TP Structure des ordinateurs et applications

Corrigé de la série de TP N°2 –*Algèbre de Boole*

Corrigé de l'Exercice N°01 :

Exercice N° 01

Règles de l'algèbre de Boole

* idempotance $\begin{cases} a.a=a \\ a+a=a \end{cases}$

* Complimentarité $\begin{cases} a.\bar{a}=0 \\ a+\bar{a}=1 \end{cases}$

* identité $\begin{cases} 1+a=a \; ; \; 1+a=1 \\ 0+a=0 \; ; \; 0+a=a \end{cases}$

* Distributivité $\begin{cases} a(b+c)=ab+ac \\ a+b.c=a+b) \end{cases}$

1) $F_1 = a.(a+b)$

$= a.a + a.b$; $\begin{cases} \text{On sais que } a.a=a \\ \text{et} \\ 1+b=1 \; ; \; a.1=a \end{cases}$

$= a + ab$

$= a(1+b)$

$= a.1$

$\boxed{F_1 = a}$

2) $F_2 = a + \underline{abc + \bar{a}bc} + \bar{a}b + \underline{ad + a\bar{d}}$

$= a \;\; bc(\underbrace{a+\bar{a}}_{1}) + \bar{a}b + a(\underbrace{d+\bar{d}}_{1})$

$= \underline{a} + bc + \bar{a}b + \underline{a}$; $\underline{a+a=a}$

$= a + bc + \bar{a}b$

$= a + b(\underbrace{c+\bar{a}}_{x})$

$= a + bx \Rightarrow a+bx = (a+b)(a+x)$ avec $x = c+\bar{a}$

$= (a+b)(a+\underbrace{c+\bar{a}}_{x})$; $\underline{a+\bar{a}=1}$

$= (a+b)(\underbrace{c+1}_{1})$

$\boxed{F_2 = a+b}$

3) $F_3 = (a+b)(\bar{a}+\bar{b})$

$= \underbrace{a\bar{a}}_{0} + a\bar{b} + b\bar{a} + \underbrace{b\bar{b}}_{0}$

$= a\bar{b} + b\bar{a}$

$\boxed{F_3 = a \oplus b}$

4) $F_4 = (a+b)(\bar{a}+b)$

$= \underbrace{a\bar{a}}_{0} + ab + b\bar{a} + \underbrace{bb}_{b}$

$= ab + b\bar{a} + b$

$= (\underbrace{b + \bar{a} + 1}_{1})b = 1 . b$

$\boxed{F_4 = b}$

5) $F_5 = abc + a\bar{b}c + \bar{a}$

$= ac(\underbrace{b+\bar{b}}_{1}) + \bar{a}$; $b+\bar{b} = 1$

$F_5 = ac + \bar{a}$; $\bar{a} + ac = (\bar{a}+a)(\bar{a}+c)$

$= (\underbrace{\bar{a}+a}_{1})(\bar{a}+c)$

$\boxed{F_5 = \bar{a} + c}$

6) $F_6 = (\bar{a}+b)(a+b+d)\bar{d}$

$= (\bar{a}+b)(a\bar{d} + b\bar{d} + \underbrace{d\bar{d}}_{0})$

$= \underbrace{\bar{a}a}_{0}\bar{d} + \bar{a}b\bar{d} + ba\bar{d} + \underbrace{bb}_{b}\bar{d}$

$= \bar{a}b\bar{d} + ba\bar{d} + b\bar{d}$

$= (\underbrace{\bar{a}+a+1}_{1})b\bar{d}$

$\boxed{F_6 = b\bar{d}}$

Corrigé de l'Exercice N°02 :

• Exercice N°02 : (Formes Canoniques)

* 1ère forme Canonique $\Sigma\Pi$ C'est une forme Numérique de disjonction (Σ min termes)
* 2ème forme Canonique $\Pi\Sigma$ C'est une forme Numérique de Conjonction (Π max termes)

1) $F_1 = ab + bc + ac$

1er min terme ab ; il manque la variable c

On peut écrire : $ab(c+\bar{c})$ car $c+\bar{c}=1$

et on fait la même chose pour les deux autres.

