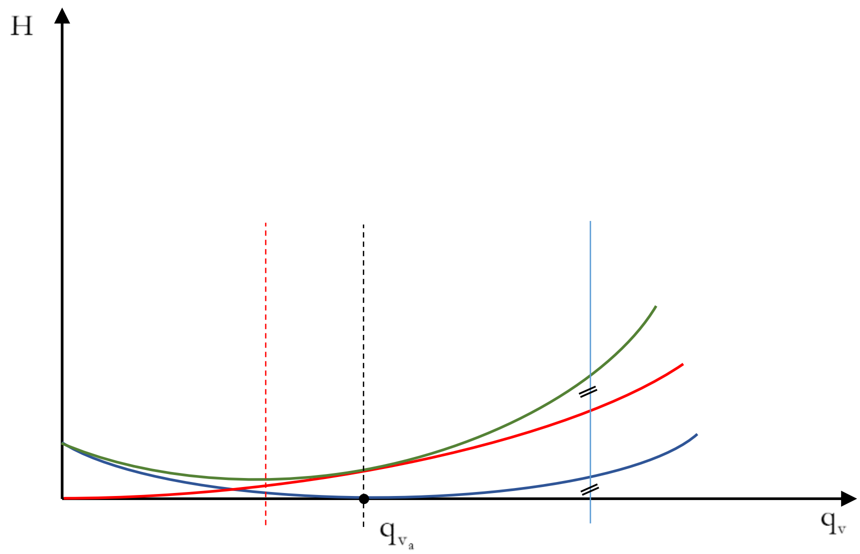


Figure 9. Losses in the blade passages of centrifugal pumps.

g. Characteristic curves / Head-Flow relationship.

One of the fundamental characteristics of a turbopump lies in the graphical representation of its actual head (or actual specific work) as a function of fluid flow rate. This characteristic is commonly referred to as the Head-Flow curve and plays a crucial role in the analysis and understanding of the pump's behavior. The Head-Flow curve results from both ideal energy transfer and energy losses within the machines.

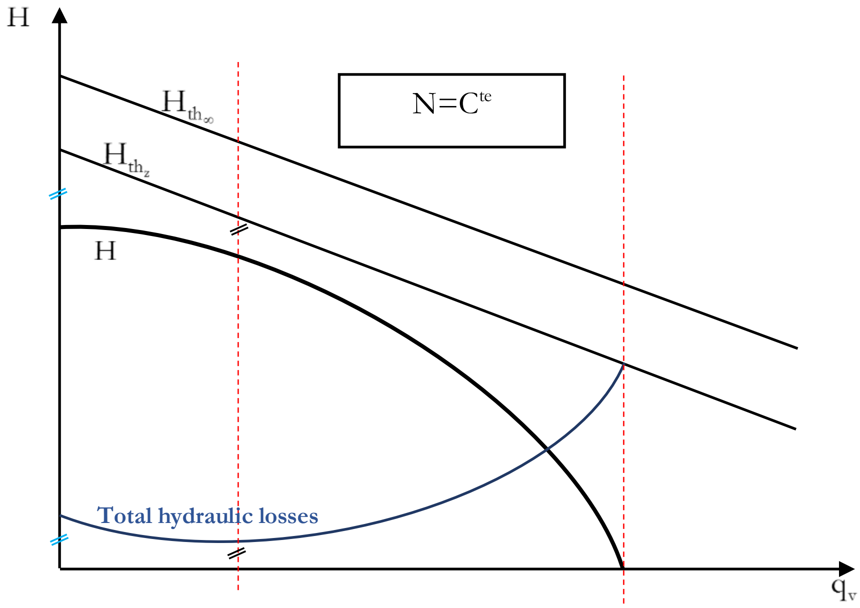


Figure 10. Head-Flow relationship.

Therefore, in the specific case of backward-curved blades $(\beta_2 < 90°)$, as illustrated in Figure 10, the performance curves (H_{th_∞}) and (H_{th_Z}) are deduced from each other using the slip factor coefficient (μ). By subtracting the total hydraulic losses from the theoretical head (H_{th_Z}), we obtain the third curve representing the actual head of the pump (H).

3. Axial pumps

An axial flow pump consists of a wheel with propeller-type blades rotating inside a casing, with meticulously adjusted clearances between the blades of the wheel and the internal walls of the casing. The fluid primarily flows in an axial direction through the movable blades of the rotor and the fixed blades of the stator. This configuration is repeated for multi-stage pumps, effectively increasing the fluid pressure.

The pump's design is optimized to guide the fluid so that it enters the rotor with a purely axial velocity. However, the impeller blades introduce a swirling component to the fluid. The stator's outlet guide vanes are specially designed to eliminate this turbulence on the outlet side and redirect the flow towards the exit.

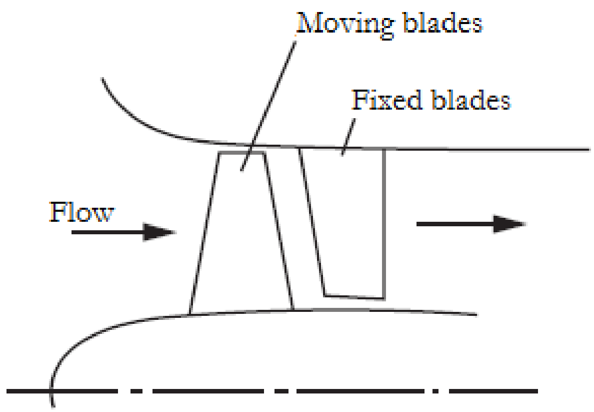


Figure 11. Axial pump configuration

a. Velocity triangles

The velocity triangles at the inlet and outlet without pre-swirl velocity (C_{U1}) are drawn for an axial flow pump (as shown in Figure 12). They are constructed using the same principle as that used for centrifugal pumps, taking into account three essential components of the pump: the impeller configuration, the inlet and outlet sections, and the absolute fluid velocity.

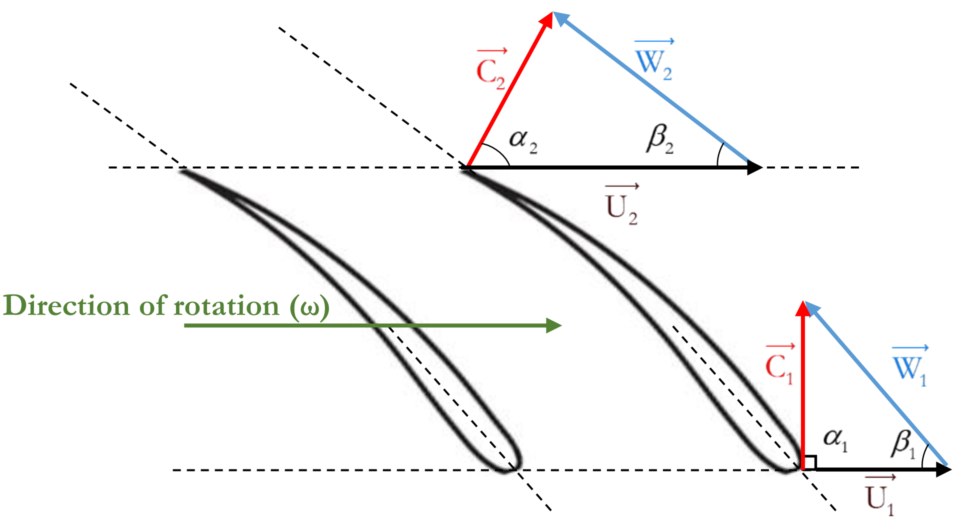


Figure 12. Principle of constructing velocity triangles for axial flow pumps $(\beta_2 < 90°)$

However, this configuration must adapt by considering the essential characteristic of axial flow pumps, which can be summarized by the axial direction of the flow, maintaining the swirling flow around the same radius between the inlet and outlet of the impeller. This feature results in the conservation of peripheral velocity $(U = U_1 = U_2)$, ultimately allowing the construction of a velocity diagram by superimposing the two triangles at the entrance and exit of the impeller, as shown in Figure 13. In this situation, the axial components (C_{a1}) and (C_{a2}) become the flow components, while the radial components (C_{r1}) and (C_{r2}) do not exist.

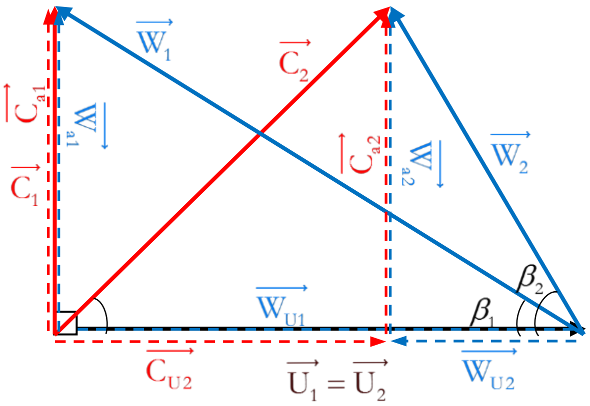


Figure 13. Diagram of velocity triangles for a purely axial inlet in an axial flow pump impeller. $(\alpha_1 = 90°)$

b. General characteristics

By revisiting the fundamentals of the theoretical Euler head, as defined by equation (I.2), and considering the specificities of the flow in the inter-blade space of the axial flow pump impeller, we can arrive at the following equation :

$$H_{th_\infty} = \frac{U\left(C_{U_2} - C_{U_1}\right)}{g}$$
$$H_{th_\infty} = \frac{C_2^2 - C_1^2}{2g} + \frac{W_1^2 - W_2^2}{2g} \tag{I.6}$$

In accordance with the velocity triangles diagram and given this scenario, liquid particles enter and exit the impeller through an annular surface defined by the size of the blades, extending from the hub radius (r_b) to the blade tip radius (r_ℓ), as shown in Figure 14. This allows us to write:

$$H_{th_\infty} = \frac{UC_{U_2}}{g}$$

with,

$$\begin{cases} C_{U_2} = U_2 - W_{U_2} \\ \tan\beta_2 = C_{a_2}/W_{U_2} \\ C_{a_2} = q_v/S \\ S = S_1 = S_2 = \pi\left(r_\ell^2 - r_b^2\right) \end{cases}$$

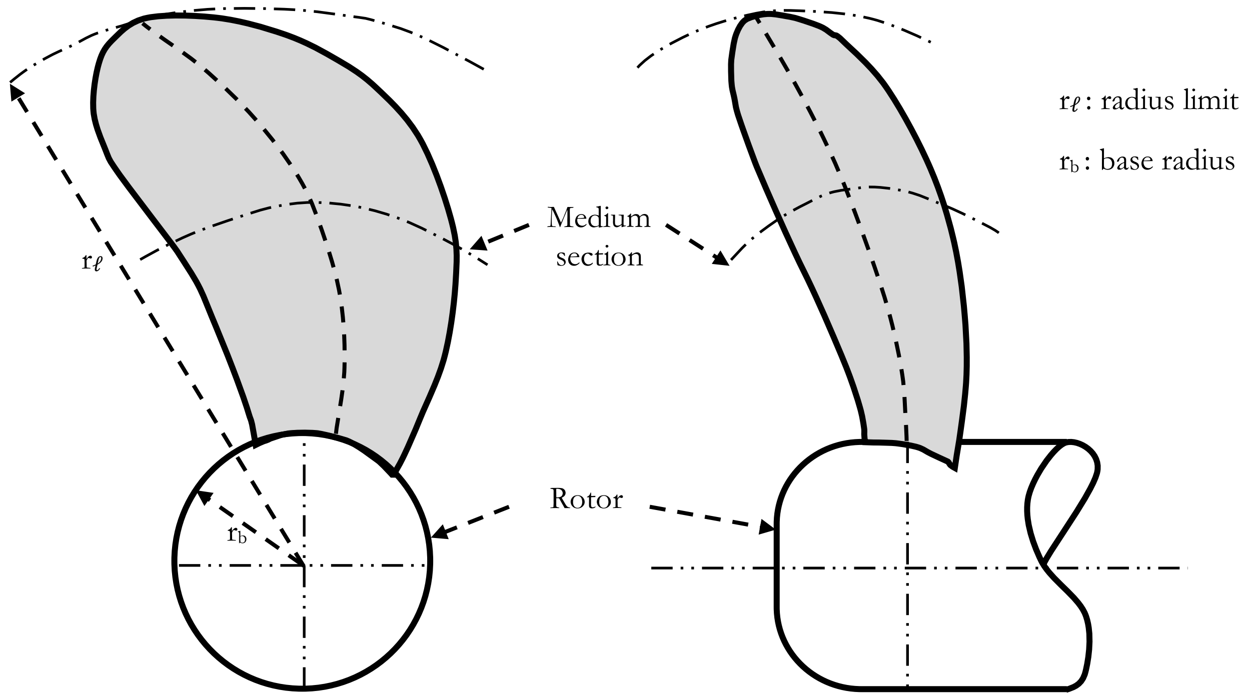


Figure 14. General shape of an axial pump blade

In this case, the expression of the theoretical Euler head for an infinite number of blades, as applied to an axial flow pump, can be formulated as follows :

$$H_{th_\infty} = \frac{U^2}{g} - \frac{U}{g}\frac{q_v}{\pi\left(r_\ell^2 - r_b^2\right)\tan\beta_2} \tag{I.7}$$

The analysis of the behavior of this characteristic resembles the study of equation (I.4), which leads us to scenarios and conclusions that share many similarities.

4. Efficiencies

The hydraulic power transmitted to the fluid by the pump, also known as fluid power, represents the mechanical energy converted into kinetic or potential energy in the circulating fluid. This hydraulic power is a crucial parameter for assessing the performance of a pump because it measures the pump's ability to provide energy to the fluid, thereby enabling the movement of the fluid through a specific system or application. It is given by :

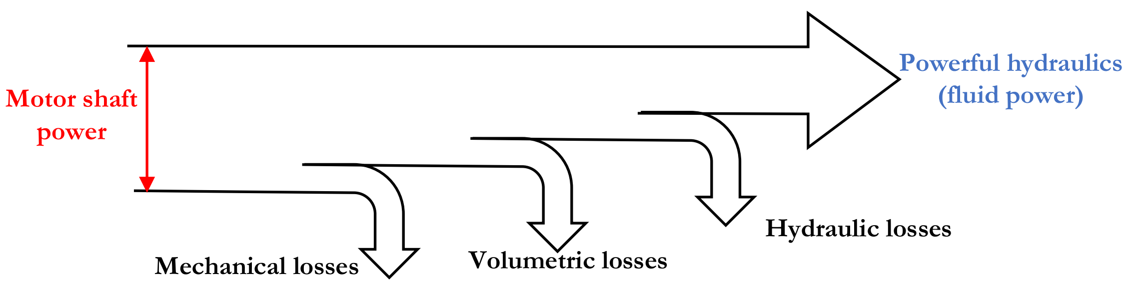
$$P_f = \rho g H q_v \tag{I.8}$$

The pump is driven by a motor whose power available on the shaft (P_a) exceeds that transferred to the fluid by the pump (P_f). It is important to note that this excess power on the motor shaft is necessary to compensate for these losses and ensure that the pump can provide the required hydraulic power to move the fluid through the system with the desired efficiency. Therefore, the power available on the motor shaft must be calculated, taking into account all these losses, to ensure that the pump operates optimally in a given context. The overall efficiency of the pump is then defined as the ratio between these two powers.

$$\eta_g = \frac{P_f}{P_a} \tag{I.9}$$

Pump losses are diverse and manifest at various levels :

- Mechanical losses : These losses are primarily related to friction, such as bearing friction and disk friction. Mechanical losses are mainly due to frictional forces that oppose the movement of the pump's moving mechanical parts.
- Volumetric losses : Losses of this type result from internal leaks within the pump. This occurs when the fluid can bypass or escape from pump components instead of following the intended path.
- Hydraulic losses : These losses stem from the behavior of the fluid itself within the pump. They include the viscous friction of the fluid along the internal walls of the pump, the head losses that occur in the inter-blade channels where the fluid flows between the impeller blades, as well as losses resulting from fluid shocks when it abruptly changes direction or speed.



Depending on the nature of each loss, different efficiencies can be defined to assess the overall efficiency of the pump in converting mechanical energy into hydraulic energy. This allows for a better understanding of its operation and overall performance.

a. Hydraulic efficiency (manometric)

Hydraulic efficiency can be defined as the ratio between the actual hydraulic power transferred to the fluid by the pump and the maximum theoretical hydraulic power that could be transferred in the absence of hydraulic losses. In other words, it quantifies the effectiveness with which the pump converts mechanical energy into useful hydraulic energy. Therefore, it is defined as :

$$\eta_{H} = \frac{H}{H_{th_Z}} = \frac{H}{H + \Delta h_{hyd}} = \frac{H_{th_Z} - \Delta h_{hyd}}{H_{th_Z}} = 1 - \frac{\Delta h_{tot}}{H_{th_Z}} \quad (I.10)$$

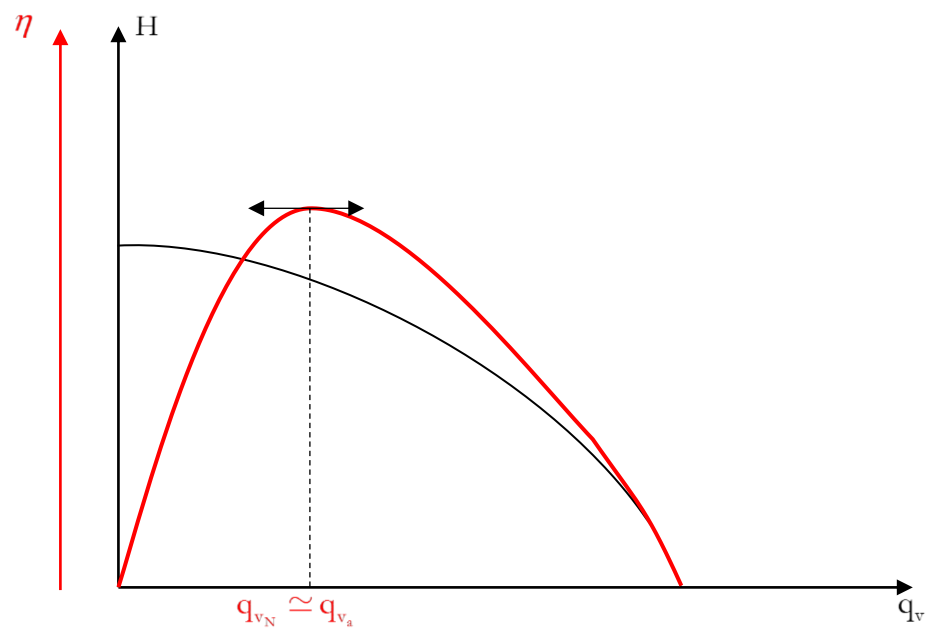


Figure 15. Hydraulic efficiency

A graph of this efficiency is presented in Figure 15. As can be observed, efficiency is highest where losses are minimized (see Figure 9). This point of maximum efficiency is commonly referred to as the "design point," and machine calibration is carried out taking this point into consideration.

b. Volumetric efficiency - losses due to leaks

Losses due to leaks occur because of the presence of flow fluid returning through the clearance between the pump impeller and the front shroud. This situation is caused by the pressure differential between the inlet and outlet of the pump impeller, as explicitly illustrated in Figure 16.

Volumetric efficiency is defined as follows :

$$\eta_v = \frac{q_v}{q_v + q_{v_f}} \tag{I.11}$$

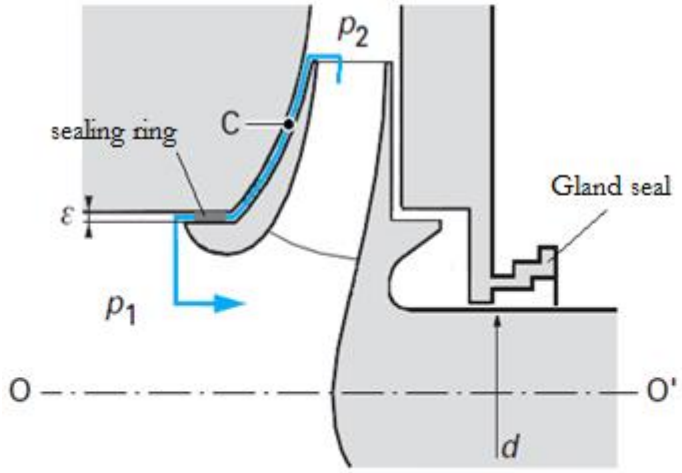


Figure 16. Losses due to leakage between the pump impeller and the front shroud.

c. Mechanical efficiency - mechanical losses

Given the machine's technology, it's important to note that the power supplied to the pump shaft by the drive motor is not entirely transferred to the pump impeller. Various elements contribute to the dissipation of a portion of this mechanical power in the form of heat.

