Chapitre 4
Multiplexage et Techniques d'accès multiples (4 Semaines)

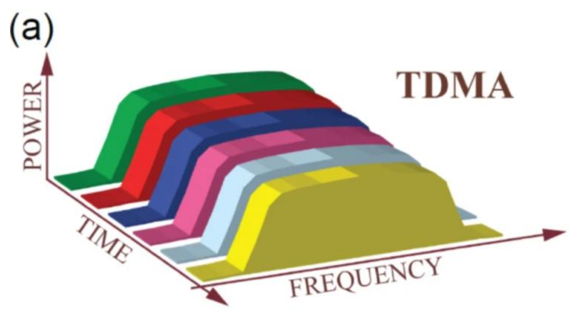
1. Introduction

Multiplexing techniques are used to **optimize utilization of available bandwidth**, reduces costs of cannels (optical fiber, cable,..), allowing multiple signals to share the same physical transmission medium and improves the overall system capacity. These principles based on the resource sharing are called access methods:

- Time Division Multiple Access (TDMA)
- Frequency Division Multiple Access (FDMA)
- Code Division Multiple Access (CDMA)
- Orthogonal Frequency Division Multiplexing (OFDM)

2. Time Division Multiple Access (TDMA)

The key idea of TDMA is that different users are assigned different time slots: multiple users share the same frequency channel, but they transmit in different time slots. This prevents signal overlap since each user is allocated a specific time slot for communication.



- Applications: Widely used in GSM (2G) mobile networks.

2.1. Frame Structure in TDMA

The duration of the frame is the total time it takes for one cycle of all time slots to complete. After one frame ends, the process repeats with the same structure.

Prevent Overlap: *Guard times* create a small buffer period between the end of one user's transmission and the start of the next user's transmission.

Frame start is defined by a reference signal or synchronization information that tells all devices when the frame begins.

Frame end coincides with the end of the last time slot before the frame repeats itself. There's no special pause or time gap at the end of the frame, but rather the system cycles immediately into the next frame.

"frame start" or "frame end" are important key for frame synchronization.

GSM Frame Structure Example:

In **GSM**, the frame is **4.615 ms** long and consists of 8 time slots, with each slot lasting **577 microseconds (µs)**. A **guard time** (GT)is inserted between consecutive time slots to ensure that signals from different users don't overlap. The typical guard time in GSM is **30.5 µs**.

```
| Time Slot 1 |GT| Time Slot 2 |GT| Time Slot 3|GT | Time Slot |GT 4 | Time Slot|GT 5 | Time Slot 6|GT | Time Slot|GT 7 | Time Slot 8 |

    ↑                                                                                                                    ↑

Frame Start                                                                                                        Frame End

| <------------------------------------------------------------ 4.615 ms ------------------------------------------------------------> |
```

2.2. The number of available channels

N in a TDMA system based on system bandwidth, guard bands and coherence bandwidth.

$$\mathcal{N} = \frac{K\,(B_{tot} - 2B_{guard})}{B_c}$$

N : number of channels
K : number of TDMA users per ratio channel or number of time slots (Since each frequency channel can support K users through TDMA.)
$\mathbf{B_{tot}}$: total spectrum allocation (total badwidth)
$\mathbf{B_{guand}}$: Guard Band
$\mathbf{B_C}$: Bandwidth of each individual channel, also known as the coherence bandwidth

If there are multiple **frequency channels** available in the same **time slot**, then **multiple users** can transmit simultaneously within that single time slot, as long as each user is assigned a unique frequency channel. This setup is known as **Frequency-Time Division Multiple Access (FTDMA)** or sometimes referred to as **multi-carrier TDMA**.

2.3. The Shannon capacity

The capacity is the **maximum achievable data rate** (in bits per second) that a single user can achieve over the bandwidth $\mathbf{B_k}$ assigned to him, given the power P, noise N0, and the time-slot sharing in a TDMA system.