On aura : $F_1 = ab(c+\bar{c}) + (a+\bar{a})bc + a(b+\bar{b})c$

$F_1 = abc + ab\bar{c} + abc + \bar{a}bc + abc + a\bar{b}c$ $\qquad a+a+a=a$

$$\boxed{F_1 = \underset{111}{abc} + \underset{110}{ab\bar{c}} + \underset{011}{\bar{a}bc} + \underset{101}{a\bar{b}c}}$$

$$\boxed{F_1 = \Sigma(7, 6, 3, 5)} \rightarrow \Sigma\Pi \text{ 1ère forme Canonique.}$$

$$\boxed{\bar{F_1} = \Sigma(0, 1, 2, 4)}$$

$\bar{F_1} = \Sigma(\underset{0}{\bar{a}\bar{b}\bar{c}}, \underset{1}{\bar{a}\bar{b}c}, \underset{2}{\bar{a}b\bar{c}}, \underset{4}{a\bar{b}\bar{c}})$

$\bar{\bar{F}} = F_1 = (a+b+c)\cdot(a+b+\bar{c})\cdot(a+\bar{b}+c)\cdot(\bar{a}+b+c)$

$F_1 = \Pi(0, 1, 2, 4)$

$(\bar{a}+b+c)$, $(a+\bar{b}+c)$, $(a+b+\bar{c})$, $(a+b+c)$

$$\boxed{F_1 = \underset{\substack{000\\0}}{(a+b+c)}\cdot\underset{\substack{001\\1}}{(a+b+\bar{c})}\cdot\underset{\substack{010\\2}}{(a+\bar{b}+c)}\cdot\underset{\substack{100\\4}}{(\bar{a}+b+c)}} \rightarrow \Pi\Sigma \text{ 2ème forme Canonique.}$$

$$\boxed{F_1 = \Pi(0, 1, 2, 4)}$$

* Méthode de Tableau (table de vérité) :

a	b	c	ab	bc	ac	F_1
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	1
1	0	0	0	0	0	0
1	0	1	0	0	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

* $F_1 = \underset{\substack{011\\3}}{\bar{a}bc} + \underset{\substack{101\\5}}{a\bar{b}c} + \underset{\substack{110\\6}}{ab\bar{c}} + \underset{\substack{111\\7}}{abc}$

$$\boxed{F_1 = \Sigma(3, 5, 6, 7)} \rightarrow \Sigma\Pi \text{ 1ère forme Canonique.}$$

* $F_1 = \underset{\substack{000\\0}}{(a+b+c)}\underset{\substack{001\\1}}{(a+b+\bar{c})}\underset{\substack{010\\2}}{(a+\bar{b}+c)}\underset{\substack{100\\4}}{(\bar{a}+b+c)}$

$$\boxed{F_1 = \Pi(0, 1, 2, 4)} \rightarrow \Pi\Sigma \text{ 2ème forme Canonique}$$

* Vérification avec logiciel Logic Friday

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Entered:
f1 = (a*b)+(b*c)+(a*c);

Unminimized:
f1 = a' b c + a b' c + a b c' + a b c;