The mechanical efficiency will then be defined as :

$$\eta_m = \frac{\text{Wheel power}}{\text{Power at the motor shaft}} = \frac{\text{Power at the motor shaft} - \sum \text{Mechanical losses}}{\text{Power at the motor shaft}}$$

$$\eta_m = \frac{P_a - P_m}{P_a} \tag{I.12}$$

In the end, by aggregating all of these losses, we achieve:

$$P_a = \rho g (H + \Delta h)(q_v + q_{v_f}) + P_m \tag{I.13}$$

Finally, based on all of the above, the overall efficiency can be expressed as follows :

$$\eta_g = \frac{P_f}{P_a} = \frac{\rho g H q_v}{\rho g (H + \Delta h)(q_v + q_{v_f}) + P_m}$$

$$\eta_g = \frac{\rho g H q_v}{\rho g (H + \Delta h) q_v} \frac{\rho g (H + \Delta h) q_v}{\rho g (H + \Delta h)(q_v + q_{v_f})} \frac{\rho g (H + \Delta h)(q_v + q_{v_f})}{\rho g (H + \Delta h)(q_v + q_{v_f}) + P_m}$$

$$\eta_g = \eta_H \cdot \eta_V \cdot \eta_m \quad (I.14)$$

5. Pumps assembly

Sometimes, the pressure or flow rate requirements cannot be met by a single pump. In such cases, it is advisable to reconsider the design by using multiple standard (or commercially available) pumping units. Two of the commonly used configurations to meet these requirements are pumps in series and pumps in parallel.

a. Pumps operating in parallel

In the case of multiple pumps operating in parallel, their inputs and outputs are interconnected. In this scenario, the net head of each pump remains constant at all times, and the resulting total flow rate is the sum of the partial flow rates from each pump. When examining the head-flow characteristic curve of the entire set of pumps in parallel, it is observed that this curve is obtained by adding the abscissas corresponding to the same ordinate. In other words, each pump in parallel independently contributes to increasing the total flow rate while maintaining the net head constant. This configuration offers valuable flexibility to meet specific flow rate requirements while maintaining the required pressure in the system.

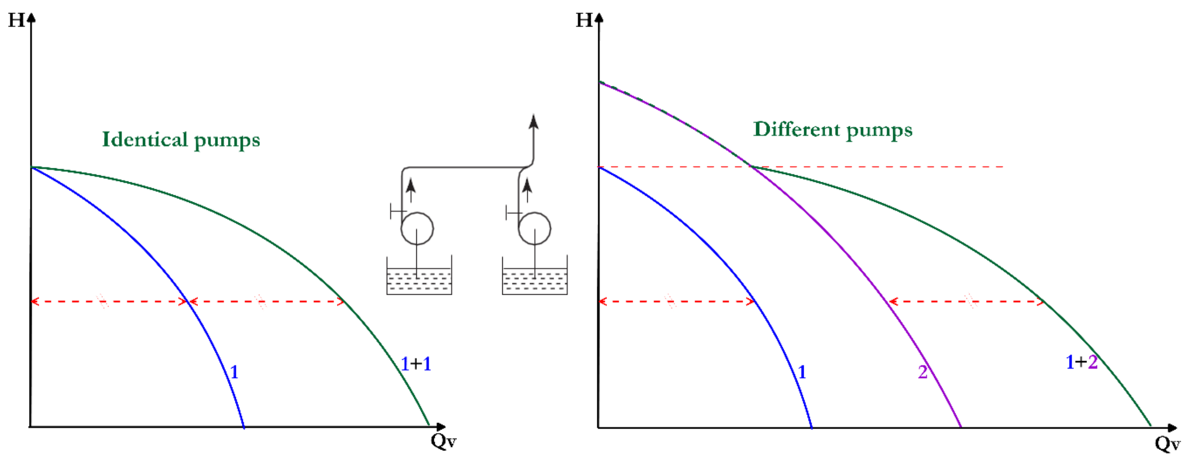


Figure 17. Pumps in parallel

b. Pumps operating in series

Two or more pumps are considered to be in series when the discharge outlet of one pump is connected to the suction inlet of the next pump. This configuration implies that all pumps in the series operate with the same flow rate, and the net heads of each pump add up. When examining the head-flow characteristic curve of the entire set of pumps in series, it is observed that this curve is obtained by adding the ordinates corresponding to the same abscissa. In other words, each pump in series contributes cumulatively to increasing the total head while maintaining a constant flow rate. This arrangement is commonly used in multistage pumps, where multiple pump stages are arranged in series to significantly increase the total pressure while maintaining a constant flow.

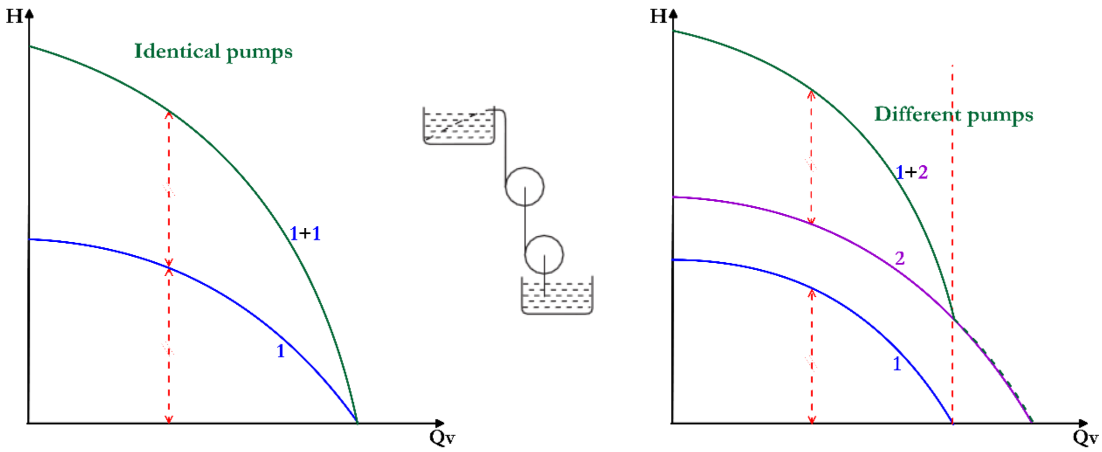


Figure 18. Pumps in series

6. Exercises

- **Exercise n°1 :**

The outer diameter of a centrifugal pump wheel is 1.3 meters, the peripheral tip speed of the wheel is 10 m/s, and the radial flow velocity at the wheel's exit is 1.6 m/s.

At the exit, the blade angle is 30 degrees relative to the tangent at the periphery of the wheel.

To achieve a flow rate of 3,5 m^3/min for a radial inlet, calculate the torque of the wheel

With : $\begin{cases} \rho = 1000 \text{ kg/m}^3 \\ H = H_{th\infty} \end{cases}$

Solution :

$D_2 = 1,3 m$ $\quad U_2 = 10 \, m/s$ $\quad q_v = 3,5 \, m^3/min$

$C_{r_2} = 1,6 \, m/s$ $\quad \beta_2 = 30°$ $\quad \rho = 1000 \, kg/m^3$

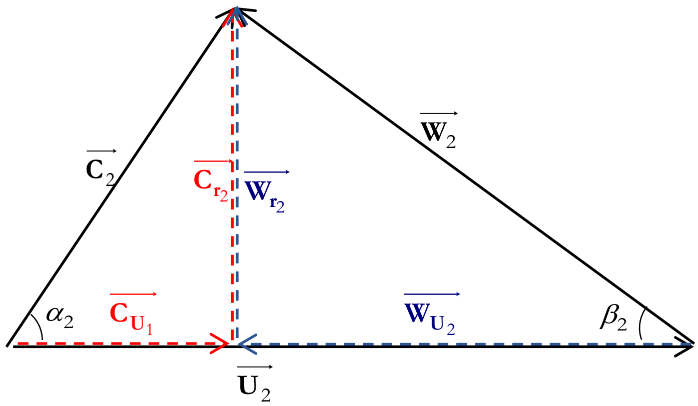
- Wheel torque (M) :

The power received by the fluid, in an ideal case (without friction, impact, or dissipation)

$$P = \omega \cdot M = \rho \cdot g \cdot H \cdot q_v$$

Theoretical Euler head with an infinite number of blades $\rightarrow H_{th_\infty} = \dfrac{U_2 C_{U_2} - U_1 C_{U_1}}{g}$

For a radial inlet : $C_{U_1} = 0(\alpha_1 = 90°) \rightarrow H_{th_\infty} = \dfrac{U_2 C_{U_2}}{g}$



$$H_{th_\infty} = \frac{U_2 \cdot C_{U_2}}{g} = \frac{U_2}{g} \cdot (U_2 - W_{U_2}) = \frac{U_2^2}{g} - U_2 \cdot W_{U_2}$$

$$H_{th_\infty} = \frac{U_2^2}{g} - \frac{U_2 \cdot C_{r_2}}{g \tan \beta_2} = \frac{10^2}{9,81} - \frac{10 \cdot 1,6}{9,81 \cdot \tan 30°}$$

$$\rightarrow \boxed{H_{th_\infty} = 7,369 \text{m} = H}$$

Furthermore,

$$U_2 = r_2 \cdot \omega \Rightarrow U_2 = \frac{D_2}{2} \cdot \omega \rightarrow \omega = \frac{2 \cdot U_2}{D_2} = \frac{2 \cdot 10}{1,3}$$

$$\rightarrow \boxed{\omega = 15,385 \, \text{rad}/\text{s}}$$

Finally,

$$M = \frac{\rho \cdot g \cdot H \cdot q_v}{\omega} = \frac{3,5 \cdot 1000 \cdot 9,81 \cdot 7,369}{60 \cdot 15,385} \qquad \rightarrow \boxed{M = 274,1 \text{ N.m}}$$

- **Exercise n°2 :**

The outer diameter of a centrifugal pump impeller is 40 cm, and its width is 5 cm. The pump operates at 800 rpm to provide a head of 16 m. The blade angle at the outlet is 40 degrees relative to the wheel's periphery. Assuming the manometric efficiency is 75% (hydraulic efficiency).

Considering that the fluid enters radially, determine the discharge flow rate of the pump.

The slip coefficient $\mu = 1$

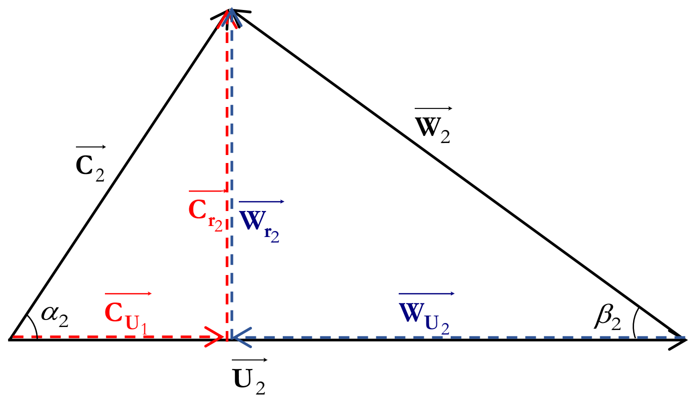
Solution :

$D_2 = 0,4\text{m}$ $b_2 = 0,05\text{m}$ $N = 800\text{rpm}$

$H = 16\text{m}$ $\beta_2 = 40°$ $\eta_H = \frac{H}{H_{thZ}} = 75\%$ $\mu = \frac{H_{thZ}}{H_{th\infty}} = 1$

1. Discharge flow rate

$$q_v = C_{r_2} \cdot S = C_{r_2} \cdot 2\pi r_2 b_2$$



2. Determination of the radial flow velocity $\left(C_{r_2}\right)$

Theoretical Euler head with an infinite number of blades $\rightarrow H_{th_\infty} = \frac{U_2 C_{U_2} - U_1 C_{U_1}}{g}$

For a radial inlet : $C_{U_1} = 0\left(\alpha_1 = 90°\right) \rightarrow H_{th_\infty} = \frac{U_2 C_{U_2}}{g}$

$$H_{th_\infty} = \frac{U_2 \cdot C_{U_2}}{g} = \frac{U_2}{g} \cdot \left(U_2 - W_{U_2}\right) = \frac{U_2^2}{g} - U_2 \cdot W_{U_2} \rightarrow W_{U_2} = U_2 - \frac{gH_{th_\infty}}{U_2}$$

$$\left.\begin{aligned} \eta_H = \frac{H}{H_{thZ}} \rightarrow H_{thZ} = \frac{H}{\eta_H} \\ \mu = \frac{H_{thZ}}{H_{th_\infty}} \rightarrow H_{thZ} = \mu \cdot H_{th_\infty} \end{aligned}\right\} \Rightarrow \frac{H}{\eta_H} = \mu \cdot H_{th_\infty} \rightarrow H_{th_\infty} = \frac{H}{\mu \cdot \eta_H} \rightarrow \boxed{H_{th_\infty} = 21,334\text{m}}$$

Outer peripheral velocity :

$$U_2 = r_2 \cdot \omega = \frac{D_2}{2} \cdot \frac{2\pi N}{60} \rightarrow \boxed{U_2 = 16,755\, \text{m}/\text{s}}$$

So,

$$H_{th_\infty} = \frac{U_2 C_{U_2}}{g} \Rightarrow C_{U_2} = \frac{g \cdot H_{th_\infty}}{U_2} \rightarrow \boxed{C_{U_2} = 12,491\, \text{m}/\text{s}}$$

$$W_{U_2} = U_2 - \frac{g \cdot H_{th_\infty}}{U_2} \rightarrow \boxed{W_{U_2} = 4,264\, \text{m}/\text{s}}$$

Thus,

$$\tan\beta_2 = \frac{W_{r_2}}{W_{U_2}} = \tan\beta_2 = \frac{C_{r_2}}{W_{U_2}} \Rightarrow C_{r_2} = W_{U_2}\tan\beta_2 \rightarrow \boxed{C_{r_2} = 3{,}578\,m/s}$$

Finally, $q_v = C_{r_2} \cdot S = C_{r_2} \cdot 2\pi r_2 b_2 \rightarrow \boxed{q_v = 0{,}225\,m^3/s = 810\,m^3/h}$

- **Exercise n°3 :**

A centrifugal pump provides a head of 20m while rotating at a speed of 600 rpm. The blades are curved backward to maintain the radial flow velocity at the inlet and outlet of the impeller constant at 2m/s.

For a radial inlet, calculate the impeller diameter when 50% of the kinetic energy at the outlet of the impeller is converted into pressure energy.

$$\begin{cases} \beta_1 = \beta_2 = 45° \\ \mu = 1 \end{cases}$$

Solution :

$N = 600\,rpm$ $H = 20m$ $\beta_1 = \beta_2 = 45°$ $C_{r_1} = C_{r_2} = 2\,m/s$

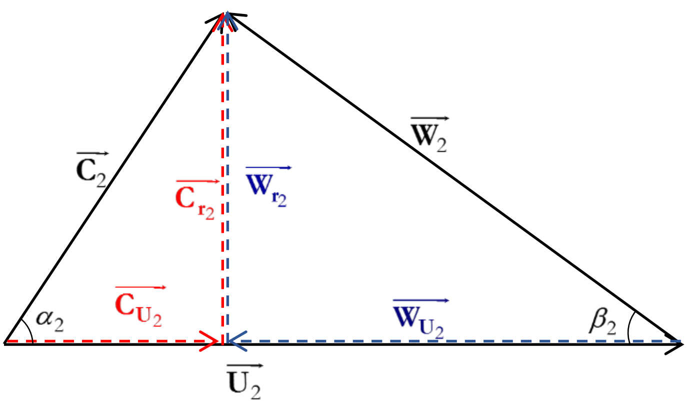
- Diameter D_2 :

50% of the kinetic energy is transformed into piezometric energy →the flow inside the impeller is not perfect, so the following hydraulic efficiency can be defined:

$$\eta_H = \frac{H}{H_{th_Z}}$$

This efficiency signifies that :

$$\rightarrow H = H_{th_Z} - \sum \text{hydraulic losses} = H_{th_Z} - \sum \text{Kinetic energy losses}$$



Theoretical Euler head with an infinite number of blades $\rightarrow H_{th_\infty} = \dfrac{U_2 C_{U_2} - U_1 C_{U_1}}{g}$

Radial inlet $(\alpha_1 = 90°)$: $C_{U_1} = 0 \rightarrow H_{th_\infty} = \dfrac{U_2 C_{U_2}}{g}$, with $H_{th_\infty} = H_{th_Z}$ $(\mu = 1)$

So, $H = \dfrac{U_2 C_{U_2}}{g} - 50\% \dfrac{C_2^2}{2g}$

Furthermore, we know that :

$$U_2 = C_{U_2} + W_{U_2} = C_{U_2} + \frac{C_{r_2}}{\tan\beta_2} = C_{U_2} + \frac{2}{\tan 45°} \rightarrow \boxed{C_{U_2} = U_2 - 2}$$

$$\left.\begin{array}{l} C_{U_2} = U_2 - 2 \\ C_{r_2} = 2 \end{array}\right\} \rightarrow \begin{cases} C_2 = \sqrt{C_{U_2}^2 + C_{r_2}^2} = \sqrt{(U_2-2)^2 + 2^2} \\ C_2^2 = (U_2-2)^2 + 2^2 \end{cases}$$

$$H = \frac{U_2 C_{U_2}}{g} - 50\% \frac{C_2^2}{2g} = \frac{U_2(U_2-2)}{g} - 50\% \frac{(U_2-2)^2 + 2^2}{2g} = \frac{3U_2^2 - 4U_2 - 8}{4g}$$

$$H = 20 \Rightarrow 20 = \frac{3U_2^2 - 4U_2 - 8}{4g}$$

$$\rightarrow 3U_2^2 - 4U_2 - 792{,}8 = 0 \Rightarrow \Delta = 4^2 + 4\cdot 3\cdot 792{,}8 = 9529{,}6$$

$$\rightarrow \begin{cases} U_2' = \dfrac{4 - \sqrt{9529{,}6}}{2\cdot 3} = -15{,}603\, m/s < 0\,(\text{Excluded}) \\ U_2'' = \dfrac{4 + \sqrt{9529{,}6}}{2\cdot 3} = 16{,}937\, m/s \end{cases}$$

Then $\rightarrow \boxed{U_2 = 16{,}937\, m/s}$

Finally,

$$U_2 = r_2 \cdot \omega = \frac{D_2}{2}\cdot\frac{2\pi N}{60} \Rightarrow D_2 = \frac{60\cdot U_2}{\pi N} \qquad \rightarrow \boxed{D_2 = 0{,}539m}$$

- **<u>Exercise n°4 :</u>**

A centrifugal pump with an outer diameter twice the inner diameter and operating at 1200 rpm delivers a total head of 75m. The radial flow velocity through the impeller is constant at 3m/s. The blades are curved at an angle of 30° at the impeller's outlet. If the outlet diameter of the impeller is 60 cm and the width at the outlet is 5 cm

For a radial inlet, determine :

1. The blade angle at the impeller's inlet relative to the periphery.

2. The power supplied to the fluid
3. The manometric (hydraulic) efficiency of the pump with $\mu = 1$.

Solution :

$D_2 = 2D_1 = 0{,}6\text{m}$ $\quad N = 1200\text{rpm}$ $\quad H = 75\text{m}$

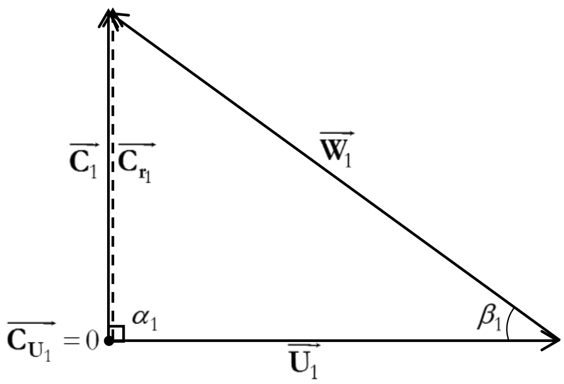
$\beta_2 = 30°$ $\quad C_{r_1} = C_{r_2} = 3\,{}^{\text{m}}/_{\text{s}}$ $\quad b_2 = 0{,}06\text{m}$

1. The blade angle at the impeller's inlet (β_1)

Peripheral velocity at the impeller's inlet

$$U_1 = r_1 \cdot \omega = \frac{D_1}{2} \cdot \frac{2\pi N}{60} \rightarrow \boxed{U_1 = 18{,}850\,{}^{\text{m}}/_{\text{s}}}$$

$$\tan\beta_1 = \frac{C_1}{U_1} \Rightarrow \beta_1 = \arctan\left(\frac{C_1}{U_1}\right) \rightarrow \boxed{\beta_1 = 9{,}04°}$$



2. Power supplied to fluid (P_f)

$$P_f = \rho g H q_v = \rho g H\left(C_{r_2} S_2\right) = \rho g H\left(C_{r_2} 2\pi r_2 b_2\right) = \rho g H C_{r_2} \pi D_2 b_2 = 1000 \cdot 9{,}81 \cdot 75 \cdot 3 \cdot \pi \cdot 0{,}6 \cdot 0{,}05$$

$$\rightarrow \boxed{P_f = 208{,}028\text{ kW}}$$

3. Manometric efficiency of the pump (η_H)

$$\eta_H = \frac{H}{H_{th_Z}}$$

Radial inlet $(\alpha_1 = 90°) \rightarrow C_{U_1} = 0$

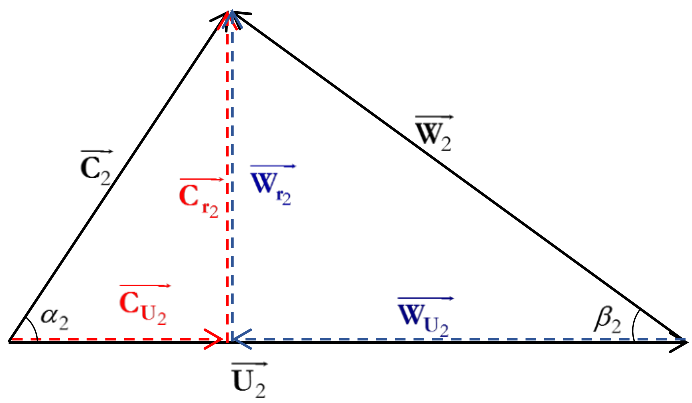
$$\mu = \frac{H_{th_Z}}{H_{th_\infty}} = 1 \rightarrow H_{th_Z} = H_{th_\infty} = \frac{U_2 C_{U_2}}{g}$$

Knowing that,

$$U_2 = r_2 \cdot \omega = D_2 \cdot \frac{\pi N}{60} \rightarrow \boxed{U_2 = 37{,}699\,{}^{\text{m}}/_{\text{s}}}$$

$$C_{U_2} = U_2 - W_{U_2} = U_2 - \frac{C_{r_2}}{\tan\beta_2} = 37{,}699 - \frac{3}{\tan 30°}$$

$$\rightarrow \boxed{C_{U_2} = 32{,}503\,{}^{\text{m}}/_{\text{s}}}$$



So,

$$H_{th_Z} = H_{th_\infty} = \frac{U_2 C_{U_2}}{g} \rightarrow \boxed{H_{th_Z} = 124,906m}$$

Finally,

$$\eta_H = \frac{H}{H_{th_Z}} = \frac{75}{124,906} \rightarrow \boxed{\eta_H = 60,05\%}$$

- **Exercise n°5 :**

A centrifugal pump has blades inclined at 30° with respect to the impeller's periphery. These blades have a depth of 20 mm at the outlet. Assuming a radial inlet and a slip factor of 0.77, determine the theoretical head developed by the pump to discharge a volume of 0.028 m³/s at a rotational speed of 1450 rpm, and then deduce the number of blades in the impeller.