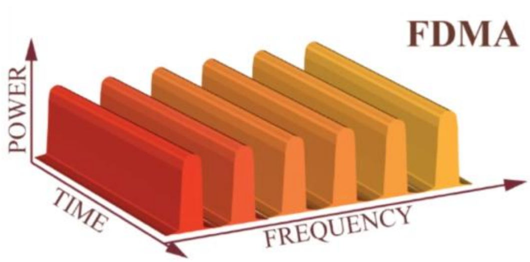
$$C_K^{\mathrm{TDMA}} = B_k \log_2\left(1 + \frac{P}{B_k \cdot N_0}\right)$$

Here:

- C_K^{TDMA} : The capacity for Kth user in the TDMA system.
- Bc : The total bandwidth available for communication.
- K : The number of users sharing the TDMA channel.
- P : The average power allocated to each user.
- N_0 : The noise power spectral density.
- $B_k = \frac{B_c}{K}$: The bandwidth allocated to each user (since the total bandwidth is divided among K users).

3. Frequency Division Multiple Access (FDMA)

FDMA works on the principle of dividing the total bandwidth of the communication channel into a number of discrete segments, and allocating each segment exclusively to a user.



3.1. Characteristics of FDMA

a) **Frequency Separation**:
   - Each user is assigned a dedicated frequency channel.
   - Users transmit simultaneously without interference, as long as adjacent channels are properly separated (guard bands).

b) **Efficiency**:
   - Guard bands between channels reduce spectral efficiency.
   - The system is less flexible for handling dynamic bandwidth demands compared to TDMA or CDMA.

c) **Power**:
   - FDMA typically requires a constant power level, as all users transmit simultaneously on their assigned frequencies.

d) **Synchronization**:
   - FDMA requires minimal synchronization compared to TDMA since users do not share time slots.

3.2. **The number of available channels:** N in a FDMA system based on system bandwidth, guard bands and coherence bandwidth.

$$N = \frac{B_t - 2B_{\text{guard}}}{B_c}$$

Bt : Total system bandwidth.
B_{guard} : Guard bands applied at the edges of the total bandwidth.
Bc : Channel bandwidth (including any intra-channel spacing if required).

3.3. Total Capacity in FDMA

If there are K users sharing the total system bandwidth B_{tot}, and each user is allocated an equal bandwidth $B_i = \frac{B_{tot}}{K}$, the **total system capacity** becomes:

$$C_{\text{FDMA, user}} = B_{\text{channel}} \cdot \log_2 \left(1 + \frac{P}{B_{\text{channel}} \cdot N_0}\right)$$

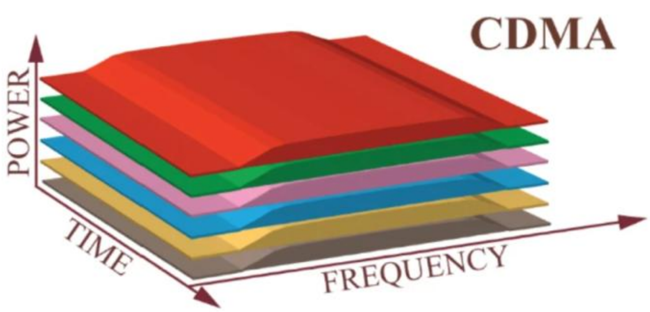
Where:

- $C_{\text{FDMA, user}}$: Maximum data rate for a single user (in bits per second, bps).
- B_{channel}: Bandwidth of the channel allocated to the user (in Hz).
- P: Transmitted power allocated to the user.
- N_0: Noise power spectral density (in W/Hz).
- $\frac{P}{B_{\text{channel}} \cdot N_0}$: Signal-to-noise ratio (SNR) for the channel.