Unminimized Product of Sums:
f1 = (a+b+c)(a+b+c')(a+b'+c)(a'+b+c);
```

2) $F_2 = (a+b)(\bar{a}+b+d)$

$$F_2 = \underset{0\ 0\ 0}{(a+b+d)}\underset{0\ 0\ 1}{(a+b+\bar{d})}\underset{1\ 0\ 0}{(\bar{a}+b+d)}$$

$\boxed{F_2 = \prod(0,1,4)}$ → ΠΣ (2ème forme Canonique)

$\boxed{\bar{F}_2 = \prod(2,3,5,6,7)}$

2 : $\bar{a}b\bar{c}$; 3 : $\bar{a}bc$; 5 : $a\bar{b}c$; 6 : $ab\bar{c}$; 7 : abc

$$\bar{F}_2 = \prod\left(\underset{2}{\overline{a+\bar{b}+c}}, \underset{3}{\overline{a+\bar{b}+\bar{c}}}, \underset{5}{\overline{\bar{a}+b+\bar{c}}}, \underset{6}{\overline{\bar{a}+\bar{b}+c}}, \underset{7}{\overline{\bar{a}+\bar{b}+\bar{c}}}\right)$$

$$\bar{\bar{F}}_2 = F_2 = \left(\overset{010}{\bar{a}b\bar{c}} + \overset{011}{\bar{a}bc} + \overset{101}{a\bar{b}c} + \overset{110}{ab\bar{c}} + \overset{111}{abc}\right)$$

$$F_2 = \sum(2,3,5,6,7)$$

$\boxed{F_2 = \sum(\bar{a}b\bar{c} + \bar{a}bc + a\bar{b}c + ab\bar{c} + abc)}$ → ΣΠ (1ère forme Canonique)

* Méthode de Tableau (Table de vérité)

a	b	d	a+b	ā	ā+b+d	F_2
0	0	0	0	1	1	0
0	0	1	0	1	1	0
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	0	0
1	0	1	1	0	1	1
1	1	0	1	0	1	1
1	1	1	1	0	1	1

* $F_2 = \underset{\underset{2}{010}}{\bar{a}b\bar{d}} + \underset{\underset{3}{011}}{\bar{a}bd} + \underset{\underset{5}{101}}{a\bar{b}d} + \underset{\underset{6}{110}}{ab\bar{d}} + \underset{\underset{7}{111}}{abd}$ → 1ère forme Canonique ΣΠ

$\boxed{F_2 = \sum(2,3,5,6,7)}$

* $F_2 = \underset{0\ 0\ 0}{(a+b+d)}\cdot\underset{0\ 0\ 1}{(a+b+\bar{d})}\cdot\underset{1\ 0\ 0}{(\bar{a}+b+d)}$ → 2ème forme Canonique ΠΣ

$\boxed{F_2 = \prod(0,1,4)}$

* Vérification avec logiciel Logic Friday

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Entered:
f2= (a+b)*(a'+b+d);

Unminimized:
f2 = a' b d' + a' b d + a b' d + a b d' + a b d;

Unminimized Product of Sums:
f2 = (a+b+d)(a+b+d')(a'+b+d);
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3)

$$F_3 = \overline{a+b+\bar{c}d}$$
$$= \bar{a}\cdot\bar{b}\cdot(c+\bar{d})$$
$$= \bar{a}\bar{b}c + \bar{a}\bar{b}\bar{d}$$
$$= \bar{a}\bar{b}c(d+\bar{d}) + \bar{a}\bar{b}(c+\bar{c})\bar{d}$$
$$= \bar{a}\bar{b}cd + \bar{a}\bar{b}c\bar{d} + \bar{a}\bar{b}c\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} \qquad a+a=a$$

$$\boxed{F_3 = \underset{0011}{\bar{a}\bar{b}cd} + \underset{0010}{\bar{a}\bar{b}c\bar{d}} + \underset{0000}{\bar{a}\bar{b}\bar{c}\bar{d}}} \sim \Sigma\Pi$$
3 2 0

$$\boxed{F_3 = \Sigma(3,2,0)} \rightarrow 1^{ère} \text{ forme canonique } \Sigma\Pi$$

a	b	c	d	$\bar{c}$	$\bar{c}d$	$a+b+\bar{c}d$	$F_3=\overline{a+b+\bar{c}d}$
0	0	0	0	1	0	0	1
0	0	0	1	1	1	1	0
0	0	1	0	0	0	0	1
0	0	1	1	0	0	0	1
0	1	0	0	1	0	1	0
0	1	0	1	1	1	1	0
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	0
1	0	0	0	1	0	1	0
1	0	0	1	1	1	1	0
1	0	1	0	0	0	1	0
1	0	1	1	0	0	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	1	1	0
1	1	1	0	0	0	1	0
1	1	1	1	0	0	1	0

$$\bar{F}_3 = \Sigma(1,4,5,6,7,8,9,A,B,C,D,E,F)$$

$\bar{a}\bar{b}\bar{c}d$, $\bar{a}b\bar{c}\bar{d}$, $\bar{a}b\bar{c}d$ ... $\bar{a}bc\bar{d}$, $\bar{a}bcd$ ... $ab\bar{c}d$, $abcd$

