Chapitre 2
Radio transmission channels

1. Introduction

The temporal and frequency behavior of radio channels is a crucial aspect of wireless communications. To understand this subject well, it is useful to focus on two main areas: the impulse response (temporal behavior) and the frequency response (frequency behavior) of the channels.

2. Temporal Behavior

The impulse response describes how a radio channel responds to a short impulse.

$$\underset{\text{Impulse Input}}{\delta[n]} \rightarrow \boxed{\text{H}} \rightarrow \underset{\text{Impulse Response}}{h[n]}$$

In practice, it shows how multiple propagation paths (due to reflection, diffraction, and scattering) affect the original signal. This response is essential for understanding the effects of propagation delay, echoes, and temporal dispersion.

2. Frequency Behavior

2.1. Channel Frequency Response

It shows how different frequencies of the signal are attenuated or amplified as they pass through the channel. The frequency response is related to the Fourier transform of the channel's impulse response.

2.2. Free-space path loss

It refers to the reduction in signal strength that occurs as electromagnetic waves propagate through free space (or air) without encountering any obstacles or reflecting surfaces.

The free-space path loss (FSPL) can be calculated using the formula:

$$\text{FSPL (dB)} = 20\log_{10}(d) + 20\log_{10}(f) + 20\log_{10}(\frac{4\pi}{c})$$

Where:

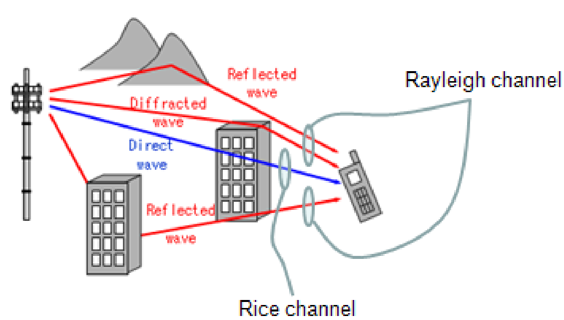
- d is the distance between the transmitter and receiver (in meters).
- f is the frequency of the signal (in Hertz).
- c is the speed of light (approximately 3×10^8 meters per second).

3. Channel Models

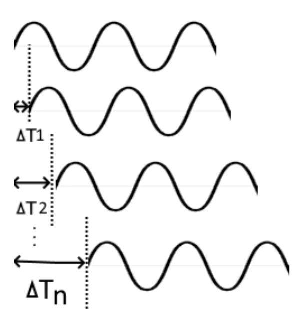
To analyze and model its behaviors, several channel models are used:

3.1. Rayleigh Model: used to model Non-Line-Of-Sight (NLOS) channels, where signal components are reflected and scattered. **Rayleigh fading model estimate the received signal strength in the NLOS situation**.

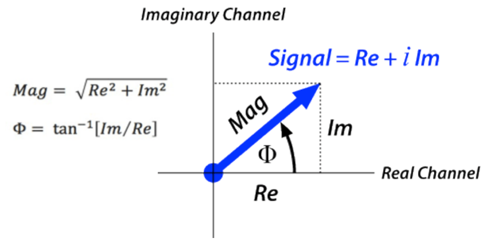
Example



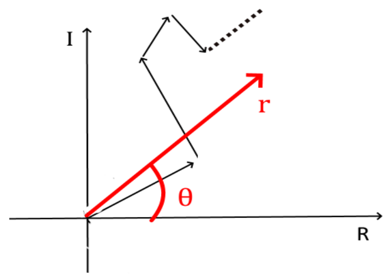
If we have a situation with a multiple paths, the signal bouncing in different objects, the receiver receives many versions of the signal with different delays.



One signal represented in the imaginary space:



All versions in the imaginary plan, all the vectors in the imaginary space will be added up as:



The *central limit theorem* states that the probability distribution function of the sum of a large number of random variables approaches a Gaussian distribution.