The input and output diameters of the wheel are given : D_1=127.5mm et D_2=250mm

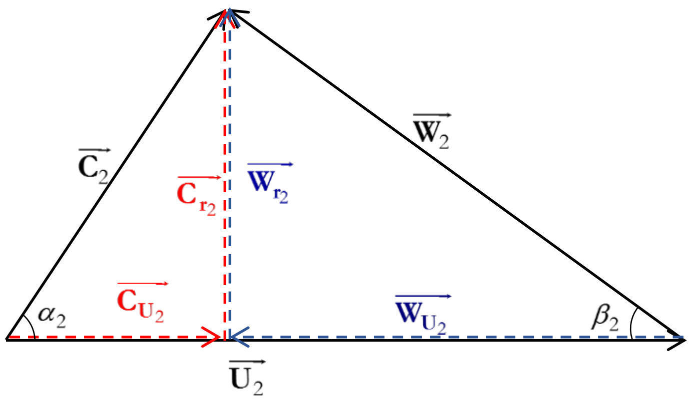
Solution :

$D_1 = 0,1275m$ $\quad D_2 = 0,25m$ $\quad N = 1450rpm$ $\quad \mu = 0,77$

$b_2 = 0,02m$ $\quad \beta_2 = 30°$ $\quad q_v = 0,028\, m^3/s$

1. Theoretical head developed by the pump

$$\mu = \frac{H_{th_Z}}{H_{th_\infty}} \rightarrow H_{th_Z} = \mu \cdot H_{th_\infty} = \mu \cdot \frac{U_2 C_{U_2}}{g} \qquad (\alpha_1 = 90° \rightarrow C_{U_1} = 0)$$



Given that,

$$U_2 = r_2 \cdot \omega = D_2 \cdot \frac{\pi N}{60}$$

$$C_{U_2} = U_2 - W_{U_2} = U_2 - \frac{C_{r_2}}{\tan \beta_2}$$

$$C_{r_2} = \frac{q_v}{2\pi r_2 b_2}$$

$$C_{U_2} = U_2 - \frac{q_v}{\pi D_2 b_2 \tan \beta_2}$$

Finally,

$$H_{th_Z} = \frac{\mu}{g}\left(D_2 \cdot \frac{\pi N}{60}\right) \cdot \left(D_2 \cdot \frac{\pi N}{60} - \frac{q_v}{\pi D_2 b_2 \tan \beta_2}\right)$$

$$H_{th_Z} = \frac{0,77}{9,81} \cdot \left(\frac{0,25\pi 1450}{60} \right) \cdot \left(\frac{0,25\pi 1450}{60} - \frac{0,028}{\pi \cdot 0,25 \cdot 0,02 \cdot \tan 30°} \right)$$

$$\rightarrow \boxed{H_{th_Z} = 23,677 m}$$

2. Number of impeller blades

STODOLA formula:

$$\mu = 1 - \frac{\pi \sin \beta_2}{Z\left(1 - \frac{C_{r_2}}{U_2} \cdot \frac{\sin \beta_2}{\cos \beta_2}\right)} = 1 - \frac{\pi \sin \beta_2}{Z\left(1 - \frac{C_{r_2}}{U_2} \cdot \cot \beta_2\right)}$$

$$\Rightarrow Z = \frac{\pi \sin \beta_2}{(1-\mu) \cdot \left(1 - -\frac{C_{r_2}}{U_2} \cdot \cot \beta_2\right)} \rightarrow \boxed{Z = 8,15 m}$$

Then,

$$Z = \begin{cases} 8 \leftrightarrow \text{Mathematical reasoning} \\ \text{Or} \\ 9 \leftrightarrow \text{Physical reasoning} \end{cases} \xrightarrow{\text{Consequence}} \begin{cases} \eta_H \leftrightarrow \text{increases} \quad \mu \leftrightarrow \text{decrease} \\ \\ \eta_H \leftrightarrow \text{decrease} \quad \mu \leftrightarrow \text{increases} \end{cases}$$

- **Exercise n°6 :**

A radial centrifugal pump is designed for the following data :

$q_v = 75\ l/s$	$b_1 = 25\ mm$	$b_2 = 23\ mm$	$q_{vF} = 2,25 l/s$ (Leakage flow rate)
$H = 30\ m$	$D_1 = 100\ mm$	$D_2 = 290\ mm$	$P_m = 2,7056 kW$ (Mechanical losses)
$N = 1750\ rpm$	$\alpha_1 = 90°$	$\beta_2 = 30°$	$\eta_g = 50\%$

$\rho = 1000\ kg/m^3$

The slip coefficient : $\mu = 1$

Determine :

1. The blade angle at the impeller's inlet
2. The angle at which water exits the impeller
3. The absolute velocity of water at the impeller's exit
4. The efficiency:

   4.1 Hydraulics (η_H)

   4.2 Volumetric (η_V)

   4.3 Mechanics (η_m)

Solution :

$$\begin{vmatrix} q_v = 75\ l/s \\ H = 30\ m \\ N = 1750 rpm \end{vmatrix} \quad \begin{vmatrix} b_1 = 25\,mm \\ D_1 = 100\,mm \\ \alpha_1 = 90° \end{vmatrix} \quad \begin{vmatrix} b_2 = 23\,mm \\ D_2 = 290\,mm \\ \beta_2 = 30° \end{vmatrix} \quad \begin{vmatrix} q_{vF} = 2{,}25 l/s \\ P_m = 2{,}7056 kW \\ \eta_g = 50\% \end{vmatrix}$$

$$\rho = 1000\ kg/m^3$$

1. Blade angle at the impeller's inlet (β_1)

Peripheral velocity at the impeller's inlet

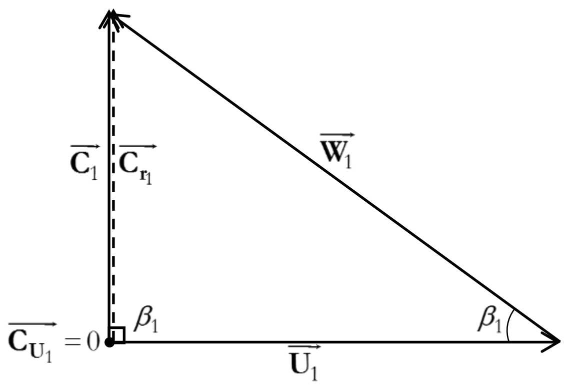
$$U_1 = r_1 \cdot \omega = \frac{D_1}{2} \cdot \frac{2\pi N}{60}$$

$$C_1 = C_{r_1} = \frac{\tilde{q}_v}{2\pi r_2 b_2} = \frac{\tilde{q}_v}{\pi D_1 b_1} = \frac{q_v + q_{vF}}{\pi D_1 b_1}$$

$$\tan \beta_1 = \frac{C_1}{U_1}$$

$$\Rightarrow \beta_1 = \arctan\left(\frac{C_1}{U_1}\right) = \arctan\left(\frac{60 \cdot (q_v + q_{vF})}{(D_1 \pi \cdot N) \cdot (\pi D_1 b_1)}\right)$$

$$\rightarrow \boxed{\beta_1 = 47{,}03°}$$



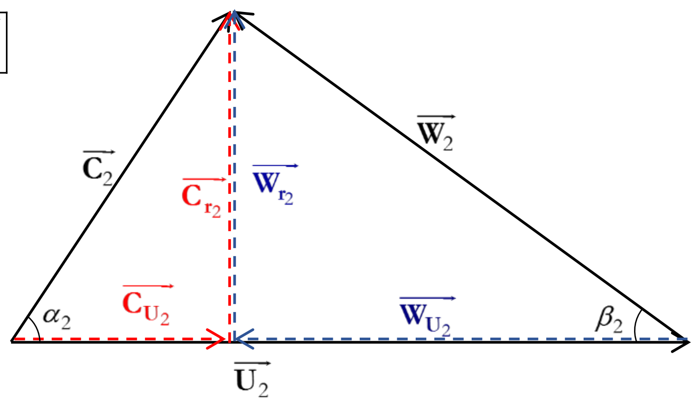
2. Angle of the flow at the impeller's outlet (α_2)

$$C_{r_2} = \frac{\tilde{q}_v}{2\pi r_2 b_2} = \frac{q_v + q_{vF}}{\pi D_2 b_2} \rightarrow \boxed{C_{r_2} = 3{,}687\ m/s}$$

$$U_2 = r_2 \cdot \omega = D_2 \cdot \frac{\pi N}{60} \rightarrow \boxed{U_2 = 26{,}573\ m/s}$$

$$C_{U_2} = U_2 - \frac{C_{r_2}}{\tan \beta_2} \rightarrow \boxed{C_{U_2} = 20{,}187\ m/s}$$

$$\alpha_2 = \arctan\left(\frac{C_{r_2}}{C_{U_2}}\right) \rightarrow \boxed{\alpha_2 = 10{,}37°}$$



3. Absolute velocity at impeller outlet (C_2)

$$C_{r_2} = C_2 \sin \alpha_2 \Rightarrow C_2 = \frac{C_{r_2}}{\sin \alpha_2} \rightarrow \boxed{C_2 = 20{,}483\ m/s}$$

4. Efficiencies
   1. Hydraulic (manometric) efficiency

$$\eta_H = \frac{H}{H_{th_Z}}$$

As : $\mu = \frac{H_{th_Z}}{H_{th_\infty}} = 1 \rightarrow H_{th_Z} = H_{th_\infty} = \frac{U_2 C_{U_2}}{g}$ $\left(\alpha_1 = 90° \rightarrow C_{U_1} = 0\right)$

$H_{th_Z} = 54{,}682m$

$$\eta_H = \frac{H}{H_{th_Z}} = \frac{30}{54{,}682} \rightarrow \boxed{\eta_H = 54{,}863\%}$$

2. Volumetric efficiency

$$\eta_V = \frac{q_v}{\tilde{q}_v} = \frac{q_v}{q_v + q_{vF}} \rightarrow \boxed{\eta_V = 97{,}087\%}$$

3. Mechanical efficiency

Method 1

$$\eta_M = \frac{\text{Impeller power}}{\text{Absolute power}} = \frac{P_a - P_m}{P_a}$$

$$\eta_g = \frac{P_f}{P_a} \Rightarrow P_a = \frac{P_f}{\eta_g} = \frac{\rho g H q_v}{\eta_g} = \frac{1000 \cdot 9{,}81 \cdot 30 \cdot 75 \cdot 10^{-3}}{0{,}5}$$

$\rightarrow \boxed{P_f = 22072{,}5W}$ et $\boxed{P_a = 44145W}$

$$\eta_m = \frac{P_a - P_m}{P_a} = \frac{44145 - 2705{,}6}{44145} \rightarrow \boxed{\eta_m = 93{,}871\%}$$

Method 2

$$\eta_g = \eta_V \eta_H \eta_m \Rightarrow \eta_m = \frac{\eta_g}{\eta_V \eta_H}$$

$$\rightarrow \boxed{\eta_m = 93{,}871\%}$$

- **Exercise n°7 :**

The characteristics of a centrifugal pump are given by the following relationship :

$$H = -301 q_V^2 + 55{,}91 q_V + 59{,}45$$

Where H is the head in [m] and q_V is the volumetric flow rate in [m^3/s]

This pump circulates water in a system whose resistance characteristic is given by :

$$H_r = 42 q_V^2 + 9{,}72 q_V + 17{,}22$$

H_r in [m] and q_V in [m^3/s]

Wheel dimensions with outer radius : r_2=200mm and b_2=31mm

The pump speed is N=1450 rpm and the hydraulic efficiency is 0,8.

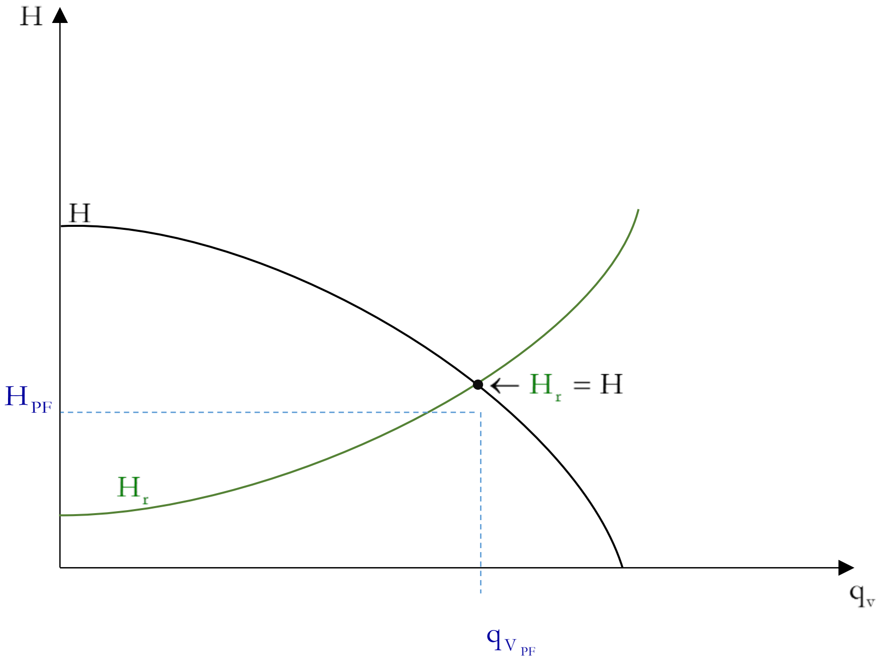
Questions: Assuming radial flow entry:

1. Plot both characteristics on the same diagram (H q_V) and then determine the operating point characteristics.
2. Determine the outlet angle of the moving channel

ρ=1000kg/m^3, g=9,81 m/s^2 et μ=1

Solution :

1. Representation of both characteristics on the same diagram (H, qv)



- Operating point characteristics.

$$H = H_r \Rightarrow -301q_V^2 + 55,91q_V + 59,45 = 42q_V^2 + 9,72q_V + 17,22$$
$$\Rightarrow -343q_V^2 + 46,19q_V + 42,23 = 0$$

$$\Delta = 60073,8 \rightarrow \begin{cases} q_V^* = 0,425\,m^3/s \\ q_V^{**} = -0,290\,m^3/s \end{cases}$$

Flow rate at the operating point

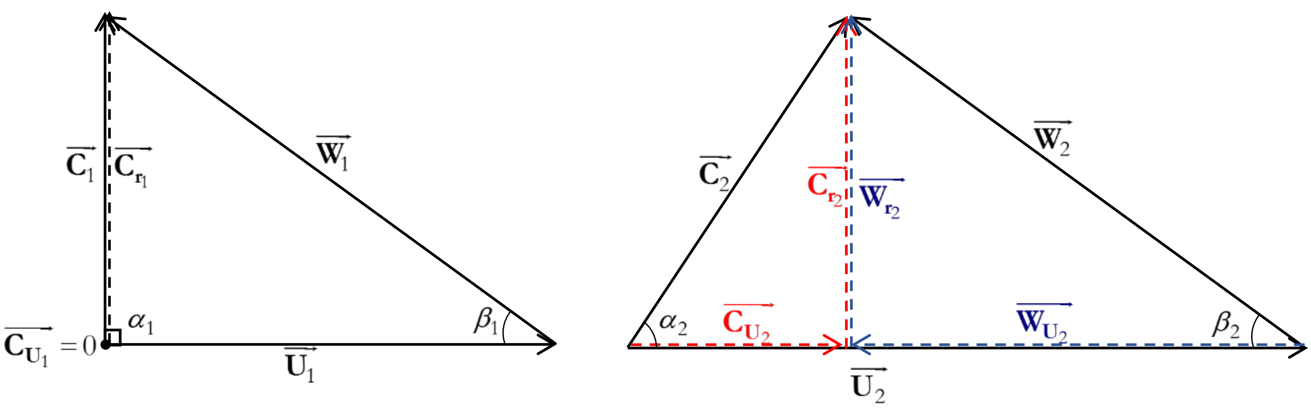
$$q_{V_{PF}} = q_V^* = 0,425\,m^3/s$$

Head at the operating point

$$H_{PF} = H\left(q_{V_{PF}}\right) = H_r\left(q_{V_{PF}}\right) = 28,92m$$

2. Mobile channel outlet angle

Supposition β2<90°



$U_2 = r_2\omega = 30,369\,m/s$

Theoretical Euler head with an infinite number of blades

$$H_{th\infty} = \frac{U_2C_{U2} - U_1C_{U1}}{g} \qquad C_{U1} = 0 \text{ because } \alpha_1 = 90° \leftrightarrow \text{Radial fluid inlet}$$

$$H_{th\infty} = \frac{U_2C_{U2}}{g} \Rightarrow C_{U2} = \frac{gH_{th\infty}}{U_2}$$

$$\begin{cases} \mu = \dfrac{H_{thZ}}{H_{th\infty}} \\ \eta_H = \dfrac{H}{H_{thZ}} = \dfrac{H_{PF}}{H_{thZ}} \end{cases} \Rightarrow H_{th\infty} = \frac{H}{\mu\eta_H} = 36,15m \Rightarrow C_{U2} = 11,677\,m/s$$

$C_{U2} < U_2 \Rightarrow \beta_2 < 90°$ (Velocity triangle at the outlet verified)

$\Rightarrow W_{U2} = U_2 - C_{U2} = 18,892\,m/s$

$$q_v = Q_{V_{PF}} = C_{r2}2\pi r_2 b_2 \Rightarrow C_{r2} = \frac{q_v}{2\pi r_2 b_2} = 10,910\,m/s = W_{r2}$$

$$\beta_2 = \arctan\left(\frac{W_{r2}}{W_{U2}}\right) = 30,27°$$

- **Exercice n°8 :**

The head H, efficiency η, and hydraulic system characteristic H_r associated with a centrifugal pump are given by the equations.

$$H = 20 + 0,8333q_V - 0,1667q_V^2$$
$$\eta = 32,643q_V - 3,2143q_V^2$$
$$H_r = 10 + 0,2116q_V^2$$

The head is given in meters, the flow rate is expressed in l/s, and the efficiency in %.

1.1. Verify that the operating point has the following coordinates :

$$\left(H_{PF} = 18,5576\,m \quad \text{et} \quad q_{V_{PF}} = 6,3594\,\ell/s\right)$$

1.2. Calculate the absolute power at this operating point

In order to increase the flow rate in this pipeline, two identical pumps are connected in parallel, and their characteristics are given by the previous equations.