- $\bar{a}+\bar{b}+\bar{c}+\bar{d}$ (1)
- $\bar{a}+\bar{b}+\bar{c}+d$ (2)
- $\bar{a}+\bar{b}+c+\bar{d}$ (3)
- $\bar{a}+\bar{b}+c+d$ (4)
- $\bar{a}+b+\bar{c}+\bar{d}$ (5)
- $\bar{a}+b+\bar{c}+d$ (6)
- $\bar{a}+b+c+\bar{d}$ (7)
- $\bar{a}+b+c+d$ (8)
- $a+\bar{b}+\bar{c}+\bar{d}$ (9)
- $a+\bar{b}+\bar{c}+d$ (10)
- $a+\bar{b}+c+\bar{d}$ (11)
- $a+\bar{b}+c+d$ (12)
- $a+b+c+\bar{d}$ (13)

$$\boxed{F_3 = (1)\cdot(2)\cdot(3)\cdot(4)\cdot(5)\cdot(6)\cdot(7)\cdot(8)\cdot(9)\cdot(10)\cdot(11)\cdot(12)\cdot(13)}$$

$\rightarrow \Pi\Sigma$ - $2^{ème}$ forme canonique.

$$\boxed{F_3 = \Pi(1,4,5,6,7,8,9,A,B,C,D,E,F)}$$

* $F_3 = \underset{0000}{\bar{a}\bar{b}\bar{c}\bar{d}} + \underset{0010}{\bar{a}\bar{b}c\bar{d}} + \underset{0011}{\bar{a}\bar{b}cd}$
0 2 3

$$\boxed{F_3 = \Sigma(0,2,3)} \sim 1^{ère} \text{ forme canonique } \Sigma\Pi$$

* $F_3 = \underset{0001}{(a+b+c+\bar{d})}\cdot\underset{0100}{(a+\bar{b}+c+d)}\cdot\underset{0101}{(a+\bar{b}+c+\bar{d})}\cdot\underset{0110}{(a+\bar{b}+\bar{c}+d)}\cdot$
$\underset{0111}{(a+\bar{b}+\bar{c}+\bar{d})}\cdot\underset{1000}{(\bar{a}+b+c+d)}\cdot\underset{1001}{(\bar{a}+b+c+\bar{d})}\cdot\underset{1010}{(\bar{a}+b+\bar{c}+d)}\cdot$
$\underset{1011}{(\bar{a}+b+\bar{c}+\bar{d})}\cdot\underset{1100}{(\bar{a}+\bar{b}+c+d)}\cdot\underset{1101}{(\bar{a}+\bar{b}+c+\bar{d})}\cdot\underset{1110}{(\bar{a}+\bar{b}+\bar{c}+d)}\cdot$
$\underset{1111}{(\bar{a}+\bar{b}+\bar{c}+\bar{d})}$

1 4 5 6 7 8 9 A B C D E F

$$\boxed{F_3 = \Pi(1,4,5,6,7,8,9,A,B,C,D,E,F)}$$

$\rightarrow \Pi\Sigma$ $2^{ème}$ forme canonique.

* Vérification avec logiciel Logic Friday

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Entered:
        F3=(a+b+c'd)';

Unminimized:
F3 = a' b' c' d' + a' b' c d' + a' b' c d;

Unminimized Product of Sums:
F3 = (a+b+c+d') (a+b'+c+d) (a+b'+c+d') (a+b'+c'+d) (a+b'+c'+d') (a'+b+c+d)
(a'+b+c+d') (a'+b+c'+d) (a'+b+c'+d') (a'+b'+c+d) (a'+b'+c+d') (a'+b'+c'+d)
(a'+b'+c'+d');
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Corrigé de l'Exercice N°03 :

Exercice N° 03

1) expression de S en fonction de A, B, C

$$S = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C}$$

A	B	C	S
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

2) Simplification de S :

$$S = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C}$$

$$= \bar{A}B(\underbrace{\bar{C}+C}_{1}) + A\bar{B}C + AB\bar{C}$$

$$= \bar{A}B + AB\bar{C} + A\bar{B}C$$

$$= B(\bar{A}+A\bar{C}) + A\bar{B}C = B\left((\overbrace{\bar{A}+A}^{1})(\bar{A}+\bar{C})\right) + A\bar{B}C$$

$$(\bar{A}+A\bar{C}) =$$

$$S = B(\bar{A}+\bar{C}) + A\bar{B}C$$

$$\boxed{S = B\bar{C} + B\bar{A} + A\bar{B}C}$$

S: C \ AB	00	01	11	10
0	0	1	1	0
1	0	1	0	1

$$S = \bar{A}B + B\bar{C} + A\bar{B}C$$

$$S = B(\bar{A}+\bar{C}) + A\bar{B}C$$

* Vérification avec logiciel Logic Friday

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Entered by truthtable:
S = A' B C' + A' B C + A B' C + A B C';

Minimized:
S = A' B  + B C' + A B' C;
```