If we consider the variable R and I are independent they join probability is $P(R,I) = P(R)xP(I)$

1. Individual Probability density function (PDF)

For each component R and I:

- PDF of R:

  Since $R \sim \mathcal{N}(0, \sigma^2)$, its PDF is:

$$f_R(R) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{R^2}{2\sigma^2}}$$

- PDF of I:

  Similarly, since $I \sim \mathcal{N}(0, \sigma^2)$, its PDF is:

$$f_I(I) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{I^2}{2\sigma^2}}$$

2. Joint PDF

Since R and I are independent, their joint PDF $P(R, I)$ is the product of their individual PDFs:

$$P(R, I) = f_R(R) \cdot f_I(I)$$

Since R and I are independent, their joint PDF $P(R, I)$ is the product of their individual PDFs:

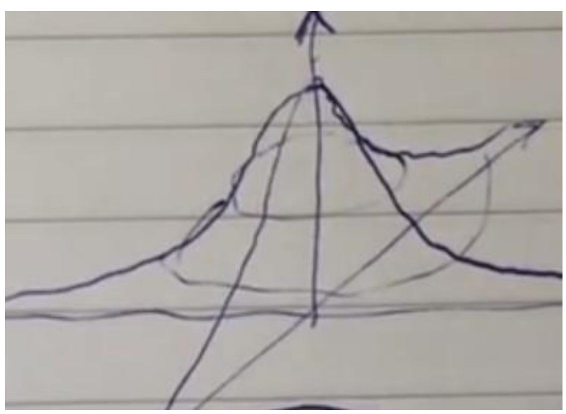
$$P(R, I) = f_R(R) \cdot f_I(I)$$

Substitute the individual PDFs:

$$P(R, I) = \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{R^2}{2\sigma^2}} \right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{I^2}{2\sigma^2}} \right)$$

Combine terms:

$$P(R, I) = \frac{1}{2\pi\sigma^2} e^{-\frac{R^2+I^2}{2\sigma^2}}$$



Changing of reference

Radius: $r = \sqrt{R^2 + I^2}$

Phase: $\theta = \text{atan}\left(\frac{I}{R}\right)$

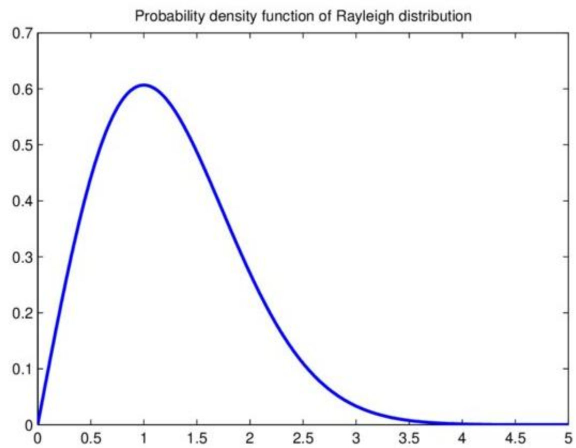
Both R and I are normally distributed with mean 0 and variance σ^2

To find density of probability of the variable (r, θ) we use the Jacobian

$$P(r,\theta) = |J|\; P(R,I)$$

The computation to find the **density of probability function** (PDF) of each variable r and theta:

$$P(r) = \frac{r}{\sigma^2} e^{\frac{-r^2}{2\sigma^2}}, \quad r \geq 0$$



$P(\theta) = constant,\; 0 < \theta < 2\pi$

σ : Standard deviation,

- In a heavily obstructed environment, the value of σ would be higher, indicating more significant fading. In contrast, in environments with fewer obstacles or a dominant LOS path, σ would be smaller, and the fading would be less severe.

The received signal

In wireless communications, the transmitted signal s(t) is modulated with a carrier frequency fc (typically expressed in terms of in-phase and quadrature components), and the channel introduces Rayleigh fading. The received signal can be represented as:

$$r(t) = R(t)e^{j\theta t}s(t)$$

Where s(t) is the transmitted baseband signal (which might include modulation such as QAM, PSK, etc.). The Rayleigh fading process R(t) and θ(t) account for the random fluctuations in the received signal's amplitude and phase due to multipath effects.

3. Instantaneous Power

The **instantaneous power** of the received signal Pr(t) is defined as the square of the amplitude of the signal at any time t

$$P_r(t) = |R(t)e^{j\theta(t)}|^2 = R(t)^2$$

The average power of the received signal in a Rayleigh fading channel is given by:

$$E[Pr]=2\sigma^2$$

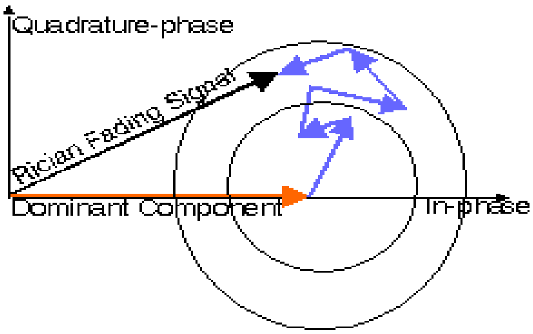
This average power reflects the mean level of the received signal's power in the presence of Rayleigh fading

Remarque

In many cases, we assume that the transmitted signal s(t) has unit average power (i.e., $E[|s(t)|^2]=1$). This is a common assumption when analyzing fading channels because it simplifies the analysis by focusing on how the channel affects the signal.