4. Code Division Multiple Access (CDMA)

4.1. Definitions

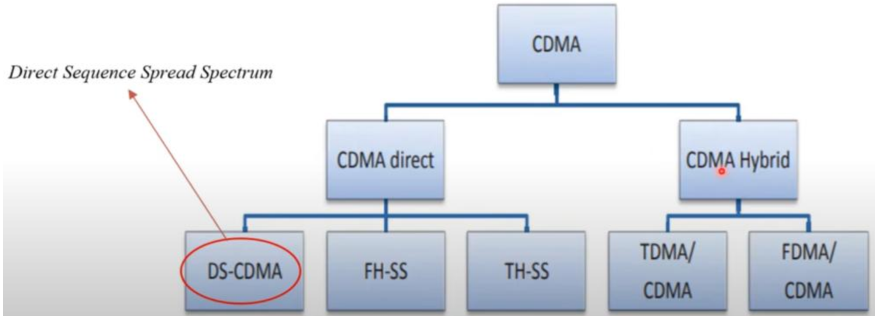
CDMA: the entire spectrum is utilized to encode the information from all users and different users are distinguished with their own unique codes.



CDMA uses **the spread spectrum techniques**; the principle is to spread a signal over a wider frequency band using unique spreading code.

The **spreading of the signal over a wider bandwidth** in **spread spectrum communication** is achieved using mathematical operations that combine the original signal with a high-rate **pseudo-random code** (also called a **spreading code**).

Many techniques of spreading band are proposed

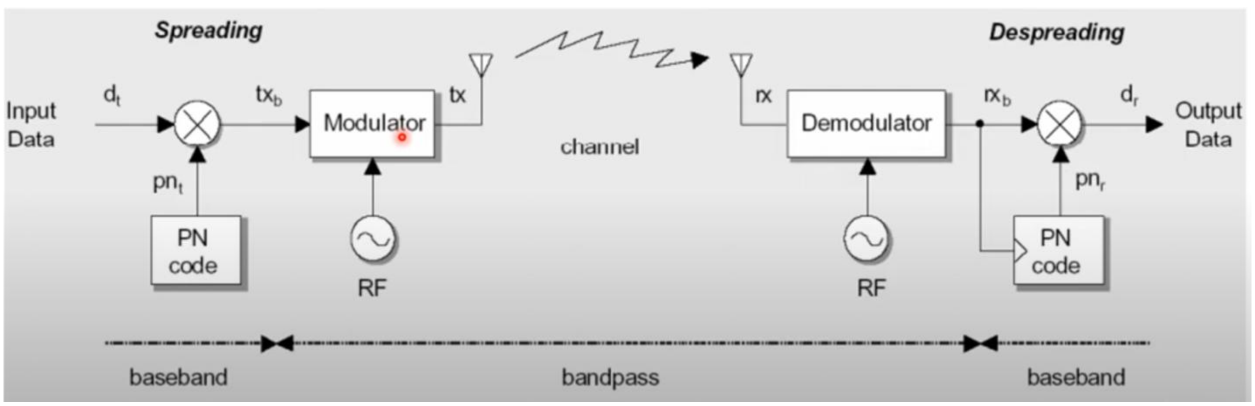


In this chapter we focus on the widely used Direct Sequence Spread Spectrum (DS-CDMA)

4.2. Direct Sequence Spread Spectrum (DS-CDMA)

The technique modulates the signal with a spreading code, resulting in spread signal. The receiver correlates the composite signal with the spreading code of the desired user treating other user's signals as noise.

The principal spreading band using **Direct Sequence** CDMA



The system has two main processes:

1. **Spreading** (at the transmitter side): Expands the bandwidth of the input signal using a pseudo-noise (PN) code.
2. **Despreading** (at the receiver side): Recovers the original signal by using the same PN code

A) *In the transmitter*

- **d(n):** Original data signal of the user (represented as a binary stream.).
- **PN(n):** Spreading code for the user (**Pseudo-Noise** or **Pseudo-Random Number**).
- **Mixing (XOR):** The data signal d is multiplied (XORed) with the PN code to generate the spread signal (**t_{xb}**). This spreads the input data across a wider frequency band
- **Modulation**:The spread signal tbt_btb is modulated onto a radio frequency (RF) carrier for transmission.
- **Transmission**:The modulated RF signal txt_xtx is transmitted over the wireless channel.

B) *Channel*

- The signal txt_xtx travels through the channel and may encounter noise, interference, or distortion.