2.1. What is the value of the flow rate in the pipeline?

2.2. Deduce the power absorbed by each pump.

$\rho = 1000\,kg/m^3$

Solution :

$$H = 20 + 0,8333q_V - 0,1667q_V^2$$
$$\eta = 32,643q_V - 3,2143q_V^2$$
$$H_r = 10 + 0,2116q_V^2$$

1.1. Checking the operating point

The operating point is defined by $(H = H_r)$:

$$H{=}H_r \Leftrightarrow 10 + 0,8333q_V - 0,3783q_V^2 = 0$$

$$\Delta = 15,8264 \Rightarrow \begin{cases} q_{V_1} = 6,3594\,\ell/s \\ q_{V_2} = -4,1567\,\ell/s \quad (\text{Excluded}) \end{cases}$$

The operating point :

$$\begin{cases} q_{V_{PF}} = 6,3594\,\ell/s \\ H_{PF} = H\left(q_{V_{PF}}\right) = H_r\left(q_{V_{PF}}\right) = 18,5576m \end{cases}$$

$\rightarrow$ The coordinates of the operating point are verified

1.2. Absolute power at the operating point

$$P_a = \frac{P_f}{\eta_{PF}} = \frac{\rho g H_{PF} q_{V_{PF}}}{\eta_{PF}} \qquad \eta_{PF} = \eta\left(q_{V_{PF}}\right) = 77,6\%$$

$$P_a = \frac{1000 \cdot 15,5576 \cdot 9,81 \cdot 6,3594 \cdot 10^{-3}}{0,776} \Rightarrow \boxed{P_a = 1491,98W}$$

2. Parallel grouping

For a parallel grouping, the net head is the same at each moment for each pump, and the resulting total flow is the sum of the individual flows.

2.1. Flow rate in the pipeline

Calculating the new operating point :

$$H_t = H_1 = H_2 = H \qquad \text{et} \qquad q_{V_t} = q_{V_1} + q_{V_2} = 2q_V \ \left(q_{V_1} = q_{V_2} \text{ "Identical pump"}\right)$$

$$H = 20 + 0,8333q_V - 0,1667q_V^2 \Rightarrow H_t = 20 + 0,8333\frac{q_{V_t}}{2} - 0,1667\left(\frac{q_{V_t}}{2}\right)^2$$

$$\rightarrow \boxed{H_t = 20 + 0,41665q_{V_t} - 0,041675q_{V_t}^2}$$

$$H_t{=}H_r \Leftrightarrow 10 + 0,41665q_V - 0,253275q_V^2 = 0$$

$$\Delta = 10,3046 \Rightarrow \begin{cases} q_{V_1} = 7,1597\,\ell/s \\ q_{V_2} = -5,5146\,\ell/s \quad (\text{Excluded}) \end{cases}$$

The new operating point :

$$\begin{cases} q_{V_{OP}} = 7,1597\,\ell/s \\ H_{OP} = H_t\left(q_{V_{OP}}\right) = H_r\left(q_{V_{OP}}\right) = 20,8468\,m \end{cases}$$

So, the flow rate in the pipeline is $7,1597\,\ell/s$

2.2. Power absorbed by each of the pumps

$$q_{V_{totale}} = 7,1597\,\ell/s \Rightarrow \text{Each pump discharges} \rightarrow q_V = \frac{q_{V_t}}{2} = 3,57935\,\ell/s$$

$$P_a = \frac{P_f}{\eta_{PF}} = \frac{\rho g H_{pump} q_{V_{pump}}}{\eta_{pump}} \rightarrow \begin{cases} q_{V_{pump}} = 3,57935\,\ell/s \\ H_{pump} = H_{totale}\left(q_{V_{pump}}\right) = H_r\left(q_{V_{pump}}\right) = 20,8468\,m \\ \eta_{pump} = \eta\left(q_{V_{pump}}\right) = 75,66\% \end{cases}$$

$$P_a = \frac{1000 \cdot 9,81 \cdot 20,8468 \cdot 3,57935 \cdot 10^{-3}}{0,7566} \rightarrow \boxed{P_a = 967,56\,W}$$

Chapter IV

Cavitation in Pumps

Table of contents

1. Description and Origin of Cavitation

Cavitation is a physical phenomenon that occurs in liquids and is characterized by the local formation of vapor due to a drop in static pressure. As the fluid passes through the pump impeller, its pressure gradually increases, transitioning from P_1 to P_2. However, this increase is not uniform, and near the impeller's inlet (point K), a region of dynamic depression ΔP forms, thus reaching the minimum pressure within the system at that specific location (see Figure 1). The dynamic depression is directly influenced by the leading edge contour

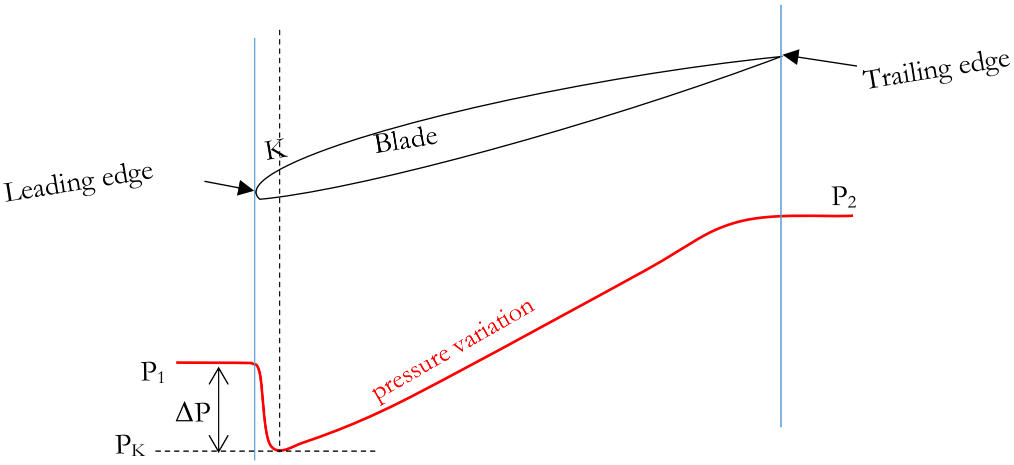
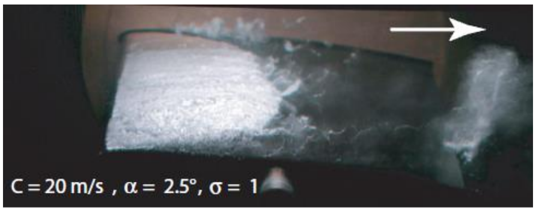


Figure 1. Evolution of pressure from impeller inlet to outlet

When installation conditions are unfavorable, especially in the presence of a high suction head, the pressure at point K can reach the limit for a liquid, known as vapor pressure (P_V). In these particular circumstances, point K will be the site of localized vaporization, resulting in the momentary formation of a vapor bubble that immediately collapses. From point K, the pressure then continues to gradually increase. Unlike the classic boiling phenomenon, it is important to note that cavitation is not caused by an increase in temperature exceeding the boiling point, but rather by a decrease in pressure, which becomes lower than the vapor pressure for the local temperature conditions.

When suction conditions deteriorate further, point K moves towards the trailing edge of the impeller, causing the release of unstable bubbles following random trajectories. These bubble releases, often referred to as 'bubble shedding,' are responsible for the characteristic erosion phenomenon associated with cavitation. This erosion results from the implosion of the bubbles on the surface of the impeller blades, the outer casing, and potentially the pump hub. (Figure 2)



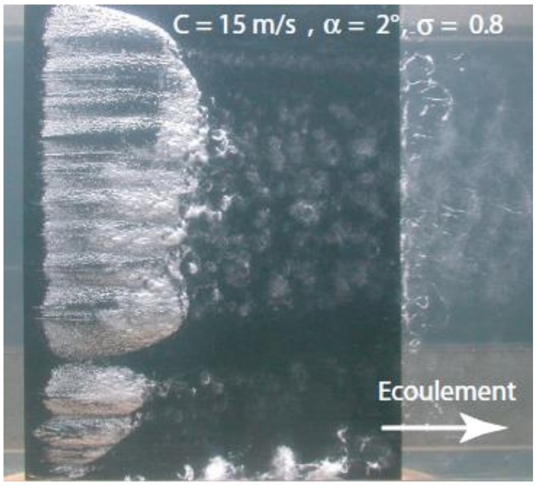


Figure 2. Development of attached cavitation on the extrados of a blade.

2. Cavitation Criteria

Cavitation has a significant impact on machine performance. This phenomenon typically occurs when the cavitation pocket reaches the region between two machine blades. The presence of this pocket leads to an increase in relative velocity (W_2), resulting in a reduction of the machine's rotational speed (C_{U2}), as shown in Figure 3. Consequently, this results in a decrease in theoretical head (in accordance with the Euler's equation).

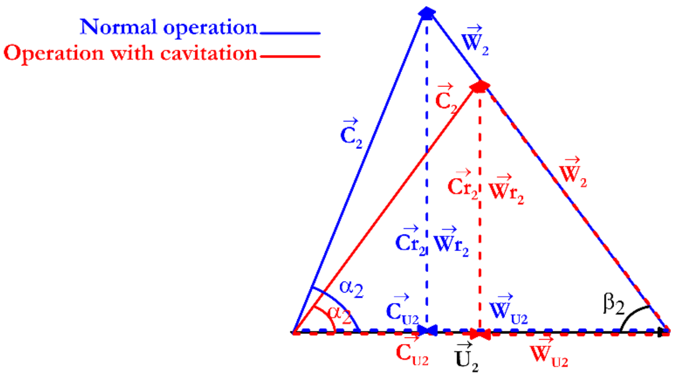


Figure 3. Impact of cavitation on the velocity triangle at the outlet of the impeller

The cavitation characteristic is presented in three stages:

- Physical cavitation : refers to the initial appearance of the first cavitation bubble at the periphery of the impeller.
- Erosive cavitation : relates to the repeated formation and release of bubbles.
- Industrial cavitation : is characterized by a noticeable decrease in the machine's head-flow characteristic.

Between the first and second stages, a distinct noise is heard, similar to the sound of rolling stones. This noise emitted by the machine gradually increases until it reaches a maximum during industrial cavitation, then decreases due to the significant presence of vapor in the liquid, which acts as a sound damper. The only stage that can be predicted through calculation is physical cavitation; the other stages must be determined through experimentation or statistical analysis

3. Manifestations of Cavitation

The different types of cavitation can be characterized by observing their physical appearance.

a. Separated-bubble cavitation

This type of cavitation typically occurs on blade profiles with a low incidence angle relative to the flow. The cavitation structures resemble more or less spherical vapor bubbles that form randomly in the liquid. For these bubbles to form, it is essential that nuclei, such as microair bubbles, exist in the liquid or on the blade surface, from which the bubbles develop.

Figure 4. Bubble cavitation on a blade profile

b. Pocket cavitation

In this context, cavitation is characterized by the formation of a single vapor-phase cavity, which remains attached to the blade profile from which it originates. This pocket forms from a detachment observed on the profile itself. Behind this pocket, unstable structures detach, are carried by the flow, and eventually collapse downstream.

Figure 5. Pocket cavitation on pump impeller blades

c. Spiral bubble-release cavitation

This phenomenon can be termed as pocket cavitation localized on the blades of the pump impeller. It is a specific situation where thin and stable structures, resembling shiny blades, form and adhere to an edge, such as the leading edge of a blade profile. This type of cavitation typically occurs when the angle of attack of a profile is excessive, resulting in a very pronounced depression.

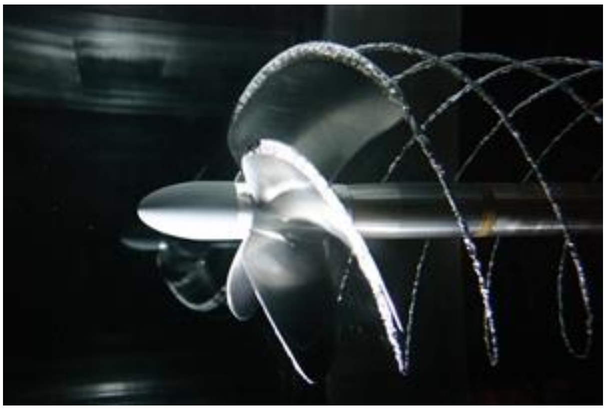


Figure 6. Spiral bubble release cavitation on axial pump blades

4. Cavitation Mechanism and Influential Factors

Cavitation typically manifests in the form of microvapor bubbles, with diameters generally smaller than a millimeter. These microbubbles often form around impurities in the liquid, such as microscopic suspended particles. Subsequently, these vapor bubbles are reabsorbed, meaning they implode back into a liquid state when the pressure reaches a sufficient level. This occurs notably in compression machines, where the pressure is lower at the inlet before increasing as the fluid passes through the roto.

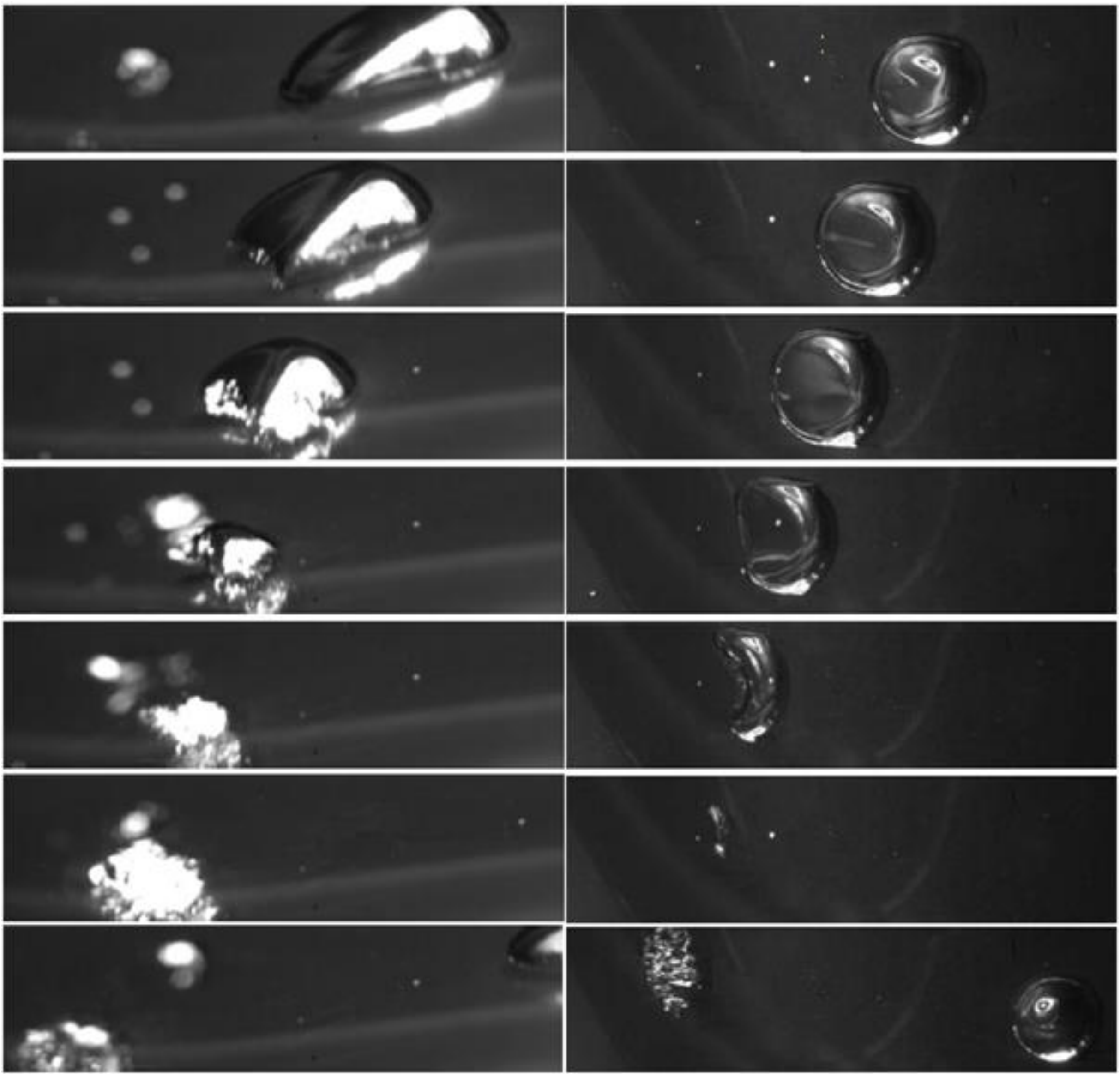


Figure 7. Implosion of a bubble on the wall of a blade profile

Static pressure can decrease at the inlet for various reasons, including :

- Increase in the pump's suction head:

If we apply Bernoulli's theorem between the free surface of the suction basin and the pump's inlet (Figure 8), we obtain :

$$\frac{P_{atm}}{\rho g} + \underset{\text{Zero level}}{0} + \underset{\text{Zero velocity}}{0} = \frac{P_e}{\rho g} + \frac{C_e^2}{2g} + h_{suction} + \underset{\text{Suction head losses}}{k\frac{C_e^2}{2g}}$$

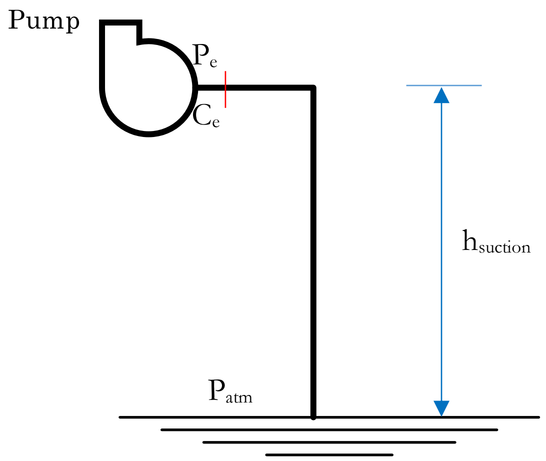


Figure 8. Pumping above a suction basin

Hence, the value of static pressure at the pump inle:

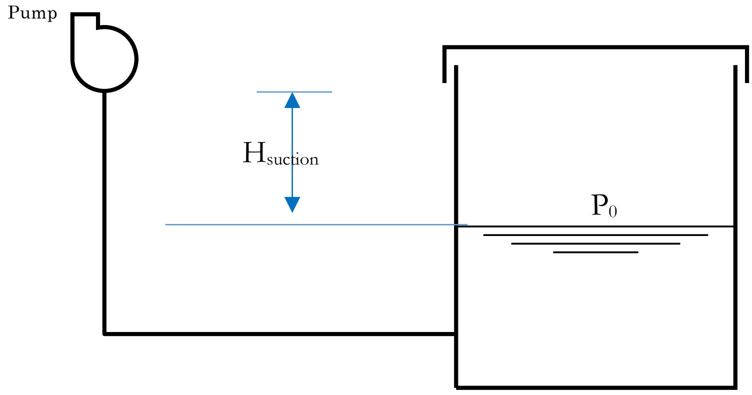
$$\frac{P_e}{\rho g} = \frac{P_{atm}}{\rho g} - \frac{C_e^2}{2g} - h_{suction} - k\frac{C_e^2}{2g} \tag{II.1}$$

The inlet pressure will be lower as the suction head increases.

- Decrease in atmospheric pressure due to an increase in altitude

If we return to equation (II.1), it is important to note that, as atmospheric pressure has decreased, static pressure will also undergo an equivalent decrease for a given operating flow rate.

- Decrease in absolute pressure in suction receiver



The pressure within the suction basin, denoted as (P_0), if ($P_0 < P_{atm}$), the maximum suction head will be even lower.

- Increase in velocity C_e

  It is through an increase in the pump's rotational speed that one achieves an increase in the inlet flow velocity, hence an elevated risk of cavitation.

- Increase in the head loss at the inlet:

  Closing a suction valve or a manifold with many singularities will increase the coefficient (k) of pressure drop and therefore reduce the static pressure at the inlet.

5. NPSH (required and available)

Rather than exclusively fixating on static inlet pressure, a more comprehensive approach to assess cavitation involves the evaluation of the pump's Net-Positive-Suction-Head (NPSH), also referred to as "total pressure head." NPSH serves as a crucial parameter intimately associated with the occurrence of cavitation.

In simple terms, the NPSH available at the machine inlet can be calculated according to the shape and characteristics of the suction line. The machine, in turn, requires a certain level of NPSH to operate without cavitation. To avoid cavitation, the following condition must be met.

$$NPSH_{available} \geq NPSH_{required} \tag{II.2}$$

The NPSH required by a machine is determined through testing and is influenced by various factors such as the machine type, its rotational speed, and the operating flow rate. However, it is equally crucial to be able to accurately calculate the NPSH available within the installation. To do so, we can refer to the following literal definition:

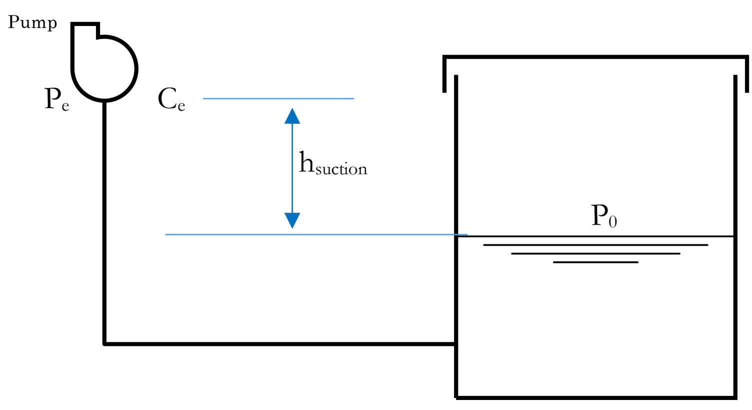
$$NPSH_{available} = \frac{P_e}{\rho g} + \frac{C_e^2}{2g} - \frac{P_v}{\rho g} \quad \text{(II.3)}$$

The quantity $\left(\frac{P_e}{\rho g} + \frac{C_e^2}{2g}\right)$ represents the net suction head of the machine. This parameter can be calculated relatively simply based on several key factors, including the suction head, head losses in the suction system, and the atmospheric pressure above the liquid surface.