Term	A	B	C	=>	S
0	0	0	0		0
1	0	0	1		0
2	0	1	0		1
3	0	1	1		1
4	1	0	0		0
5	1	0	1		1
6	1	1	0		1
7	1	1	1		0

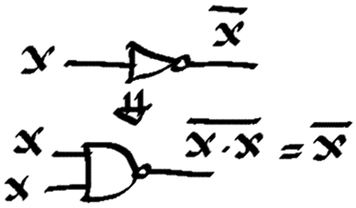
Corrigé de l'Exercice N°04 :

Exercice N° 04 :

a) Réalisation des f^{cts} logiques NOT, AND, OR, XOR en utilisant que des portes NAND :

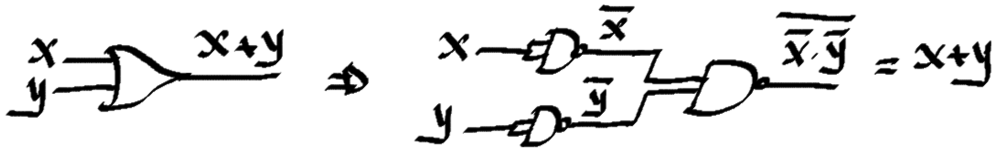
NOT $\overline{x} = \overline{x \cdot x}$

