Exo 01 $\frac{04}{04}$ $x(t) = 2t+1$ et $y(t) = 4t(t-1)$

1/ equation de la trajectoire :

$$y(x) = x^2 - 4x + 3 \cdot \quad 1$$

2/ Composante de vitesse $\vec{V}$ et $\vec{a}$:

$$\begin{cases} x(t) = 2t+1 \\ y(t) = 4t(t-1) \end{cases} \longrightarrow \begin{cases} V_x(t) = 2 \quad 0.25 \\ V_y(t) = 4(2t-1) = 8t-4 \quad 0.25 \end{cases} \Longrightarrow \begin{cases} a_x = 0 \quad 0.25 \\ a_y = 8 \quad 0.25 \end{cases}$$

$$\vec{V} = 2\vec{i} + (8t-4)\vec{j} \qquad \vec{a} = 8\vec{j}$$

3/ Composante tangentielle et acceleration :

on a $V = \sqrt{V_x^2 + V_y^2} = 2\sqrt{16t^2 - 16t + 5} \Rightarrow V = \boxed{\sqrt{64t^2 - 64t + 20}}$ 0.5

$$a_t = \frac{d\|V\|}{dt} = \boxed{16 \frac{2t-1}{\sqrt{16t^2 - 16t + 5}}} = \boxed{\frac{32(2t-1)}{\sqrt{64t^2 - 64t + 20}}} \quad 0.5$$

composante normale de l'acceleration :

on a : $\|a\| = \sqrt{a_x^2 + a_y^2} = 8 \longrightarrow a_n = \sqrt{a^2 - a_t^2} = \boxed{\frac{8}{\sqrt{16t^2 - 16t + 5}}}$ 0.5 (1)

Rayon de corbure : $a_n = \frac{\|\vec{a}\| \times \|\vec{V}\|}{\vec{V}} \longrightarrow$

on a : $a_n = \frac{V^2}{R} \Longrightarrow R = \frac{V^2}{a_n} = \boxed{\frac{(16t^2 - 16t + 5)^2}{2}}$ 0.5

02 : $\frac{3,5}{3,5}$ $\left(2, \frac{\pi}{3}, \frac{\pi}{4}\right)$

$$\begin{cases} \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \\ \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \end{cases}$$

$$\begin{cases} \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \\ \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \end{cases}$$

1/ les cordonné cartisiennes :

$$\begin{cases} x = r \sin\theta \cos\varphi & 0,5 \\ y = r \sin\theta \sin\phi & 0,5 \\ z = r \cos\theta & 0,5 \end{cases}$$

$$\begin{cases} x = 2 \sin\left(\frac{\pi}{3}\right) \cos\left(\frac{\pi}{4}\right) = \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}}{\sqrt{2}} & 0,5 \\ y = 2 \sin\left(\frac{\pi}{3}\right) \sin\left(\frac{\pi}{4}\right) = \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}}{\sqrt{2}} & 0,5 \\ z = 2 \cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right) = 1 & 0,5 \end{cases}$$

$$\left(x = \sqrt{\frac{3}{2}}, \; y = \sqrt{\frac{3}{2}}, \; z = 1\right)$$

2/ $\vec{OM}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

$$\vec{OM}(t) = r\left[\underbrace{\sin\theta\cos\varphi\,\vec{i} + \sin\theta\sin\phi\,\vec{j} + \cos\theta}_{\vec{e_r}}\right]$$

$$= r\vec{e}_r \quad 0,25$$

$$\left\|\frac{d\vec{OM}}{dt}\right\| = r \quad 0,25$$