In the standard installation diagram we are examining, several essential parameters can be identified. Firstly, (P_0) represents the pressure at the liquid surface in direct contact with the atmosphere. Next, (hasp) corresponds to the geometric suction head, which is the vertical distance measured from the liquid's free surface to the pump's inlet point. Additionally, (Δh) represents the head losses resulting from the suction line

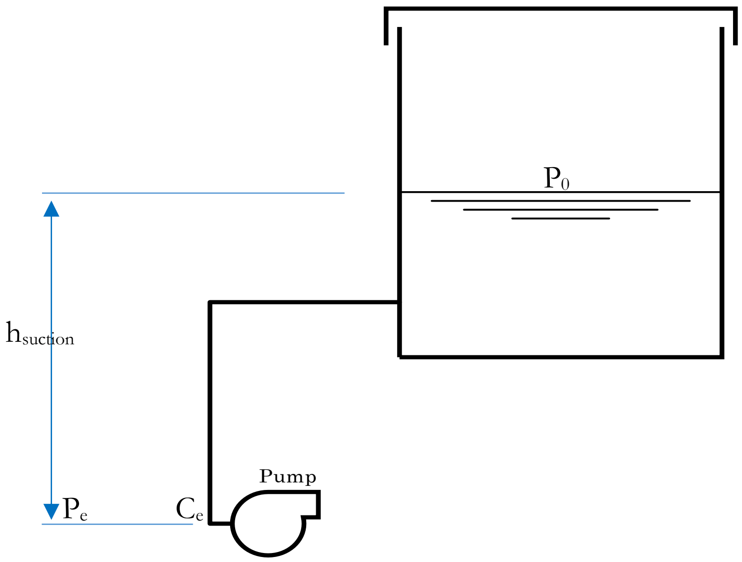
The application of Bernoulli's theorem between the liquid's free surface and the pump's inlet provides us with:

- Pump above the suction level



$$\frac{P_e}{\rho g} + \frac{C_e^2}{2g} = \frac{P_0}{\rho g} - \underbrace{\Delta h}_{\text{head losses}} - h_{suction} \quad \text{(II.4)}$$

- Pump below the suction level



$$\frac{P_e}{\rho g} + \frac{C_e^2}{2g} = \frac{P_0}{\rho g} - \underbrace{\Delta h}_{\text{head losses}} + h_{suction} \quad \text{(II.5)}$$

This leads to the second expression of the NPSH available.

$$NPSH_{available} = \frac{P_0}{\rho g} - \Delta h \pm h_{suction} - \frac{P_v}{\rho g} \quad \text{(II.6)}$$

With the sign : + if the pump is below suction level

- if the pump is above suction level

6. Cavitation Similarity

a. Similarity coefficients

The NPSH required for a pump follows the same principles of similarity as the total head (H). Similar to H, it is possible to define dimensionless coefficients that remain constant when working with pumps of similar geometry in similarity operation mode.

The analysis of the development of cavitation phenomena can also be interpreted according to the fundamental laws of fluid dynamics. The similarity laws specific to turbomachinery can be applied when comparing reduced-scale models to prototypes.

To characterize this particularity, two similar approaches can be identified among the various concepts found in the literature.

- The first approach involves directly adapting the expression of the pressure coefficient by replacing the head H with the NPSH required characteristic. This adapted coefficient will be referred to as (ψ') :

$$\psi' = \frac{gNPSH_{required}}{N^2D^2} \tag{II.7}$$

- The second approach is based on the formulation of a coefficient called the Thoma coefficient, denoted as (σ), which is defined as the ratio of the mass energy required at intake to the useful mass energy, i.e. gH :

$$\sigma = \frac{NPSH_{required}}{H} \tag{II.8}$$

b. Cavitation similarity condition

When analyzing the occurrence of cavitation, the suction conditions of a pump are significantly influenced by the NPSH available at the suction ($NPSH_{available}$). In this context, we can relate this parameter ($NPSH_{available}$) to the Thoma coefficient, denoted as σ', defined as follows :

$$\sigma' = \frac{NPSH_{available}}{H} \tag{II.9}$$

Thus, the non-cavitation condition, applicable to a specific pump, can be generalized to all pumps belonging to the same family in one of the following forms :

$$\sigma' > \sigma \tag{II.10}$$

We can conclude that two pumps of the same family, operating in similarity, are also in similarity concerning cavitation if they exhibit the same resistance to the onset of this phenomenon.

7. Cavitation Effects

Cavitation, a common phenomenon in hydraulic systems, primarily has detrimental consequences that can vary in severity. Here are the main effects of cavitation, the impact of which depends on its intensity and user concerns:

- Noise : Cavitation typically begins with the emission of noises, the intensity of which varies with its development. This noise is caused by the volume fluctuations of bubbles or vapor pockets. As cavitation worsens, the noise can become very disruptive, ranging from slight crackling to excessive sound levels. In fact, cavitation noise is the primary source of noise pollution related to liquid flows.

- Performance Loss : Another effect of cavitation is the reduction of hydraulic equipment performance. As cavitation develops, it can disrupt flows by creating obstructions, resulting in a decrease in lift height and efficiency for pumps, a decrease in thrust for propellers, increased head losses, and flow rate limitations for valves.
- Vibrations : Vibrations result from the fluctuations of cavitating structures in the presence of solid walls. These fluctuations generate unsteady forces that manifest as vibrations. For example, cavitation at the propellers of a boat can cause shaft vibrations, which are then transmitted to the boat's structure.
- Mechanical Erosion : Lastly, cavitation can lead to a phenomenon of mechanical erosion. When cavitating structures collapse rapidly, they generate very high local liquid velocities. If these collapses occur near a wall, high-energy liquid jets can form and cause erosion damage. If this phenomenon repeats, it can ultimately lead to the physical destruction of the structure.

8. Cavitation remedies

Various measures can be implemented to eliminate or mitigate the detrimental effects of cavitation. Here are some examples of solutions that can be considered:

- Circuit pressurization : To reduce the risk of cavitation, one option is to increase local pressure in areas prone to cavitation. This can be achieved by pressurizing the circuit, which helps maintain the static pressure at an adequate level.
- Strategic arrangement of components : It is possible to place components most susceptible to cavitation in the lower parts of the system. By lowering these elements, the likelihood of cavitation occurring at critical levels is reduced.
- Optimization of suction pressure and pump speed : For pumps, adjusting the suction pressure or reducing the rotational speed can help prevent cavitation. These measures aim to maintain optimal operating conditions.
- Staging of head loss : For flow control or throttling devices, one approach is to stage the head loss so that each stage operates outside the cavitation zone. Commercial valves incorporate this design to ensure cavitation-free operation.

By adopting these strategies, it is possible to minimize the detrimental effects of cavitation and maintain the proper functioning of hydraulic equipment.

9. Exercises

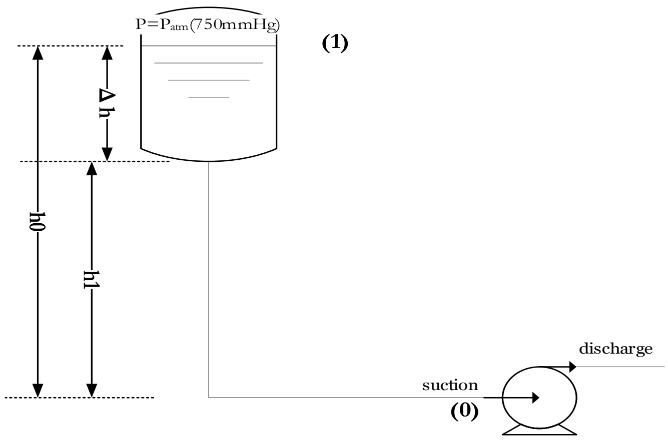
- **Exercise n°1 :**

Condensation water, at T = 90°C, is pumped by a centrifugal pump from a manifold breathing at atmospheric pressure (p_{atm} = 750 mmHg).

The required liquid head ($NPSH_{required}$) for the pump suction pipe must be at least 2.5 meters higher than the vapor pressure for water at 90 °C ($P_{v(water)}$ = 611 mmHg) to avoid cavitation.

Assuming a head loss in the suction line H_f = 1,6m and knowing that the pump is located at h_1=2,0m below the condenser.

750mmHg = 1bar

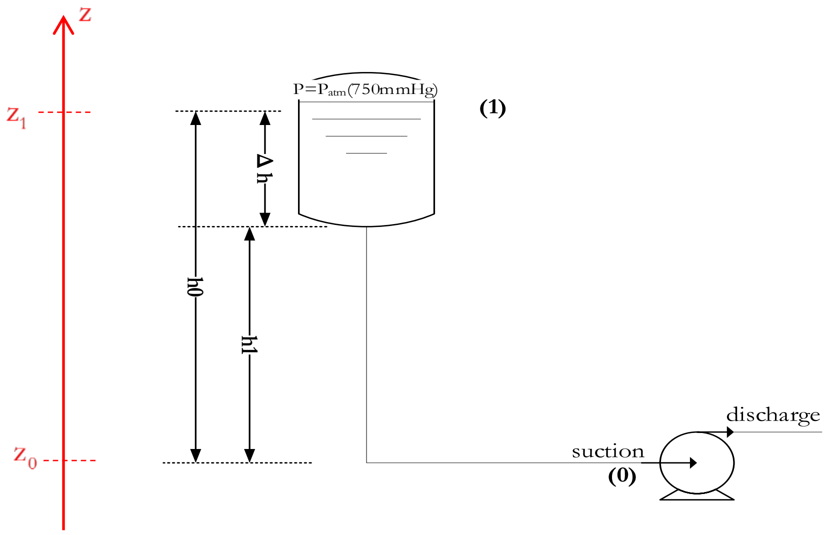


Solution :

- Determination of the height (ΔZ) to prevent cavitation :

Data :	T=90 °C	P_{atm} = 750 mmHg ≃ 1 bar
P_v = 611 mmHg	$NPSH_{required} = 2,5\,m$	$H_f = 1,6\,m$

The safety margin (minimum pressure supplement) NPSH that must be considered to prevent cavitation should meet $NPSH_{available} \geq NPSH_{required}$.



To deduce the minimum height (Δh) $\Rightarrow$ $NPSH_{available} = NPSH_{required}$, (Cavitation threshold).

With : $NPSH_{available} = \frac{P_0}{\rho g} - \frac{P_v}{\rho g} + \frac{C_0^2}{2g}$

Since the pressure at point (0) is unknown, applying Bernoulli's theorem between (0) and (1) will give us:

$$\frac{P_1}{\rho g} + z_1 + \frac{C_1^2}{2g} = \frac{P_0}{\rho g} + z_0 + \frac{C_0^2}{2g} + \underbrace{\frac{J_{1\to 0}}{\rho g}}_{\substack{=H_f \\ \text{head losses} \\ \text{between 1 and 0}}} \Rightarrow \frac{P_0}{\rho g} + \frac{C_0^2}{2g} = \frac{P_1}{\rho g} + \underbrace{(z_1 - z_0)}_{=h_0} + \frac{C_1^2}{2g} - H_f$$

By substituting in the previous equation, we arrive at :

$$NPSH_{available} = \frac{P_1 - P_v}{\rho g} + h_0 - H_f \Rightarrow h_0 = NPSH_{available} - \left(\frac{P_1 - P_v}{\rho g}\right) + H_f \Rightarrow \boxed{h_0 = 2,21 \text{ m}}$$

Knowing that :

$P_1 = 750$ mmHg $\rightarrow$ 1bar

$P_v = 611$ mmHg $\rightarrow$ $\frac{611}{750} = 0,81467$ bar

So, the minimum height that needs to be ensured in the reservoir is :

$$\Delta h = h_0 - h_1 \Rightarrow \boxed{\Delta h = 0,21 \text{ m}}$$

- **Exercise n°2 :**

In order to ensure the transfer of water from one reservoir (R1) to another reservoir (R2) with a flow rate $q_V = 23\ m^3/h$, the network connecting the two reservoirs is equipped with a centrifugal pump with a minimum NPSH at the operating point of 8,25 mH_2O.

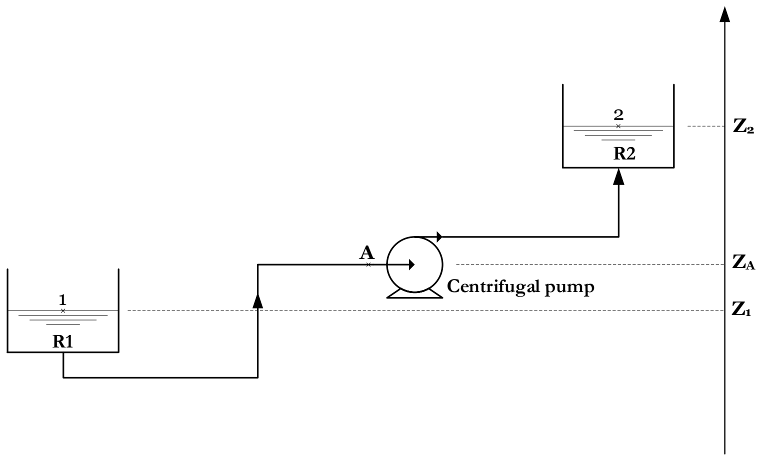
The pumped water is at 20°C, and its vapor pressure at this temperature is Pv = 0,023 bar.

1. Calculate the pressure at point A. if the total pressure drops (1⟶A) are estimated at J=0,03224bar.
2. Check if the pump is suitable for the circuit? justify your answer..

Data : $\Delta Z_1=Z_A-Z_1=1{,}5m$ $\Delta Z_2=Z_2-Z_A=2{,}25m$ $\Delta Z_3=Z_2-Z_1=3{,}75m$

$C_1=4{,}35m/s$ $P_1=P_2=1bar$ $\rho=1000kg/m^3$

*** mH_2O : meters of water column**



Solution :

Data :	$q_V = 23\,m^3/h$	$P(NPSH_{required}) = 8{,}25\,mH_2O = 8{,}25 \cdot \rho g = 0{,}809\,bar$	
$\Delta Z_1 = Z_A - Z_1 = 1{,}5\,m$	$\Delta Z_2 = Z_2 - Z_A = 2{,}25\,m$	$\Delta Z_3 = Z_2 - Z_1 = 3{,}75\,m$	$C_1 = 4{,}35\,m/s$
$P_1 = P_2 = 1\,bar$	$P_v = 0{,}023\,bar$	$g = 9{,}81\,m^2/s$	$\rho = 1000\,kg/m^3$

1. Pressure at point A with $J_{1\to A} = 0{,}03224\,bar$

According to Bernoulli between (1) and (A)

$$P_1 + \rho g z_1 + \frac{\rho C_1^2}{2} = P_A + \rho g z_A + \frac{\rho C_A^2}{2} + J_{1\to A}$$

$$\Rightarrow P_A = P_1 + \rho g (z_1 - z_A) + \frac{\rho}{2}(C_1^2 - C_A^2) - J_{1\to A}$$

Knowing that:

$$q_V = C_A S_A = C_A \frac{\pi}{4} D_A^2 \Rightarrow C_A = \frac{4q_V}{\pi D_A^2} \rightarrow \boxed{C_A = 7{,}94\, m/s}$$

The final result is : $\boxed{P_A = 0{,}6\,\text{bar}}$

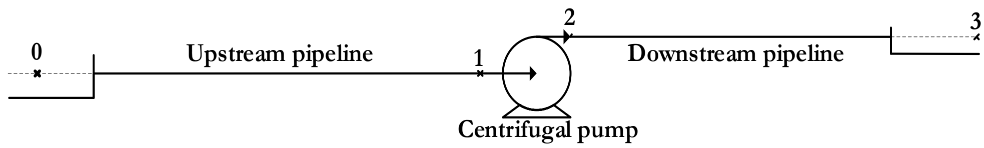
2. Verification of the proper functioning of the pump:

$$NPSH_{available} = \frac{P_A}{\rho g} - \frac{P_v}{\rho g} + \frac{C_A^2}{2g} \rightarrow \boxed{NPSH_{available} = 9{,}56\,m}$$

Since $\left(NPSH_{available} = 9{,}56\,m \geq NPSH_{required} = 8{,}25\,m\right)$ → the pump is suitable for the considered circuit.

- **<u>Exercise n°3 :</u>**

Consider a purely radial centrifugal pump located in an industrial process, as illustrated in the diagram below:



Pump and circuit characteristics are as follows :

Centrifugal pump	Upstream circuit	Downstream circuit
$H = -128{,}141 q_V + 44{,}323$	$P_0 = 1\text{bar}$	$Hr_{dwn} = 23{,}405 q_V^2 + H_f + 4{,}329$
$NPSH_{required} = 0{,}923 + 3{,}092 q_V + 2{,}952 q_V^2$	$L_{ups} = 18m$	$L_{dwn} = 33m$
$\beta_2 = 39°$	$S_{ups} = 0{,}0367 m^2$	$S_{dwn} \neq S_{ups}$
$D_2 = 135\,mm \quad \rho = 1000\,kg/m^3$	$P_V = 0{,}625\text{bar}$	
$N = 2950\,rpm \quad \mu = 1 \quad \eta_H = 1$		H and Hr in [m] et q_V in $\left[m^3/s\right]$

<u>Assumptions :</u>

- Negligible change in potential energy
- Losses of head in the pipelines $H_f = 0{,}579 L \cdot q_V$
- Velocities at points (0) and (3) are negligible
- Losses of head in the casing and the volute are not negligible.

1. Check pump operation.

To address the localized issue, it is proposed to reduce the distance between the pump and point (0) by 15m.

2. Analyze in detail the new operation, highlighting all the changes induced by the displacement of the pump (including an analysis of the velocity triangle at the impeller's exit), and then deduce the width of the impeller.

Solution :

Data :

Centrifugal pump	Upstream circuit	Downstream circuit
$H = -128,141q_V + 44,323$	$P_0 = 1bar$	$Hr_{dwn} = 23,405q_V^2 + H_f + 4,329$
$NPSH_{required} = 0,923 + 3,092q_V + 2,952q_V^2$	$L_{ups} = 18m$	$L_{dwn} = 33m$
$\beta_2 = 39°$	$S_{ups} = 0,0367m^2$	$S_{dwn} \neq S_{ups}$
$D_2 = 135\,mm \quad \rho = 1000\,kg/m^3$	$P_V = 0,625bar$	$H_f = 0,579L \cdot q_V$
$N = 2950rpm \quad \mu = 1 \quad \eta_H = 1$		H and Hr in [m] et q_V in $\left[m^3/s\right]$

1. Verification of the pump operation

Determination of the operating point

$$\mu = 1 \quad \eta_H = 1 \Rightarrow H_{th\infty} = H = -128,141q_V + 44,323$$

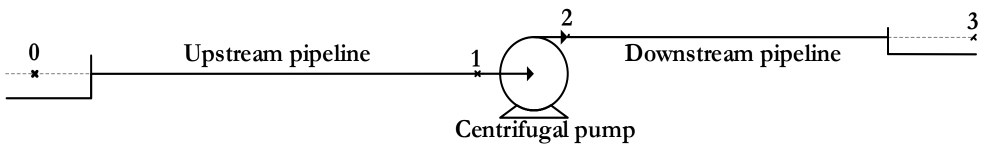
Operating point (OP1)

$$H_{th\infty} = Hr_{dwn} \Leftrightarrow -128,141q_V + 44,323 = 23,405q_V^2 + H_f + 4,329 = 23,405q_V^2 + 0,579L_{dwn}q_V + 4,329$$

$$\Delta = 25425,37 \Rightarrow \begin{cases} q_V' = 0,2608\,m^3/s \\ q_V'' = -6,552\,m^3/s < 0\,(\text{Excluded}) \end{cases}$$

$$\text{Finally} \rightarrow \begin{cases} q_{VOP1} = 0,2608\,m^3/s \\ H_{OP1} = H_{th\infty}(q_{VOP1}) = Hr_{dwn}(q_{VOP1}) = 10,9m \end{cases}$$

Cavitation verification : $NPSH_{required1} = 0,923 + 3,092q_{VOP1} + 2,952q_{VOP1}^2 = 1,9302m$



Bernoulli between (0) and (1)

$$\frac{P_0}{\rho g} + \frac{C_0^2}{2g} + z_0 = \frac{P_1}{\rho g} + \frac{C_1^2}{2g} + z_1 + 0,579L_{ups}q_V$$

$$Q_V = C_1 S_{ups} \Rightarrow C_1 = q_V / S_{ups} = 7,145\,m/s$$

$$\frac{P_1}{\rho g} = \frac{P_0}{\rho g} - \frac{C_1^2}{2g} - +0,579L_{ups}q_V = 4,874m$$

$$NPSH_{available1} = \frac{P_1}{\rho g} + \frac{C_1^2}{2g} - \frac{P_v}{\rho g} = 1,105m$$

$NPSH_{available} < NPSH_{required}$ → The operation of this pump carries a risk of damage due to cavitation.