$\overline{x} = \overline{x} + \overline{x}$ ⇒

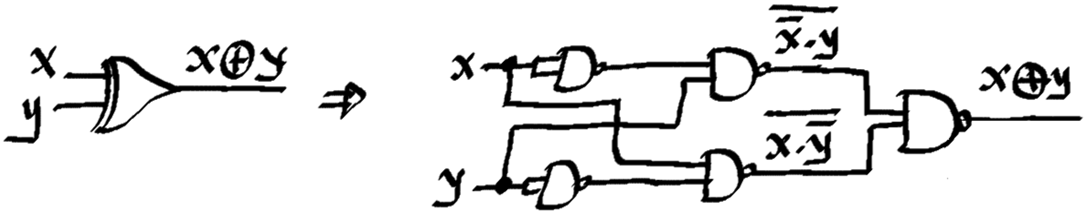


AND

OR

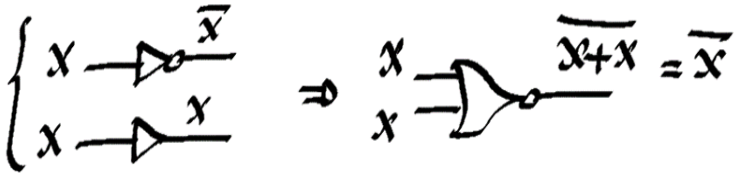


XOR

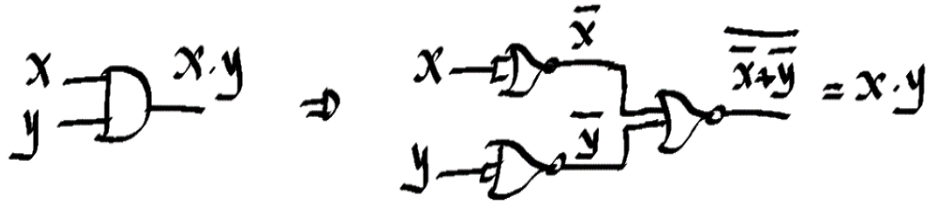


b) Réalisation des f^{cts} avec des portes NOR uniquement

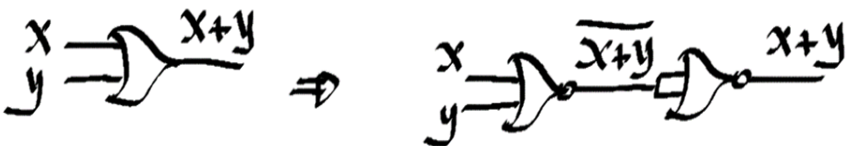
NOT →



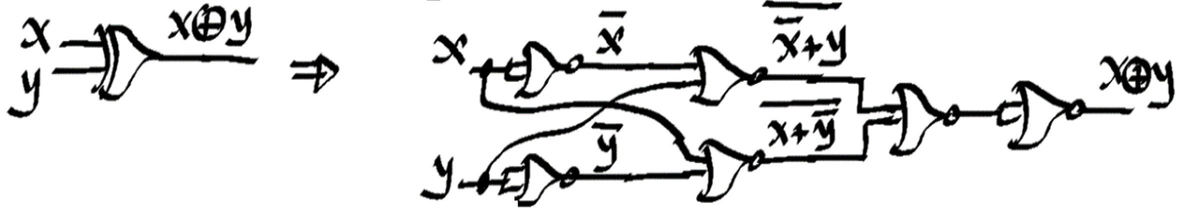
AND



OR



XOR



Corrigé de l'Exercice N°05 :

Exercice N° 05 :

Simplification à l'aide de la table de Karnaugh

$$F_1(a,b,c,d) = \bar{a}\bar{c}d + \bar{a}cb + \bar{b}\bar{c}d + \bar{a}\bar{b}cd$$

Etape 1 : écrire F_1 sous la forme canonique.

$$F_1 = \bar{a}(b+\bar{b})\bar{c}d + \bar{a}bc(d+\bar{d}) + (a+\bar{a})\bar{b}\bar{c}d + \bar{a}\bar{b}cd$$

$$= \bar{a}b\bar{c}d + \bar{a}\bar{b}\bar{c}d + \bar{a}bcd + \bar{a}bc\bar{d} + a\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd$$

$$F_1 = \underset{\substack{0101\\5}}{\bar{a}b\bar{c}d} + \underset{\substack{0001\\1}}{\bar{a}\bar{b}\bar{c}d} + \underset{\substack{0111\\7}}{\bar{a}bcd} + \underset{\substack{0110\\6}}{\bar{a}bc\bar{d}} + \underset{\substack{1001\\9}}{a\bar{b}\bar{c}d} + \underset{\substack{0011\\3}}{\bar{a}\bar{b}cd}$$

$$F_1 = \Sigma(5,1,7,6,9,3)$$

$$\boxed{F_1 = \bar{a}d + \bar{a}bc + \bar{b}\bar{c}d}$$

F_1: cd \ ab	00	01	11	10
00	(0)	(4)	(12)	(8)
01	1 (1)	1 (5)	(13)	1 (9)
11	1 (3)	1 (7)	(15)	(11)
10	(2)	1 (6)	(14)	(10)

$$F_{2(a,b,c)} = \Sigma\ 0,1,3$$

$$\boxed{F_2(a,b,c) = \bar{a}c + \bar{a}\bar{b}}$$

F_2: c \ ab	00	01	11	10
0	1 (0)	(2)	(6)	(4)
1	1 (1)	1 (3)	(7)	(5)

$$F_3(a,b,c) = \Sigma\ 0,3,4,6,7$$

$$\boxed{\begin{array}{l} F_3(a,b,c) = \bar{b}\bar{c} + ab + bc \\ F_3(a,b,c) = \bar{b}\bar{c} + b(a+c) \end{array}}$$

F_3: c \ ab	00	01	11	10
0	1 (0)	(2)	1 (6)	1 (4)
1	(1)	1 (3)	1 (7)	(5)

$F_4(a,b,c,d) = \sum 5,7,13,15$

F_4 cd \ ab	00	01	11	10
00	0	4	12	8
01	1	1 (5)	1 (13)	9
11	3	1 (7)	1 (15)	10
10	2	6	14	11

$$\boxed{F_4(a,b,c,d) = bd}$$

$F_5(a,b,c,d) = \sum 0,5,9,10$ et $\phi = 2,3,8,15$

F_5 cd \ ab	00	01	11	10
00	1			ϕ
01		1		1
11	ϕ		ϕ	
10	ϕ			1

$$\boxed{F_5(a,b,c,d) = \bar{b}\bar{d} + a\bar{b}\bar{c} + \bar{a}b\bar{c}d}$$

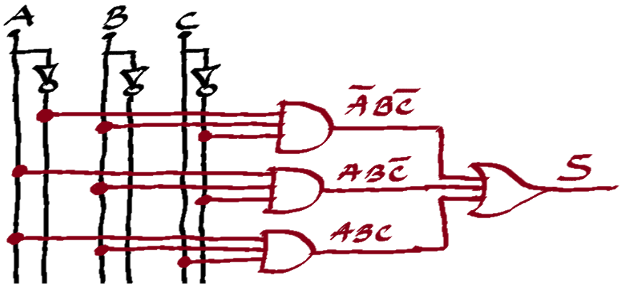
Corrigé de l'Exercice N°06 :