4.2. Rician Model

It is applicable to channels with a dominant direct path (line-of-sight, LOS). The amplitude of the received signal in a Rician fading channel follows a **Rician distribution**



The probability density function (PDF) is used to represent the fluctuations in signal strength cause by multipath propagation.

$$P(r) = \frac{r}{\sigma^2} e^{-(\frac{r^2+xA^2}{2\sigma^2})} I_0\left(\frac{rA}{\sigma^2}\right) \quad , \quad r \geq 0$$

Where:

r: the amplitude of the received signal
A: Amplitude of the LOS component (deterministic)
$\boldsymbol{\sigma^2}$**:** Variance of the scattered components,
$\boldsymbol{I_0}$: modifies Bessel function of the first kind of order zero

B. Rician Factor K

The **Rician factor** K is a critical parameter that describes the ratio of the power of the **LOS component** to the power of the **scattered multipath components**. It is defined as:

$$K\,(dB) = \frac{P_{LOS}}{\sum P_{scattered}}$$

Where:

- P_{LOSP} : Power of the direct LOS component,
- $\sum P_{scattered}$: Total power of the scattered multipath components.

The **K factor** is often expressed in decibels (dB):

$$K\ (dB) = 10log_{10}(\frac{P_{LOS}}{\sum P_{scattered}})$$

- When K=0 (i.e., no dominant LOS path), the Rician model reduces to the **Rayleigh fading model**.

- As K increases, the dominance of the LOS path increases, and the channel becomes more deterministic (less fading).

