DATA ANALYSIS

L2 - COMMUNICATION

GW - Worksheet

Mathematical reminder (part 2)

This sheet is a reminder of the main concepts and operations we will be using throughout the course. This is the second and last of two series of reminders. This reminder should be kept, as we will need it for the rest of our teaching.

Part Two

A prerequisite is the theory of the set of numbers.

Math & Statistics Recap II

Calculation of Proportions and Frequencies

In this section, the Greek letter Ω denotes a non-empty set containing a finite number n_Ω of elements.

We call Ω the *reference set*.

Definition

Let A be a subset of Ω and n_A its number of elements. The **Proportion** (also called **Frequency**) of elements in A relative to Ω is the number $P_{A/\Omega} = \frac{n_A}{n_\Omega}$. When we want to express the proportion as a frequency, we use the letter f instead of $P_{A/\Omega}$, **A** represents α % of Ω where $\alpha = P_{A/\Omega} \times 100$.

A **Percentage** is a *fraction* with a denominator of 100. We write α % instead of $\frac{\alpha}{100}$.

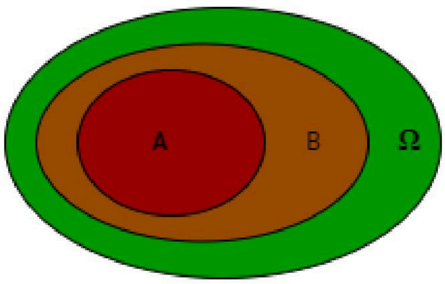
To convert a **Proportion** into a *Percentage*, the following conversion is performed:

$$P = \alpha\,\% = \frac{\alpha}{100} \iff \alpha = 100 \times P$$

Proportion of the Proportion

Let A and B be two subsets of Ω such that $A \subset B$. Let $P_{A/B}$ be the proportion of A relative to B, $P_{B/\Omega}$ the proportion of B relative to Ω, and $P_{A/\Omega}$ the proportion of A relative to Ω. The following equality holds:

$$P_{A/\Omega} = P_{A/B} \times P_{B/\Omega}$$



We say that $P_{A/B}$ and $P_{B/\Omega}$ are **scaled proportions** and $P_{A/B} \times P_{B/\Omega}$ is a **proportion of a proportion** (also called *percentage of percentage*).

Rate of Change

Let Q be a quantity whose values are strictly positive.

Consider that the quantity Q evolves from an initial value $Q_{initial}$ to a final value Q_{final}. The ***Rate of Change*** of the quantity **Q** from $Q_{initial}$ to Q_{final} (also called: *Relative Variation*) is the ratio δ expressed as: $\delta = \frac{Q_{final}-Q_{initial}}{Q_{initial}}$.

We call $\delta\%$ the **Percentage Change** from Q_{initial} to Q_{final}.

We call $C = \frac{Q_{\text{final}}}{Q_{\text{initial}}}$ the **Multiplier Coefficient** for the change from Q_{initial} to Q_{final}.

Theorem

Let δ be the **rate of change** from Q_{initial} to Q_{final}. The **multiplier coefficient** for the change in Q is noted as **C**. We then have the following:

$$\boxed{C = 1 + \delta}$$

The percentage change, noted as t %, for Q is given by the following equality:

$$\boxed{t = 100\delta}$$

From the previous theorem, we can derive the following:

$$Q_{\text{final}} = Q_{\text{initial}}(1+\delta) = Q_{\text{initial}}\left(1 + \frac{t}{100}\right)$$

In the case of positive Q values, we have:

- $\delta > 0$ ($t > 0$), if and only if, $\mathbf{C} > \mathbf{1}$, if and only if, the values of Q increase;
- $\delta < 0$ ($t < 0$), if and only if, $0 < \mathbf{C} < 1$, if and only if, the values of Q decrease.

Successive and Reciprocal Changes

The **global rate of change** is the rate that allows a direct transition from Q_{initial} to Q_{final}.

Theorem

If $\delta_1, \delta_2, \ldots, \delta_n$ are successive rates of change and Δ is the global rate of change, then we have:

$$\boxed{1 + \Delta = (1+\delta_1) \times (1+\delta_2) \times \cdots \times (1+\delta_n)}$$

Two changes are called **reciprocal** if the succession of these two changes applied to a quantity does not affect its values:

Let δ and δ' be two rates of change reciprocal to each other, then we have:

$$\boxed{(1+\delta) \times (1+\delta') = 1}$$