Exercice N°06 :

a) l'expression logique de la Sortie S :

$S = \bar{A}B\bar{C} + AB\bar{C} + ABC$

b) logigramme de S :



c) simplification algébrique de S :

$S = \bar{A}B\bar{C} + AB\bar{C} + ABC$

$S = \underbrace{(\bar{A} + A)}_{1} B\bar{C} + ABC$

$S = B\bar{C} + ABC$

$= B(\bar{C} + AC)$

$= B(\bar{C} + A)\underbrace{(\bar{C} + C)}_{1}$

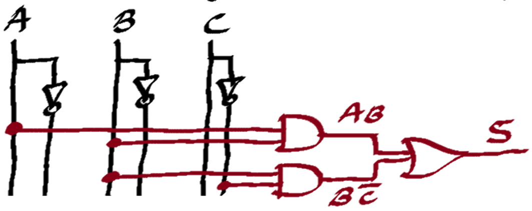
$\boxed{S = B\bar{C} + BA = B\bar{C} + AB}$

d) Simplification de S en Utilisant la table de Karnaugh :

$\boxed{S = AB + B\bar{C}}$

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	1	0

e) Circuit logique de S simplifiée :

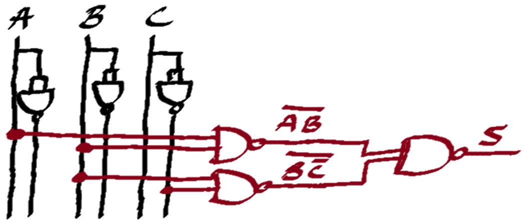


f) Circuit logique de S avec des portes NAND

$S = AB + B\bar{C}$

$\bar{\bar{S}} = S = \overline{\overline{AB + B\bar{C}}}$

$S = \overline{\overline{AB} \cdot \overline{B\bar{C}}}$



Corrigé de l'Exercice N°07 :

Exercice N°07

1) fonctions logiques de S_1, S_2, S_3, S_4

* $S_1 = \overline{E_1}E_2 + E_1\overline{E_2} + E_1E_2$
* $S_2 = \overline{E_1}\,\overline{E_2} + \overline{E_1}E_2$
* $S_3 = S_4 = S_1$

E_1	E_2	S_1	S_2	S_3	S_4
0	0	0	1	0	0
0	1	1	1	1	1
1	0	1	0	1	1
1	1	1	0	1	1

2) simplifications :

* $S_1 = \overline{E_1}E_2 + E_1(\underbrace{\overline{E_2} + E_2}_{1})$

$\boxed{S_1 = \overline{E_1}E_2 + E_1}$

* $S_2 = \overline{E_1}(\underbrace{\overline{E_2} + E_2}_{1})$

$\boxed{S_2 = \overline{E_1}}$

$\boxed{* \; S_3 = S_4 = S_1 = \overline{E_1}E_2 + E_1}$

Exercice N° 08 : (Commande d'une serrure)

A, B, C, D : 4 entrées

S : Sortie

1)

A	B	C	D	S
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

2)

$$S = \overset{0111}{\bar{A}BCD} + \overset{1011}{A\bar{B}CD} + \overset{1100}{AB\bar{C}\bar{D}} + \overset{1101}{AB\bar{C}D} + \overset{1110}{ABC\bar{D}} + \overset{1111}{ABCD}$$

$$S = \sum 7, 11, 12, 13, 14, 15$$

3) Simplification de S à l'aide de Karnaugh :

S — CD \ AB	00	01	11	10
00	0	4	1 (12)	8
01	1	5	1 (13)	9
11	3	1 (7)	1 (15)	1 (11)
10	2	6	1 (14)	10

$$S = AB + ACD + BCD$$

4) Logigramme de S avec des portes NAND uniquement

$$\bar{\bar{S}} = S = \overline{\overline{AB + ACD + BCD}}$$

$$S = \overline{\overline{AB} \cdot \overline{ACD} \cdot \overline{BCD}}$$