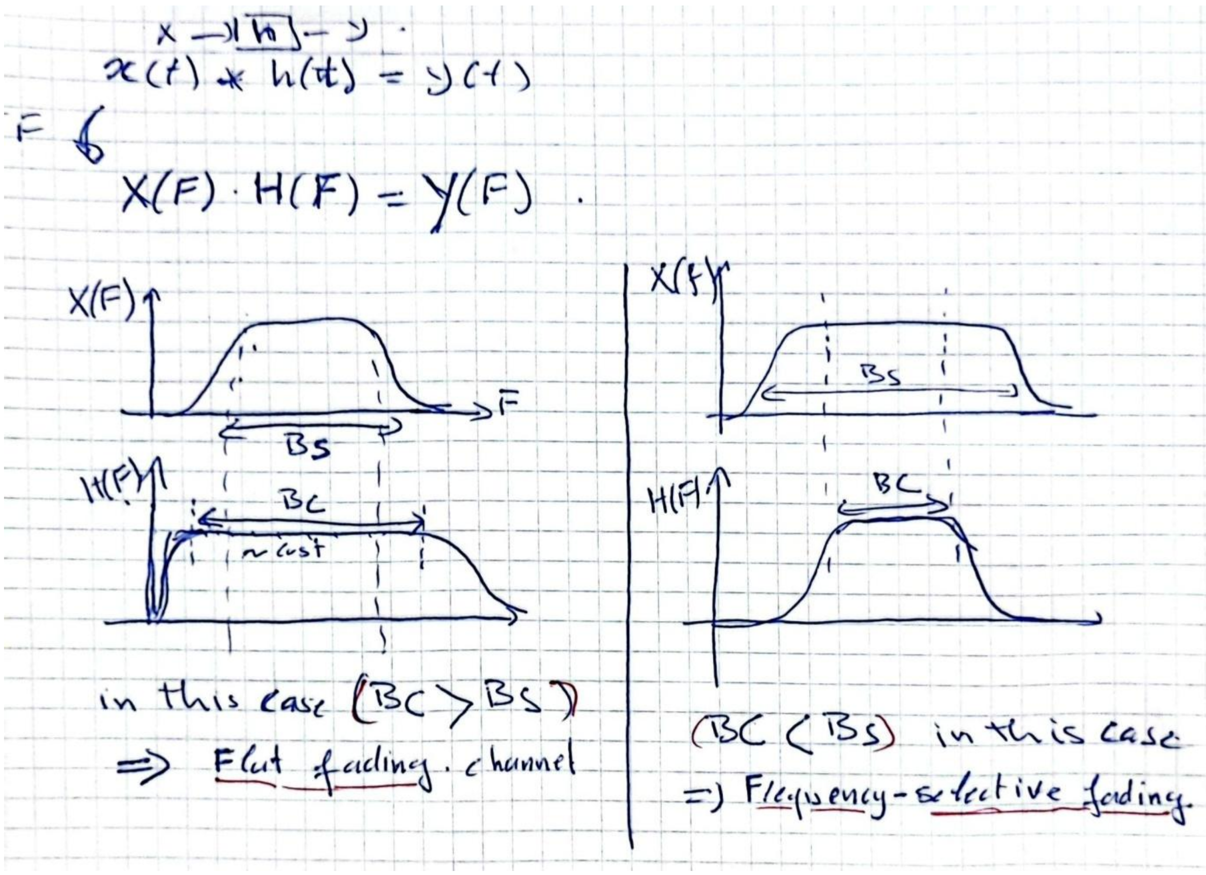
4. Coherence Bandwidth

Coherence bandwidth is a critical concept in the study of wireless communications, particularly in understanding the frequency behavior of radio channels.

Coherence bandwidth (Bc) is defined as the range of frequencies over which the channel's frequency response is relatively constant. Correspond to the range of frequencies where the cannel reacts similarly to all frequencies of the input signal (all frequencies of a signal tend to experience similar fading)

Flat Fading vs. Frequency-Selective Fading

- If the signal bandwidth is much less than the coherence bandwidth ($B \ll Bc$), the channel can be considered **flat fading**, meaning the entire signal bandwidth experiences similar fading.
- If the signal bandwidth is comparable to or greater than the coherence bandwidth ($B \geq Bc$), the channel exhibits **frequency-selective fading**, where different parts of the signal spectrum experience different levels of fading.



The coherence bandwidth can be estimated using the following approximation:

$$Bc = \frac{1}{\tau_{rms}}$$

$\Delta\tau$: is the root mean square (RMS) **delay spread** (*propagation du retard*) of the channel.

$$\tau_{\text{rms}} = \sqrt{\frac{\sum_k \left((t_k - t_a) - \tau_m\right)^2 a_k^2}{\sum_k a_k^2}}$$

Where:

- t_k = arrival time of path k,
- t_a = arrival time of the first path,
- a_k = amplitude of path k (representing the power of the signal along that path),
- τ_m = mean delay, which is the weighted average delay,
- The numerator involves the **weighted variance** of the arrival times, weighted by a_k^2, i.e., the power of each path.

The **mean delay** τm is calculated as follows:

$$\tau_m = \frac{\sum_k (t_k - t_a)\, a_k^2}{\sum_k a_k^2}$$

τ_{rms} is the maximum delay in the cannel, the correspondence with the frequency domain is the notion of coherence bandwidth (Bc).

5. Doppler Shift

Also known as Doppler Effect is the change in frequency or wavelength of a signal as observed by someone moving relative to the source of the signal. This effect is particularly important in telecommunications, especially in systems involving moving objects or users, such as mobile phones, satellites, and vehicles.

For a signal source moving relative to an observer, the observed frequency f' is given by:

$$f' = f\left(\frac{c \pm v_r}{c \pm v_s}\right)$$

where:

- f is the emitted frequency of the signal.
- C is the speed of light (or the speed of sound in the case of acoustics).
- v_r is the speed of the receiver relative to the medium.
- v_s is the speed of the source relative to the medium.
- The sign depends on whether the source and receiver are moving towards or away from each other.

Doppler Shift in Telecommunications

In wireless communication, the Doppler shift can affect the signal due to the relative motion between the transmitter and receiver. The frequency shift (Δf) observed is:

$$\Delta f = f' - f$$

Where f′ is the observed frequency and f is the emitted frequency.

Doppler Shift in a Special Case: Relative Velocity

When considering relative velocity, the Doppler shift can be simplified to a formula

$$\Delta f = \frac{v_{rel}}{c} f$$

where v_{rel} is positive if the source and receiver are moving towards each other and negative if moving away from each other.