2. Analysis of the new operation after moving the pump back by 15 meters.

$L_{ups} = 18 - 15 = 3m \qquad L_{dwn} = 33 + 15 = 48m$

Operating point (OP2)

$$H_{th\infty} = Hr_{dwn} \Leftrightarrow -128,141q_V + 44,323 = 23,405q_V^2 + 0,579L_{dwn}q_V + 4,329$$

$$\Delta = 28058,45 \Rightarrow \begin{cases} q_V^{'} = 0,2473\,m^3/s \\ q_V^{''} = -6,91\,m^3/s < 0\,(\text{Excluded}) \end{cases}$$

$$\text{Finally} \rightarrow \begin{cases} q_{VOP2} = 0,2473\,m^3/s \\ H_2 = H_{th\infty}(q_{VOP2}) = Hr_{dwn}(q_{VOP2}) = 12,63m \end{cases}$$

Cavitation verification : $NPSH_{required2} = 0,923 + 3,092q_{VOP2} + 2,952q_{VOP2}^2 = 1,868m$

Bernoulli between (0) and (1)

$$\frac{P_0}{\rho g} + \frac{C_0^2}{2g} + z_0 = \frac{P_1}{\rho g} + \frac{C_1^2}{2g} + z_1 + 0,579L_{ups}q_V$$

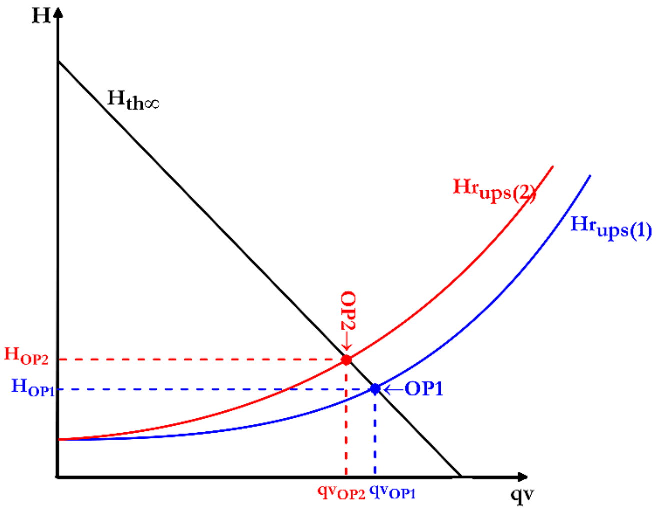
$$q_V = C_1 S_{ups} \Rightarrow C_1 = q_V / S_{ups} = 6,775\,m/s$$

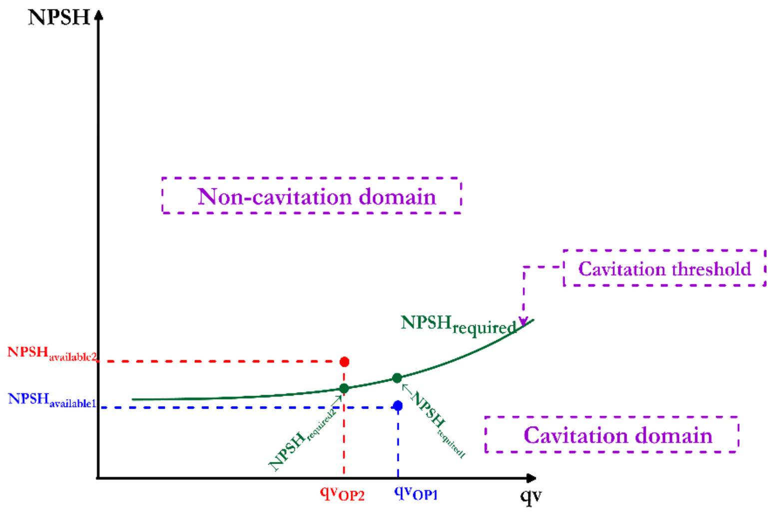
$$\frac{P_1}{\rho g} = \frac{P_0}{\rho g} - \frac{C_1^2}{2g} - +0,579L_{ups}q_V = 7,425m$$

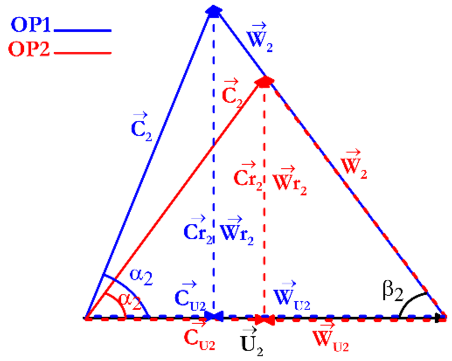
$$NPSH_{available2} = \frac{P_1}{\rho g} + \frac{C_1^2}{2g} - \frac{P_v}{\rho g} = 3,393m$$

$NPSH_{available} > NPSH_{required}$ → So, this new pump operation poses no risk of cavitation damage.

- Summary and deduction of the impeller thickness







Theoretical Euler head at infinite number of blades :

$$H_{th\infty} = \frac{U_2 C_{U2} - U_1 C_{U1}}{g} \quad \text{With } C_{U1} = 0 \left(\alpha_1 = 90°\right)$$

$U_2 = D_2 \pi N/60 = 20{,}852\,\text{m/s}$

$C_{U2} = \dfrac{gH_{th\infty}}{U_2}$

$\beta_2 = 39° < 90° \Rightarrow W_{U2} = U_2 - C_{U2}$

$\tan\beta_2 = W_{r2}/W_{U2} = C_{r2}/W_{U2} \Rightarrow C_{r2} = W_{U2}\tan\beta_2$

$\alpha_2 = \arctan\left(C_{r2}/C_{U2}\right)$

$C_2 = C_{U2}/\cos\alpha_2 = C_{r2}/\sin\alpha_2$

	q_V $[m^3/s]$	C_{U2} $[m/s]$	W_{U2} $[m/s]$	C_{r2} $[m/s]$	$\alpha_2\,[°]$	C_2 $[m/s]$	$q_V = D_2\pi b_2 C_{r2} \Rightarrow b_2 = q_V/D_2\pi C_{r2}$
OP1	0,2608	5,128	15,724	12,733	68,06	13,727	0,0483 m
OP2	0,2473	5,942	14,910	12,074	63,80	13,457	0,0483 m

Chapter V

Hydraulic Turbines

Table of contents

1. Introduction

Hydraulic turbines are devices designed to harness the energy contained in fluids, primarily water. This process relies on the availability of a fluid at a certain elevation, creating a difference in elevation or pressure. This difference is crucial as it enables the fluid to transfer its kinetic and potential energy to the turbine shaft, which is located at a lower level. The ingenuity of this technology lies in the turbine's ability to convert this mechanical energy into electrical energy. To achieve this, the turbine is coupled to a generator through a mechanical coupling system (see Figure 1).

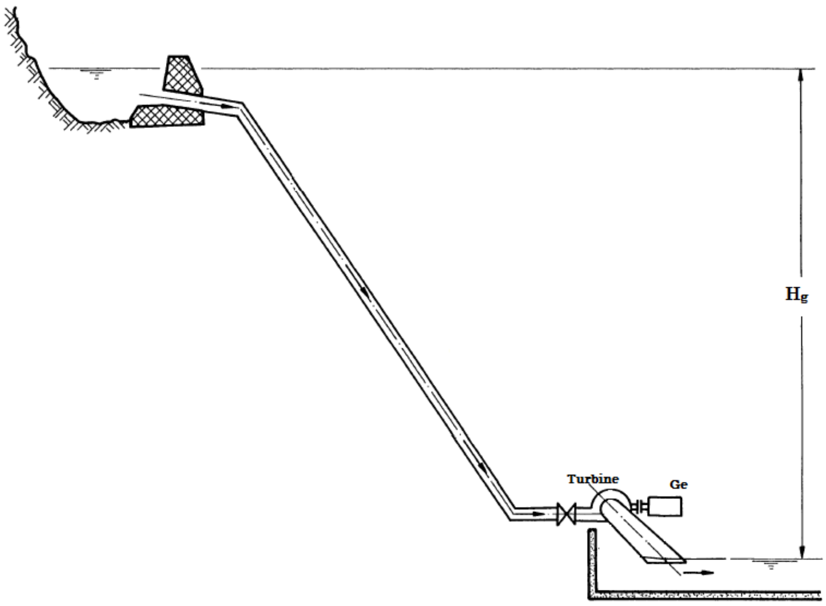


Figure 1. Diagram of a typical hydroelectric project

2. Parameters common to all turbine types

- Turbine hydraulic power :

Hydraulic power is the power supplied to the turbine by the water that feeds it. It is determined by the product of hydraulic head (gH) and mass flow rate. $\left(q_m = \rho q_v\right)$ (on Figure 1 : $H=H_g$).

$$P_{hyd} = P_f = \rho q_v \, gH \tag{V.1}$$

- Torque (M) in [Nm]:

The pressurized water entering the turbine exerts a hydrodynamic force on the blades or buckets of the wheel. This force generates a torque that sets the wheel in motion.

- Rotation speed (N) en [rpm]:

  Once set in motion, the turbine will rotate at a rotational speed determined by the operating conditions. $\left(\omega = \frac{2\pi N}{60} \quad \text{in [rad / s]}\right)$

- Mechanical power on the turbine shaft [W] :

  According to the laws of physics, mechanical power is given by the product of torque and rotational speed :

$$P_{mec} = \omega M \tag{V.2}$$

- Efficiency :

  Any transformation of energy in a machine gives rise to losses. As a result, the power obtained at the turbine shaft, which is used to drive the generator, is less than the hydraulic power. The ratio between these two powers is the efficiency, a parameter that defines the quality of the turbine :

$$\eta_t = \eta_{turbine} = \frac{P_{mec}}{P_{hyd}} \tag{V.3}$$

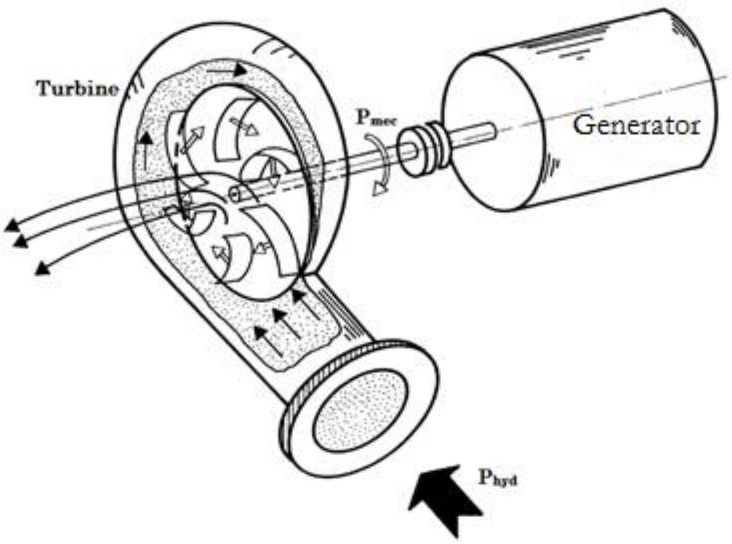


Figure 2. Basic turbine operation

3. Action turbines

a. Operating principle

The action turbine is distinguished by the fact that all the available energy for the blades is in the form of kinetic energy. In this type of turbine, the exchange of energy between the water and the blades occurs at constant pressure, which is often equivalent to atmospheric pressure in the context of hydraulic turbines. This fundamental characteristic enables the action turbine to efficiently convert the kinetic energy of the water into mechanical energy.

b. Pelton Turbine (Lester Allan Pelton (1829-1908)

The Pelton turbine consists of a bucket wheel set in motion by a jet of water from an injector. The buckets are profiled for maximum efficiency, while allowing water to escape at the sides of the wheel. They feature a notch to ensure optimum jet progression through the trough. The injector is designed to produce a cylindrical jet that is as homogeneous as possible, with minimum dispersion. See Figure 3.

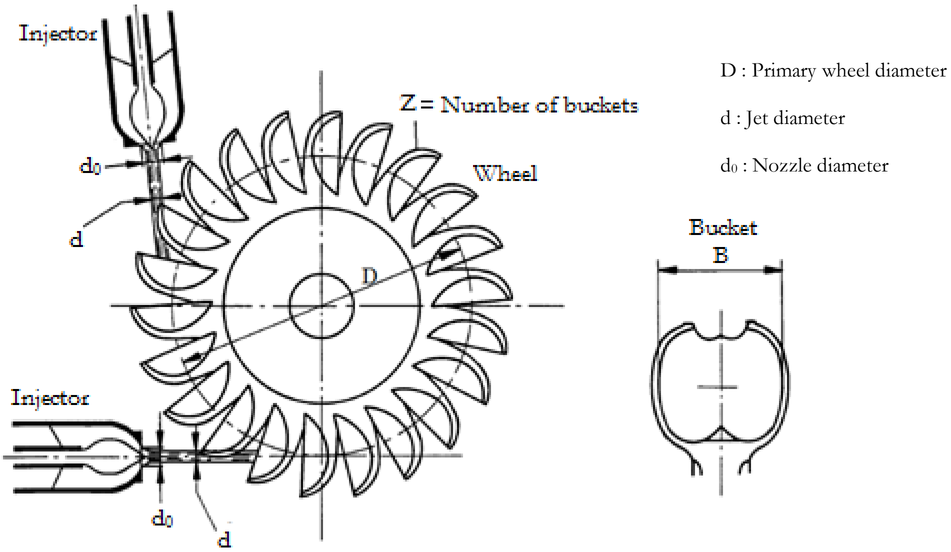


Figure 3. Schematic view of a Pelton wheel with two nozzles

c. Injecteurs

Pelton turbines can be configured with multiple nozzles to provide greater flexibility in adjusting the water flow. This adjustment is made using a movable needle integrated inside each nozzle, allowing for precise control of the amount of water entering the turbine. This feature is particularly useful for adapting energy production to changes in demand or environmental conditions. Another important component of the Pelton turbine is the deflector. This device can be quickly positioned between the nozzle and the turbine wheel to divert the water jet. Its primary role is to assist in rapidly stopping the turbine by diverting the water jet away from the Pelton cups. By doing so, the deflector temporarily allows the jet to function as a brake, effectively helping to halt the turbine's rotation when needed. This functionality is crucial for the safety and control of the Pelton turbine in various situations. (See Figure 4)

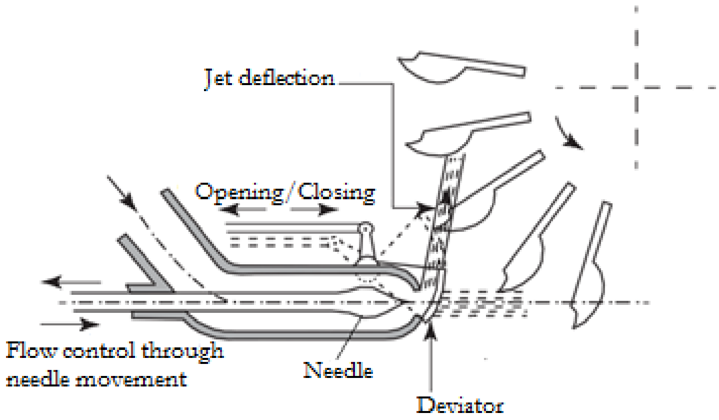


Figure 4. Injector flow control

d. Analysis of Turbine Operation

The total head, denoted as H_g, at the project site represents the variation in water level between the reservoir and the tailrace canal. This measurement is crucial for determining the availability of exploitable hydraulic energy by the system.

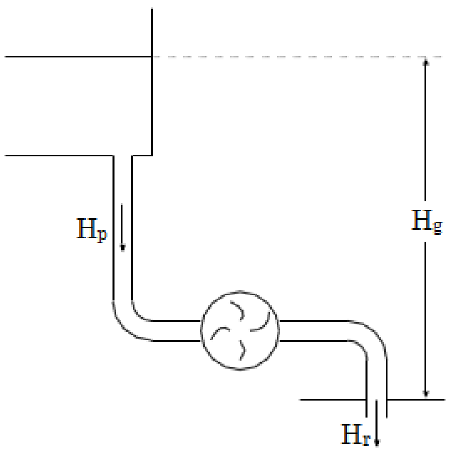


Figure 5. Diagram of different heights

However, it's important to note that the actual usable head at the buckets, denoted as H, needs to account for various factors. Firstly, consideration must be given to the singular and regular head losses occurring in the piping, represented as (H_P). These head losses reduce the effective head that water can provide for generating mechanical energy. Additionally, it is essential to take into account the height of the buckets above the level of the tailrace canal, denoted as (H_r). This component is crucial as it indicates the vertical distance between the turbine buckets and the point where the water is discharged into the tailrace canal. This height directly contributes to the amount of energy the turbine can extract from the water flow. See Figure 5.

All this means :

$$H_g = H + H_p + H_r \tag{V.4}$$

Therefore, the velocity of the water jet at the injector outlet or at the rotor bucket inlet is expressed as follows :

$$C_1 = k_c\sqrt{2gH} \tag{V.5}$$

Where (k_c) is the velocity correction coefficient..

The wheel's peripheral speed is :

$$U = \frac{\pi DN}{60} \tag{V.6}$$

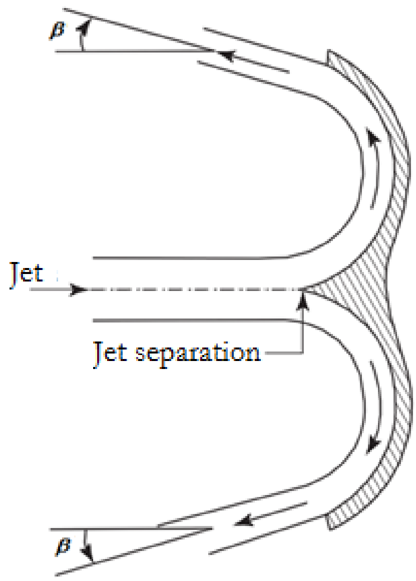
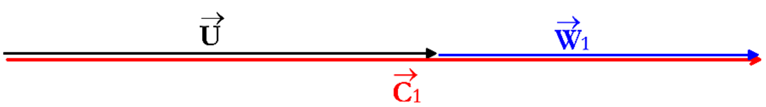


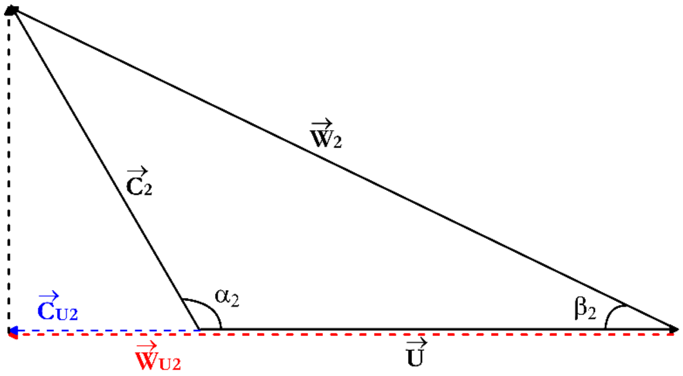
Figure 6. Flow of a jet over the bucket of a Pelton turbine

Respecting the configuration of Figure 6; the velocity triangles will appear as follows :

<u>Input</u>



Output



e. **Efficiencies**

- Hydraulic efficiency :

  Hydraulic efficiency is defined as the ratio between the ideal power supplied by the fluid to the rotor and the power available in the flow of water entering the buckets. $\left(\eta_h = \frac{P_{rf}}{P}\right)$. Thus, we can write :

$$\eta_h = \frac{U_1 C_{U_1} - U_2 C_{U_2}}{C_1^2/2} = \frac{U\left(C_{U_1} - C_{U_2}\right)}{C_1^2/2} \tag{V.7}$$

- Volumetric efficiency:

  The definition of volumetric efficiency is as follows

$$\eta_v = \frac{q_v - q_{v_f}}{q_v} \tag{V.8}$$

  In the case of the Pelton turbine, $\left(q_{v_f}\right)$ is associated with the "inefficient" volume flow present in the outer layers of the water jet, which may be less efficient than the part of the jet that directly impacts the blades.

- Mechanical efficiency:

  This efficiency is defined as. :

$$\eta_m = \frac{P_r - P_m}{P_r} \tag{V.9}$$

  Where $\left(P_r\right)$ is the shaft power and $\left(P_m\right)$ represents all mechanical losses.