C) *In the receiver*

- **Reception**: The received signal rxr_xrx is captured and demodulated back to baseband.
- **PN Code Generator**: The receiver generates the same PN sequence pn_r as the transmitter.
- **Despreading**: The received signal r_x is multiplied (XORed) with the PN sequence pn_r to reverse the spreading process and recover the original baseband data dr.
- **Output Data**: The despread signal dr matches the original input data dt, provided synchronization and decoding are correct.

Spread Signal:

Tx=d(n)·PN(n)

At the receiver:

rx=tx+N

- The received composite signal rx contains signals from all users:

$$rx(n) = \sum_{i=1}^{K} d_i(n) \cdot Pn(n) + N(n)$$

Where K is the number of users and N(n) is noise.

Despreading:

- Correlate rx(n) with the desired user's spreading code PN(n):

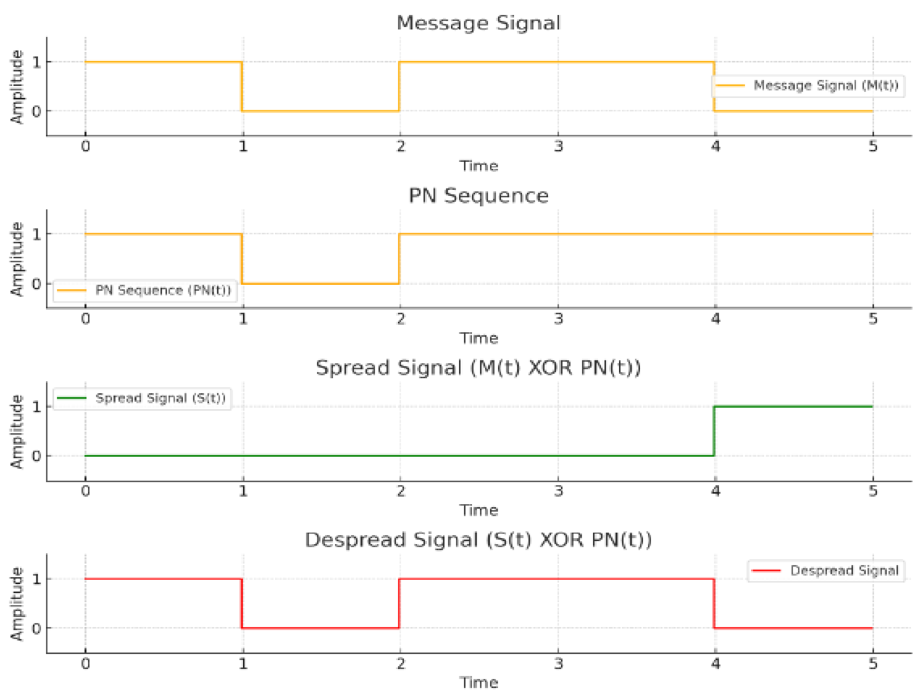
Output=rx(t)·PN(n)

This extracts the desired user's signal while minimizing interference from other users.

Example

Data sequence M(t)=[1,0,1,1,0]

Séquence PN : PN(t)=[1,0,1,1,1]



- **Message Signal M(t)**: The original data signal.
- **PN Sequence PN(t)**: The pseudo-noise sequence used for spreading.
- **Spread Signal S(t)**: The spread signal obtained by M(t)⊕PN(t).
- **Despread Signal**: The despread signal obtained by S(t)⊕PN(t), which recovers the original message M(t).

4.3. Spreading Code Proprieties

- The signal of the code has unit energy: $\sum_0^N p_n^2 = 1$

Adjusting the PN Signal

Original PN Sequence: Binary values ±1

Normalization: Divide each value by $\sqrt{N}$

For a PN sequence PN=[1,−1,1,1] (N=4):

$$PN_{\text{norm}} = \left[\frac{1}{\sqrt{4}}, \frac{-1}{\sqrt{4}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{4}}\right] = [0.5, -0.5, 0.5, 0.5].$$

- Codes are orthogonal to ensure that signal from different users do not interfere with each other.

$$\sum_0^N C_1(n)C_2(n) = 0$$

C1: code user 1
C2: code user 2

- Over a code period, the number of +1 and −1 in the spreading sequence should be nearly equal.

$$\sum_{i=1}^N c_i \approx 0$$

- The cross-correlation between different spreading codes should be as low as possible.

$$R_{c1,c2(\tau)} = \sum_0^N C_1(n)C_2(n-\tau) = 0$$

4.4. Code de Walsh

Walsh codes are derived from **Hadamard matrices**, which are square matrices where all elements are either +1 or −1, and the rows (or columns) are mutually orthogonal.