4. Reaction Turbines

A reaction turbine is a closed (flooded) machine that takes advantage of both water velocity, representing kinetic energy, and a pressure difference. This combination of kinetic energy and pressure variations enables the turbine to efficiently convert hydraulic energy into mechanical energy.

a. Francis turbine

The operation of a Francis turbine can be compared to that of a reversed centrifugal pump, classifying it as a centripetal turbomachine. Water from a conduit enters a spiraling outer casing. At the inlet of the wheel, the turbine is equipped with a set of guide vanes that ensures a uniform distribution of water inwards. These guide vanes act as nozzles, transforming a portion of the water's pressure energy into kinetic energy. The water is then directed towards the rotor blades, where it gains both kinetic and pressure energy. The rotor blades capture these forms of energy and propel the water into the draft tube. This tube is specially designed to decelerate the fluid, increase its pressure, and thus prevent cavitation, which could be detrimental to the turbine's operation. (Figure 7)

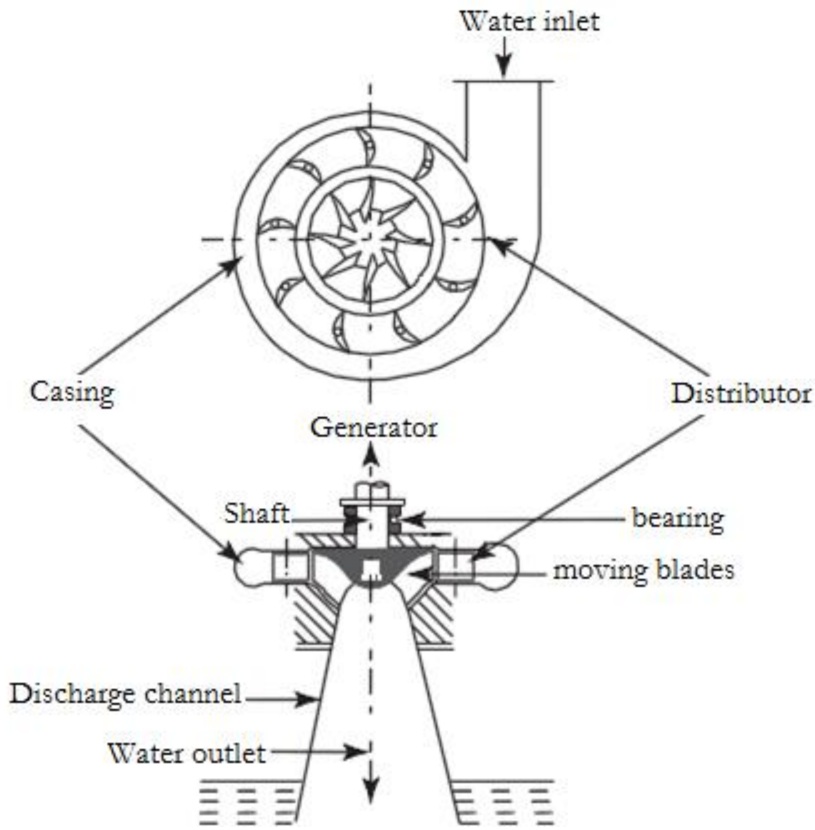


Figure 7. Francis turbine diagram

i. Analysis of turbine operation

For the Francis turbine, the flow is typically perpendicular to the axis of the movable blades at the inlet, and then it changes orientation to become parallel to the axis of turbine rotation, creating an axial discharge flow.

In this context, the complete set of velocity triangles is illustrated in Figure 8, providing a better understanding of how water interacts with the turbine components to produce energy.

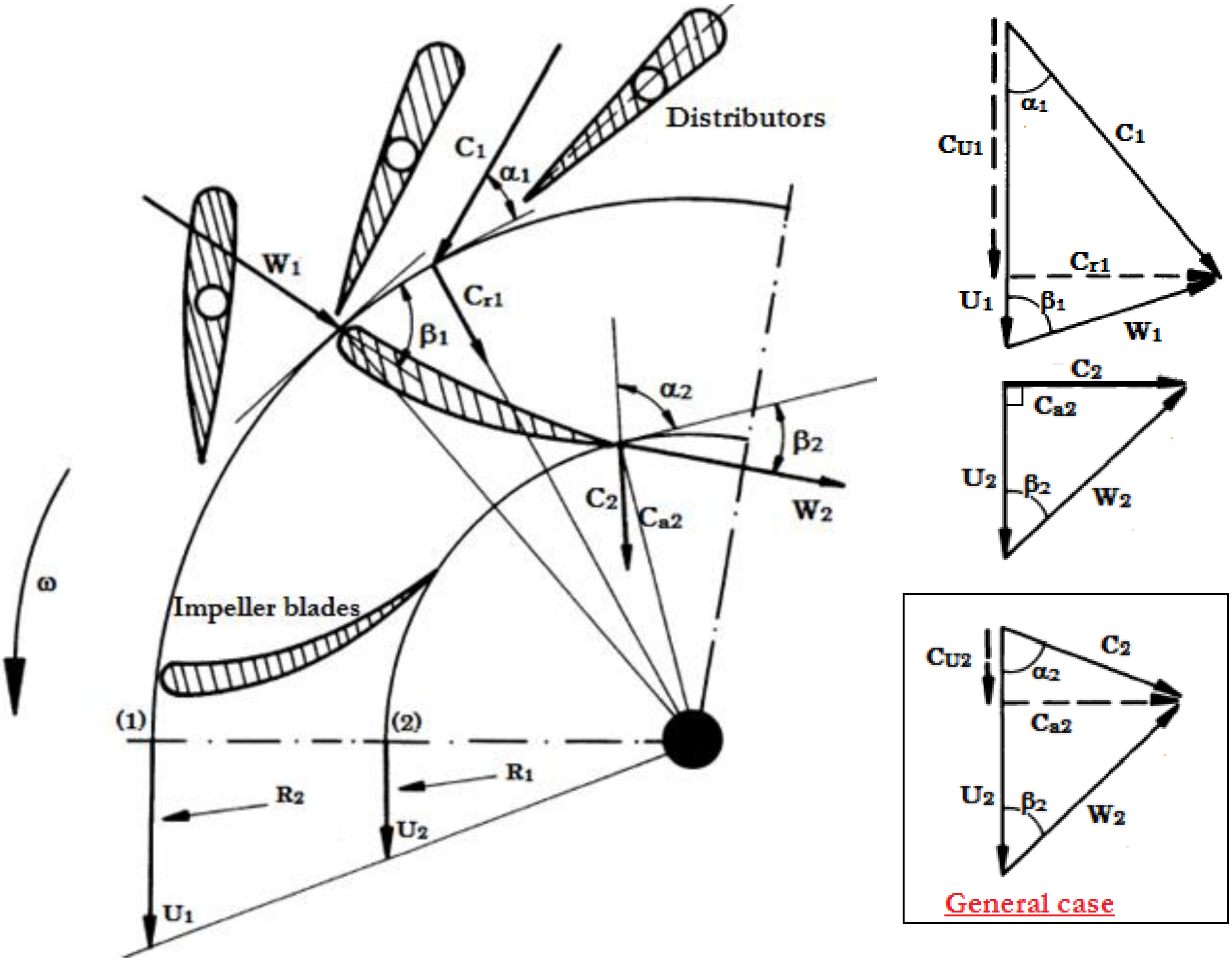


Figure 8. Velocity triangle of a Francis turbine

The velocity triangle at the exit of the blades with $(\alpha_2 = 90°)$, should be taken into consideration in the hydraulic efficiency formula. As for other efficiency expressions, such as volumetric efficiency, mechanical efficiency, and overall efficiency, which were detailed for the Pelton turbine in the previous section, they are also applicable to the Francis turbine. These expressions remain valid for the Francis turbine as they are independent of the specific geometric elements and energy transfer mechanism of these turbines, providing a strong foundation for the analysis of their performance.

b. **Kaplan turbine (Victor Kaplan (1876-1934)**

The main components of a Kaplan turbine are very similar to those of a Francis turbine. They include a spiral casing, a nozzle equipped with guide vanes, a rotor and a diffuser.

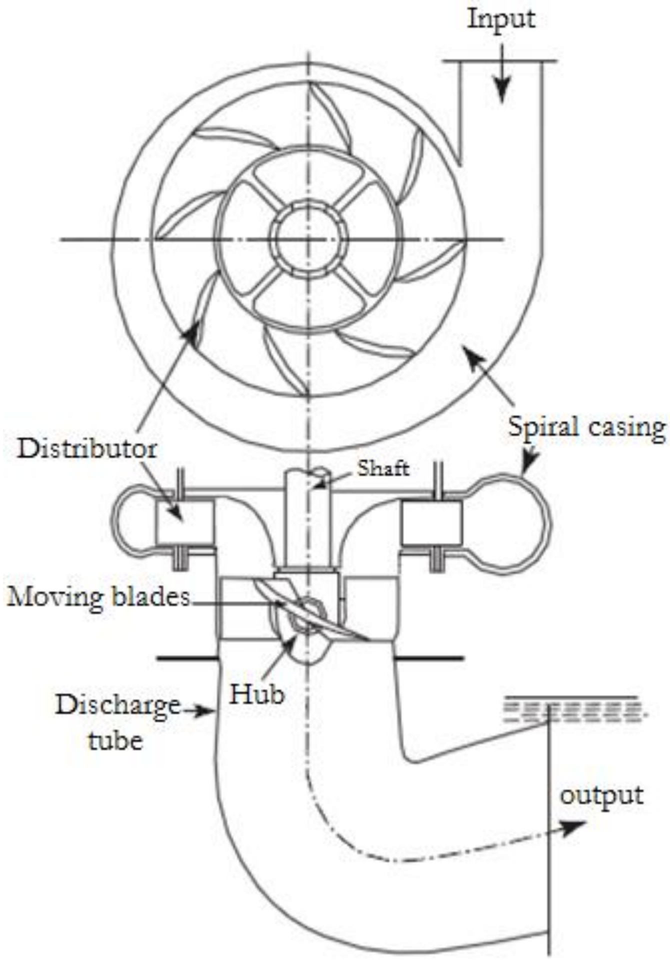


Figure 9. Kaplan turbine components

i. Analyse du fonctionnement de la turbine

In this type of turbine, the flow is predominantly axial, which means that the peripheral velocity at the inlet and outlet of the rotor remains essentially constant $(U_1 = U_2 = U)$.

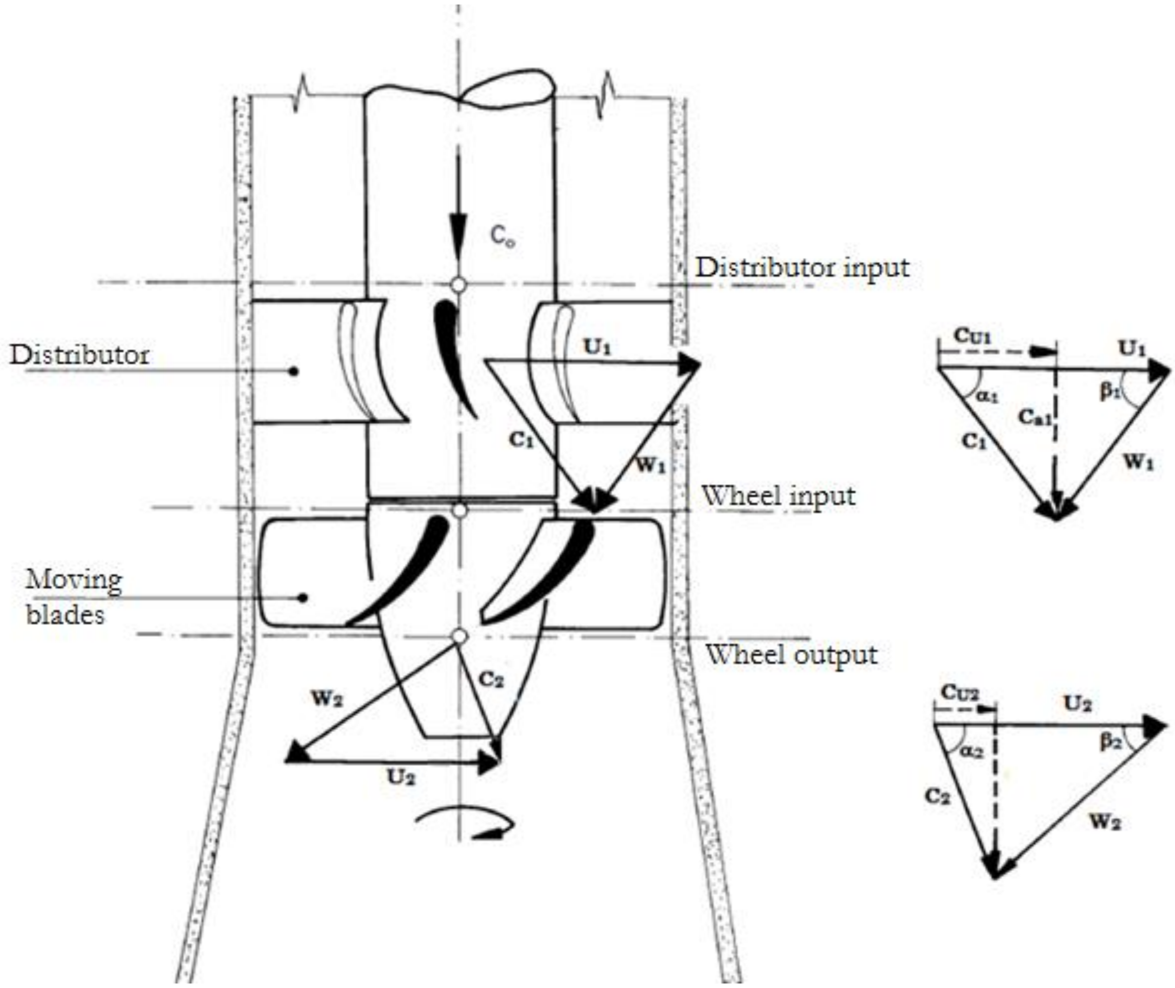


Figure 10. Keplan turbine velocity triangle

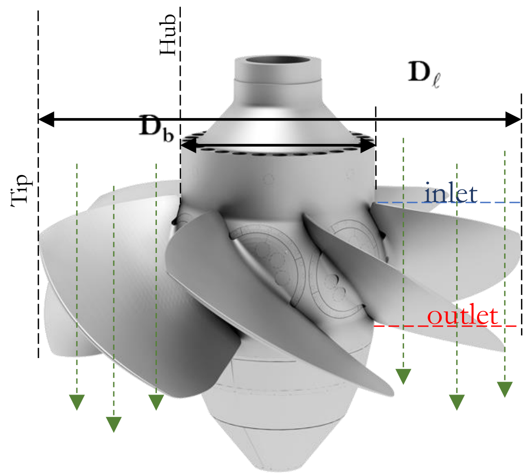
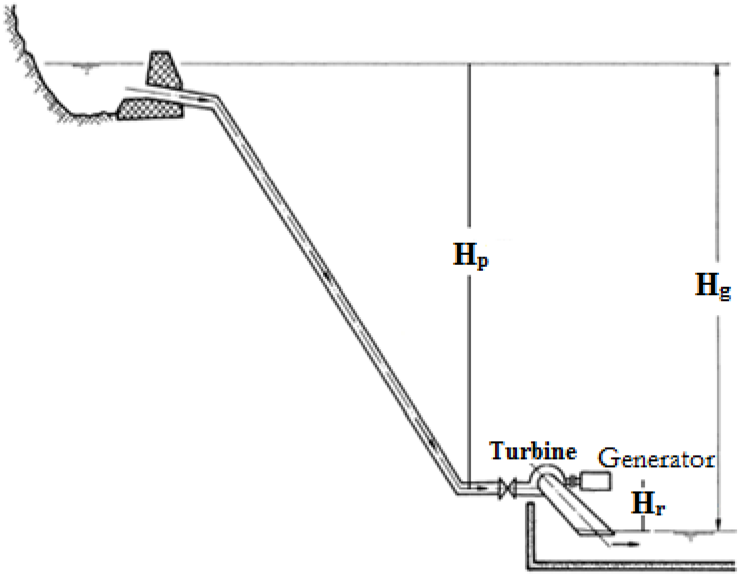


Figure 11. Schematization of the flow domain within a Kaplan turbine wheel

5. Exercises

- **Exercise n°1 :** Pelton turbine

In order to study the operation of a Pelton turbine with a rotational speed of 850 rpm located at a site with an available head of 750m (see the figure below).



We are provided with the following information :

- The expected power output from the site is 3750 kW.
- The total head losses in the pipes upstream and downstream of the turbine are estimated to be 7% and 1% of the gross head, respectively (Hp = 7% Hg and Hr = 1% Hg).
- The friction factor for the inlet pipe, which has a length of 650 meters, is estimated to be 0.0072.
- The overall efficiency of the turbine is 80%.
- The velocity ratio ($\phi = U/C_1 = 0,48$) and the nozzle coefficient ($k_C = 0,985$).
- Mechanical losses around the rotating components are approximately 522,6716 kW

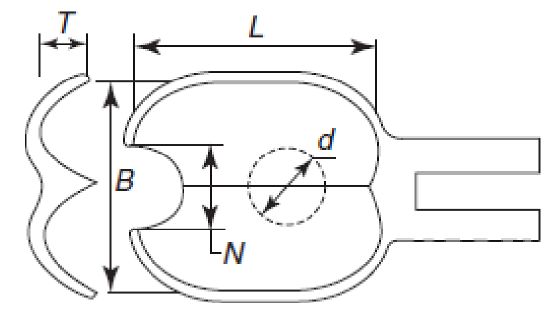
Assuming that:

- Flow over the bucket is frictionless ($W_1=W_2$), with an exit inclination $\beta_2=15°$.
- The flow exit inclination from the bucket is characterized by $\alpha_2 > 90°$.
- Inefficiency of water in the system is estimated at 0,02 m^3/s.
- The Pelton turbine has only a single nozzle

Questions :

1. Deduct the turbine (process) operating flow rate.

2. Calculate the outgoing jet velocity from the injector.
3. Determine the diameter of the pipe upstream of the turbine.
4. Calculate impeller diameter and jet diameter.
5. Calculate the dimensions of the trough (see diagram below)
6. Calculate hydraulic, volumetric and mechanical efficiency.



T=2.3d (Depth)
B=2,8d (Width)
L=0,6d (Length)

$$Z = \frac{0,5D}{d} + 15 \text{ (Number of buckets)}$$

Bucket proportion

Solution :

Data :

$$N = 850\,\text{rpm} \qquad H_g = 750\,\text{m} \qquad P_t = 3750\,\text{kW} \qquad \eta = 80\%$$

1. Turbine operating flow rate

$$\eta = \frac{P_t}{P_h} \Rightarrow P_t = \eta \cdot P_h = \eta \cdot \rho g H q_V \Rightarrow q_V = \frac{P_t}{\eta \cdot \rho g H}$$

H : Available height on the distributors.

$$H = H_g - H_p - H_r = H_g - 7\%H_g - 1\%H_g$$

H_p : Pressure losses in the inlet pipeline.