In digital communications, the binary representation of Walsh codes is typically 0 and 1 instead of −1 and +1.

When using **Walsh matrices**, the operation used to combine the spreading codes with the transmitted signals is **multiplication** (not XOR).

Walsh codes are of length $N=2^m$, where m is an integer

Walsh codes are generated recursively using the **Hadamard matrix**:

$$H_1 = [1], \quad H_N = \begin{bmatrix} H_{N/2} & H_{N/2} \\ H_{N/2} & -H_{N/2} \end{bmatrix}$$

For N=2, the Walsh matrix is:

$$H_2 = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}$$

For N=4, the Walsh matrix is:

$$H_4 = \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix}$$

Generation of the codes

- Replace +1→0 and -1→1 for binary Walsh codes.
- For H4 in binary:

$$\text{Walsh Codes:} \begin{matrix} W_0 = [0,0,0,0] \\ W_1 = [0,1,0,1] \\ W_2 = [0,0,1,1] \\ W_3 = [0,1,1,0] \end{matrix}$$

Example of Using Walsh Codes in CDMA

Imagine a simple **synchronous CDMA** system where we want to transmit data from 4 users over the same frequency band using **Walsh codes** for user separation. Each user has a data bit they wish to transmit. Assume the following data: data of **user 1,2,3,4**: [+1], [-1], [+1], [-1] respectively.

Here's how it works:

- **Walsh matrix** of size H4 (length N=4), we assign one unique code to each user

$$H_4 = \begin{bmatrix} W_0 & W_1 & W_2 & W_3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

User 1: W0=[0,0,0,0]
User 2: W1=[0,1,0,1]
User 3: W2=[0,0,1,1]
User 4: W3=[0,1,1,0]

Here, binary 0 and 1 correspond to -1 and +1 in actual signal processing

- **Spreading**

Each user's data bit is multiplied with its corresponding **Walsh code** to spread the signal. Let's recall the Walsh codes (H_4):

Now, spreading is done by multiplying each data bit with the corresponding Walsh code.

For each user:

User 1: s1=d1·W0 = (+1)·[+1,+1,+1,+1]=[+1,+1,+1,+1]

User 2: s2=d2·W1 = (−1)·[+1,−1,+1,−1]=[−1,+1,−1,+1]

User 3: s3=d3·W2 = (+1)·[+1,+1,−1,−1]=[+1,+1,−1,−1]

User 4: s4=d4·W3 = (−1)·[+1,−1,−1,+1]=[−1,+1,+1,−1]

- **Combining Signals**

The composite signal S(t) is formed by summing the spread signals:

S(t)=s1+s2+s3+s4

Substitute the values:

S(t)=[+1,+1,+1,+1]+[−1,+1,−1,+1]+[+1,+1,−1,−1]+[−1,+1,+1,−1]
S(t)=[0,+4,0,0]

- **Despreading (**recovering the original signal**)**

At the receiver, each user **despreads** the signal S(t) by multiplying it with their own Walsh code and summing the result over the code length (N=4).

The receiver calculates: $d = \frac{1}{N} \cdot \sum S(t) \cdot W$

User 1: d0=1/4·([0,+4,0,0]·[+1,+1,+1,+1])=4/4 =1

User 2: d1=1/4·([0,+4,0,0]·[+1,-1,+1,-1])=-4/4 =-1

User 3: d1=1/4·([0,+4,0,0]·[+1,+1,-1,- 1])=-4/4 =1

User 4: d1=1/4·([0,+4,0,0]·[+1,-1,-1,+1])=-4/4 =-1

This correctly recovers all user data