H_r : Pressure losses in the discharge pipeline

$$\rightarrow \boxed{H = 690\,\text{m}}$$

Finally,

$$\rightarrow \boxed{q_V = 0,6925\, m^3/s}$$

2. Flow velocity at the jet outlet

$$C_1 = k_c\sqrt{2gh} \qquad h = H = 690\,\text{m} \qquad \rightarrow \boxed{C_1 = 114,607\,m/s}$$

3. Diameter of the piping upstream of the turbine

$$H_p = k_f \frac{C_p^2 \ell}{2gD_p}$$

ℓ=650m k_f=0,0072

C_p : Pipeline flow velocity

$$q_V = C_p S \Rightarrow C_p = \frac{q_V}{S} = \frac{4q_V}{\pi D_p^2}$$

$$\rightarrow H_p = k_f \frac{16\ell q_V^2}{2g\pi^2 D_p^5} \Rightarrow D_p = \sqrt[5]{k_f \frac{8\ell q_V^2}{g\pi^2 H_p}} = \sqrt[5]{k_f \frac{8\ell q_V^2}{g\pi^2 7\% H_g}} \qquad \rightarrow \boxed{D_p = 0,323m}$$

4. 1. Wheel diameter

$$U = r\omega = \frac{D\pi N}{60} \Rightarrow D = \frac{60U}{\pi N} \qquad \left(\text{knowing that: } \phi = U/C_1 = 0,48\right)$$

$$\Rightarrow D = \frac{60\phi C_1}{\pi N}$$

$$\rightarrow \boxed{D_r = 1,236m} \qquad \boxed{U = 55,011 m/s}$$

2. Jet diameter

$$qv_{tubine} = \left(q_V - qv_f\right) = \frac{\pi}{4} d^2 C_1 \Rightarrow d = \sqrt{\frac{4\left(q_V - qv_f\right)}{\pi C_1}} \qquad \rightarrow \boxed{d = 0,086m}$$

5. Buckets dimensions

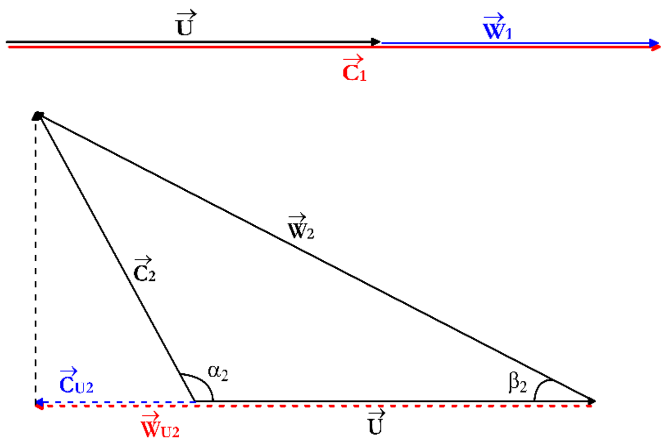
$$T = 2,3 \times d \quad (\text{Depth}) \quad \rightarrow \boxed{T = 0,1978m}$$

$$B = 2,8 \times d \quad (\text{Width}) \quad \rightarrow \boxed{B = 0,2408m}$$

$$L = 0,6 \times d \quad (\text{Length}) \quad \rightarrow \boxed{L = 0,0516m}$$

$$Z = 0,5 \times \frac{D}{d} + 15 \quad (\text{Number of buckets}) \quad \rightarrow \boxed{Z = 22,186 = 23}$$

6. 1. Hydraulic efficiency



$$\eta_H = \frac{U_1 C_{U_1} - U_2 C_{U_2}}{C_1^2/2} \qquad \left(C_{U_1} = C_1, U = U_1 = U_2\right)$$

$$\eta_H = \frac{2U\left(C_1 - C_{U_2}\right)}{C_1^2} = \frac{2U}{C_1^2}\left(C_1 - \left(W_2 \cos\beta_2 - U\right)\right) \qquad \left(W_2 = W_1\right)$$

$$= \frac{2U}{C_1^2}\left(C_1 - \left(W_1 \cos\beta_2 - U\right)\right) = \frac{2U}{C_1^2}\left(C_1 + U - W_1 \cos\beta_2\right)$$

$W_1 = C_1 - U$: see diagram below

$$\eta_H = \frac{2U}{C_1^2}\left(C_1 + U - \left(C_1 - U\right)\cos\beta_2\right) = \frac{2U}{C_1}\frac{\left(C_1 + U - \left(C_1 - U\right)\cos\beta_2\right)}{C_1}$$

$$= 2\phi\left(\left(1+\phi\right) - \left(1-\phi\right)\cos\beta_2\right) \qquad \left(\phi = \frac{U}{C_1}\right)$$

$$\rightarrow \boxed{\eta_H = 0{,}93861}$$

2. Volumetric efficiency

$$\eta_V = \frac{q_V - q_{v_f}}{q_V} \qquad \left(q_{v_f} = 0{,}02\, m^3/s\right)$$

$$\rightarrow \boxed{\eta_V = 0{,}97112}$$

3. Mechanical efficiency

Method 1

$$\eta = \eta_H \cdot \eta_V \cdot \eta_M \Rightarrow \eta_M = \frac{\eta}{\eta_H \cdot \eta_V}$$

$$\rightarrow \boxed{\eta_M = 0{,}87767}$$

Method 2

$$\eta_M = \frac{P_r - P_m}{P_r} \qquad \left(P_m : \text{Mechanical losses} = 522{,}6716\text{kW}\right)$$

P_r = Rotor power

$$P_r = \eta_V \cdot \eta_H \cdot P_h = \eta_V \cdot \eta_H \cdot \rho g H q_V$$

$$\boxed{P_r = 4272{,}637\text{kW}}$$

$$\eta_M = \frac{P_r - P_m}{P_r} \rightarrow \boxed{\eta_M = 0{,}87767}$$

- **<u>Exercise n°2 :</u>** <u>Francis turbine</u>

A Francis turbine designed for a site with a net head of 10m operates at 300 rpm and has a flow rate of 2 m^3/s.

Given that:

- The velocity ratio $(\phi = U_1/C_1)$ is assumed to be 0.8.
- Hydraulic losses in the turbine represent 15% of the available energy.
- The ratio (D_2/D_1) is estimated to be 0.7.
- The outflow is purely axial.
- Water exits the rotor with no swirl ($C_{r1}=C_{a2}$).

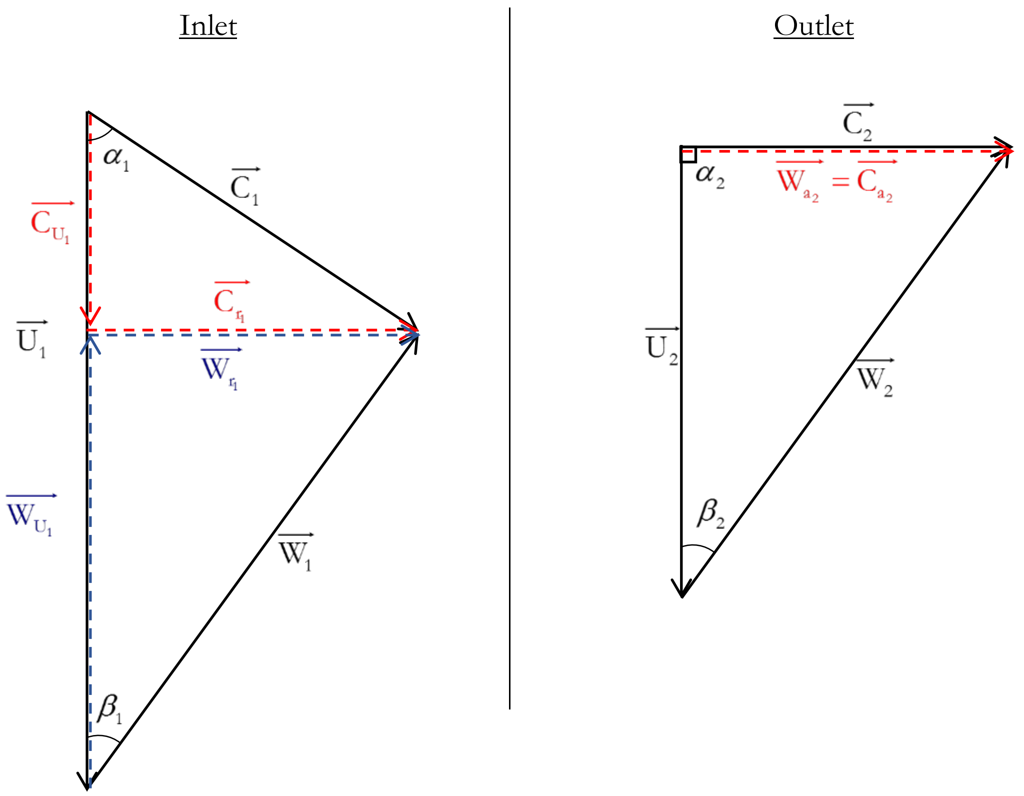
Questions :

1. Draw the velocity triangles at the inlet and outlet of the rotor.
2. Calculate the dimensions of the rotor and all elements of each velocity triangle.
3. Determine the thickness at the inlet and outlet of the rotor

Solution :

$N=300\,rpm \qquad h=10\,m \qquad q_V = 2\,m^2/s$

1. Velocity triangle



2. 1. Rotor dimensions

$C_1 = \sqrt{2gh} \qquad \rightarrow \boxed{C_1 = 14{,}01\,m/s}$

$$\phi = \frac{U_1}{C_1} \Rightarrow U_1 = \phi \cdot C_1 \qquad \rightarrow \boxed{U_1 = 11,21\,m/s}$$

$$U_1 = r_1\omega = \frac{D_1 \pi N}{60} \Rightarrow D_1 = \frac{60 U_1}{\pi N} \qquad \rightarrow \boxed{D_1 = 0,713m}$$

$$\frac{D_2}{D_1} = 0,7 \Rightarrow D_2 = 0,7 \cdot D_1 \qquad \rightarrow \boxed{D_2 = 0,499m}$$

2. Elements of the velocity triangles

- Inlet:

$$\eta_H = 1 - 15\% = 85\% = \frac{U_1 C_{U_1} - U_2 C_{U_2}}{C_1^2/2}$$

Purely axial outlet :$\rightarrow \alpha_2 = 90°$ et $C_{U2} = 0$

$$\eta_H = \frac{2 U_1 C_{U_1}}{C_1^2} \Rightarrow C_{U_1} = \frac{\eta_H C_1^2}{2U_1} \qquad \rightarrow \boxed{C_{U_1} = 7,44\,m/s}$$

$$C_{U_1} = C_1 \cos\alpha_1 \Rightarrow \alpha_1 = \arccos\left(\frac{C_{U_1}}{C_1}\right) \qquad \rightarrow \boxed{\alpha_1 = 57,91°}$$

$$C_{r_1} = C_1 \sin\alpha_1 \qquad \rightarrow \boxed{C_{r_1} = 11,86\,m/s = W_{r_1} = C_{a_2} = C_2}$$

$$W_{U_1} = U_1 - C_{U_1} \qquad \rightarrow \boxed{W_{U_1} = 3,76\,m/s}$$

$$\tan\beta_1 = \frac{W_{r1}}{W_{U1}} \Rightarrow \beta_1 = \arctan\left(\frac{W_{r1}}{W_{U1}}\right) \rightarrow \boxed{\beta_1 = 72,40°}$$

- Outlet : $\alpha_2 = 90°$ et $C_{U2} = 0$

$\rightarrow \boxed{C_{a2} = W_{a2} = C_2 = 11,86\,m/s}$ (Already calculated previously)

$$U_2 = r_2\omega = \frac{D_2 \pi N}{60} = W_{U2} \qquad \rightarrow \boxed{U_2 = 7,84\,m/s = W_{U_2}}$$

$$\beta_2 = \arctan\left(\frac{W_{a_2}}{W_{U_2}}\right) \qquad \rightarrow \boxed{\beta_2 = 56,54°}$$

$$W_{U_2} = W_2 \cos\beta_2 \Rightarrow W_2 = \frac{W_{U_2}}{W_2} \rightarrow \boxed{W_2 = 14,23\,m/s}$$

3. Wheel thickness :

- Inlet :

$$q_V = 2\pi r_1 b_1 C_{r1} \Rightarrow b_1 = \frac{q_V}{\pi D_1 C_{r1}} \quad \rightarrow \boxed{b_1 = 0{,}075\text{m}}$$

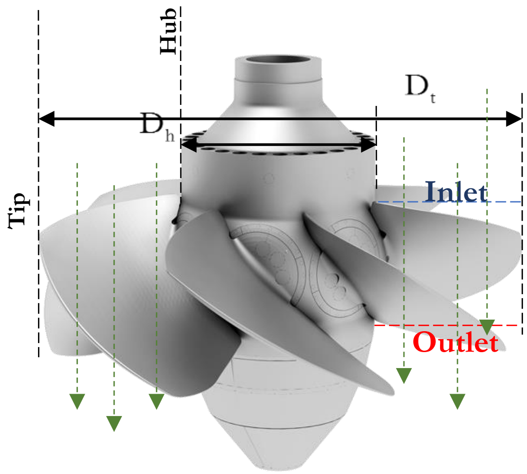
- Outlet :

$$Q_V = 2\pi r_2 b_2 C_{a2} \Rightarrow b_2 = \frac{q_V}{\pi D_2 C_{a2}} \quad \rightarrow \boxed{b_2 = 0{,}107\text{m}}$$

- **Exercise n°3 :** Kaplan turbine

The data for a Kaplan turbine are as follows :

- The velocity ratio $(\phi = U_{1t}/C_1)$ and $(\psi = C_{a_1}/C_1)$ are 2.0 and 0.65 respectively.
- The ratio of hub diameter (D_h) to tip diameter (D_t) is 0.3.
- Hydraulic efficiency and overall efficiency are estimated at 92% and 85% respectively.
- The operating height of the turbine is 4 m.
- The turbine develops up to 8,000 kW of power.
- The inlet flow is free vortex (C_U R=Cst).



Questions :

- Under the assumption that the discharge is axial, and the flow components in that direction remain constant (C_a=Cst) across the rotor (along the blade height)

Determine :

1. Turbine operating flow rate
2. Rotor diameters (hub diameter and tip diameter)

3. Values of the elements of each velocity triangle
4. Using the values obtained, draw the associated speed triangles.

Solution :

Data :

$\phi = 2 = \frac{U_t}{C_1}$ $\quad \psi = 0,65 = \frac{C_{a_1}}{C_1}$ $\quad H = 4\,m$ $\quad \eta = 85\%$

$\eta_H = 92\%$ $\quad P_t = 8000\,kW$ $\quad \frac{D_b}{D_\ell} = 0,3$

1. Turbine operating flow rate

$$\eta = \frac{P_t}{P_f} \Rightarrow P_t = \eta \cdot P_f = \eta \cdot \rho g H q_V$$

$$\Rightarrow q_V = \frac{P_t}{\eta \rho g H} \qquad \rightarrow \boxed{q_V = 239,85\,m^3/s}$$

2. Determining different diameters

$$C_1 = \sqrt{2gh} \qquad \rightarrow \boxed{C_1 = 8,859\,m/s}$$

$$\phi = \frac{U_t}{C_1} = 2 \Rightarrow U_t = \phi \cdot C_1$$

$$\rightarrow \boxed{U_t = 17,718\,m/s = U_{t1} = U_{t2}} \text{ (Axial flow)}$$

$$\psi = 0,65 = \frac{C_{a_1}}{C_1} \Rightarrow C_{a_1} = \psi \cdot C_1 \qquad \rightarrow \boxed{C_{a_1} = 5,758\,m/s}$$

$$Q_V = S \cdot C_{a_1} = \frac{\pi}{4}\left(D_t^2 - D_h^2\right)C_{a_1} = \frac{\pi}{4}\left(D_t^2 - 0,3^2 D_t^2\right)C_{a_1} = \frac{\pi}{4}\left(1 - 0,3^2\right)D_t^2 C_{a_1}$$

$$\Rightarrow D_t = \sqrt{\frac{4 q_V}{\pi\left(1 - 0,3^2\right)C_{a_1}}}$$

$$\rightarrow \boxed{D_t = 7,634\,m}$$

$$D_h = 0,3 \cdot D_t \qquad \rightarrow \boxed{D_h = 2,290\,m}$$

3. Elements of velocity triangles

Hub

Inlet

$$U_h = r_h \omega = \frac{D_h \pi N}{60} = \frac{D_h}{D_t}\frac{\pi N}{60} D_t$$

$$= \frac{D_h}{D_t} U_t = 0,3 U_t$$

$$U_h = \frac{D_h}{D_t} U_t$$

$$\rightarrow \boxed{U_h = U_{h_1} = U_{h_2} = 5,32\,\mathrm{m/s}}$$

$$C_{1_h} = 8,859\,\mathrm{m/s}$$

$$\eta_H = \frac{U_{1_h} C_{U_{1_h}} - U_{2_h} C_{U_{2_h}}}{C_{1_h}^2 / 2}$$

Sortie axiale $\rightarrow C_{U_{2_h}} = 0$

$$\Rightarrow \eta_H = \frac{U_{1_h} C_{U_{1_h}}}{C_{1_h}^2 / 2} \Rightarrow C_{U_{1_h}} = \frac{C_{1_h}^2}{2} \frac{\eta_H}{U_{1_h}}$$

$$\rightarrow \boxed{C_{U_{1_h}} = 6,79\,\mathrm{m/s}}$$

$$C_{a_{1_h}} = 5,758\,\mathrm{m/s}$$

$$\alpha_{1_h} = \arctan\left(\frac{C_{a_{1_h}}}{C_{U_{1_h}}}\right)$$

$$\rightarrow \boxed{\alpha_{1_h} = 40,29°}$$

$$W_{U_{1_h}} = C_{U_{1_h}} - U_{1_h} \qquad \left(C_{U_{1_h}} > U_{1_h}\right)$$

$$\rightarrow \boxed{W_{U_{1_b}} = 1,47\,\mathrm{m/s}}$$

$$\beta'_{1_h} = \arctan\left(\frac{C_{a_{1_h}}}{W_{U_{1_h}}}\right) = 75,62°$$

$$\rightarrow \boxed{\beta_{1_h} = 104,38°}$$

$$W_{a_{1_h}} = C_{a_{1_h}} = 5,76\,\mathrm{m/s}$$

Tip

$$U_t = U_{t_1} = U_{t_2} = 17,72\,\mathrm{m/s}$$

$$U_t = \frac{\pi N}{60} D_t \Rightarrow N = \frac{60 U_t}{\pi D_t}$$

$$\rightarrow \boxed{N = 44,32\,\mathrm{tr/min}}$$

$$C_{1_t} = 8,859\,\mathrm{m/s}$$

$$\eta_H = \frac{U_{1_t} C_{U_{1_t}}}{C_{1_t}^2 / 2} \Rightarrow C_{U_{1_t}} = \frac{C_{1_t}^2}{2} \frac{\eta_H}{U_{1_t}}$$

$$\rightarrow \boxed{C_{U_{1_t}} = 2,04\,\mathrm{m/s}}$$

$$C_{a_{1_t}} = 5,758\,\mathrm{m/s}$$

$$\alpha_{1_t} = \arctan\left(\frac{C_{a_{1_t}}}{C_{U_{1_t}}}\right)$$

$$\rightarrow \boxed{\alpha_{1_t} = 70,51°}$$

$$W_{U_{1_t}} = U_{1_t} - C_{U_{1_t}}$$

$$\rightarrow \boxed{W_{U_{1_t}} = 15,68\,\mathrm{m/s}}$$

$$\beta_{1_t} = \arctan\left(\frac{C_{a_{1_t}}}{W_{U_{1_t}}}\right) = 20,16°$$

$$W_{a_{1_t}} = C_{a_{1_t}} = 5,76\,\mathrm{m/s}$$

$$W_{1_t} = \sqrt{C_{a_{1_t}}^2 + W_{U_{1_t}}^2}$$

$$\rightarrow \boxed{W_{1_t} = 16,70\,\mathrm{m/s}}$$

$$W_{1_h} = \sqrt{C_{a_{1_h}}^2 + W_{U_{1_h}}^2}$$

$$\rightarrow \boxed{W_{1_h} = 5{,}94\,\text{m/s}}$$

Outlet

Hub

$$U_{h_2} = U_{h_1} = 5{,}32\,\text{m/s}$$

$$C_{a_{2_h}} = C_{a_{1_h}} = C_{a_h} = 5{,}76\,\text{m/s}$$

$$\alpha_{2_h} = 90° \quad (\text{axial outlet})$$

$$\beta_{2_h} = \arctan\left(\frac{C_{2_h}}{U_{2_h}}\right)$$

$$\rightarrow \boxed{\beta_{2_h} = 47{,}29°}$$

$$W_{2_h} = \sqrt{C_{2_h}^2 + U_{2_h}^2}$$

$$\rightarrow \boxed{W_{2_h} = 7{,}84\,\text{m/s}}$$

Tip

$$U_{t_2} = U_{t_1} = 17{,}72\,\text{m/s}$$

$$C_{a_{2_t}} = C_{a_{1_t}} = C_{a_t} = 5{,}76\,\text{m/s}$$

$$\alpha_{2_t} = 90° \quad (\text{axial outlet})$$

$$\beta_{2_t} = \arctan\left(\frac{C_{2_t}}{U_{2_t}}\right)$$

$$\rightarrow \boxed{\beta_{2_t} = 18°}$$

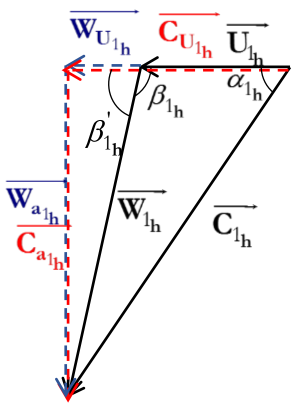
$$W_{2_t} = \sqrt{C_{2_t}^2 + U_{2_t}^2}$$

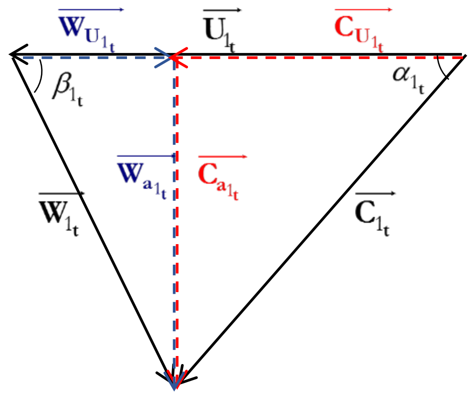
$$\rightarrow \boxed{W_{2_t} = 18{,}63\,\text{m/s}}$$

4. Velocity triangles

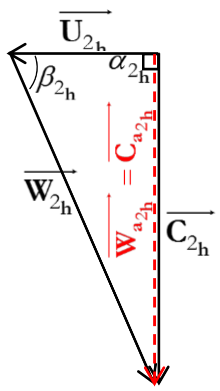
Hub | Tip

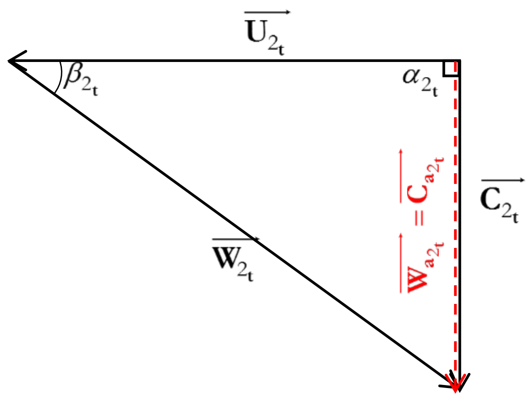
Inlet





Outlet





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