

Exercices N°02 (énoncé)

Sur le tableau suivant figurent les caractéristiques de 18 anciens véhicules, on vous demande d'effectuer une analyse en composantes principales (ACP) sur données centrées réduites, ce qui conduit à chercher les valeurs & vecteurs propres de la matrice de corrélation R .

| | obs | Modèle | CYL | PUISS | LONG | LARG | POIDS | VITESSE |
|--|-----|------------------|------|-------|------|------|-------|---------|
| | 1 | ALFASUD-TI-1350 | 1350 | 79 | 393 | 161 | 870 | 165 |
| | 2 | AUDI-100-L | 1588 | 85 | 468 | 177 | 1110 | 160 |
| | 3 | SIMCA-1307-GLS | 1294 | 68 | 424 | 168 | 1050 | 152 |
| | 4 | CITROEN-GS-CLUB | 1222 | 59 | 412 | 161 | 930 | 151 |
| | 5 | FIAT-132-1600GLS | 1585 | 98 | 439 | 164 | 1105 | 165 |
| | 6 | LANCIA-BETA-1300 | 1297 | 82 | 429 | 169 | 1080 | 160 |
| | 7 | PEUGEOT-504 | 1796 | 79 | 449 | 169 | 1160 | 154 |
| | 8 | RENAULT-16-TL | 1565 | 55 | 424 | 163 | 1010 | 140 |
| | 9 | RENAULT-30-TS | 2664 | 128 | 452 | 173 | 1320 | 180 |
| | 10 | TOYOTA-COROLLA | 1166 | 55 | 399 | 157 | 815 | 140 |
| | 11 | ALFETTA-1.66 | 1570 | 109 | 428 | 162 | 1060 | 175 |
| | 12 | PRINCESS-1800-HL | 1798 | 82 | 445 | 172 | 1160 | 158 |
| | 13 | DATSUN-2000-GL | 1998 | 115 | 469 | 169 | 1370 | 160 |
| | 14 | TAUNUS-2000-GL | 1993 | 98 | 438 | 170 | 1080 | 167 |
| | 15 | RANCHO | 1442 | 80 | 431 | 166 | 1129 | 144 |
| | 16 | MAZDA-9295 | 1769 | 83 | 440 | 165 | 1095 | 165 |
| | 17 | OPEL-REKORD-L | 1979 | 100 | 459 | 173 | 1120 | 173 |
| | 18 | LADA-1300 | 1294 | 68 | 404 | 161 | 955 | 140 |

1. LA MATRICE DES DONNÉES X :

$$X = \begin{pmatrix} 1350 & 79 & 393 & 161 & 870 & 165 \\ 1588 & 85 & 468 & 177 & 1110 & 160 \\ 1294 & 68 & 424 & 168 & 1050 & 152 \\ 1222 & 59 & 412 & 161 & 930 & 151 \\ 1585 & 98 & 439 & 164 & 1105 & 165 \\ 1297 & 82 & 429 & 169 & 1080 & 160 \\ 1796 & 79 & 449 & 169 & 1160 & 154 \\ 1565 & 55 & 424 & 163 & 1010 & 140 \\ 2664 & 128 & 452 & 173 & 1320 & 180 \\ 1166 & 55 & 399 & 157 & 815 & 140 \\ 1570 & 109 & 428 & 162 & 1060 & 175 \\ 1798 & 82 & 445 & 172 & 1160 & 158 \\ 1998 & 115 & 469 & 169 & 1370 & 160 \\ 1993 & 98 & 438 & 170 & 1080 & 167 \\ 1442 & 80 & 431 & 166 & 1129 & 144 \\ 1769 & 83 & 440 & 165 & 1095 & 165 \\ 1979 & 100 & 459 & 173 & 1120 & 173 \\ 1294 & 68 & 404 & 161 & 955 & 140 \end{pmatrix}$$

2. LE CENTRE DE GRAVITÉ g

$$g = X^T \cdot D \cdot \mathbf{1}_n = \begin{pmatrix} 1631.667 \\ 84.611 \\ 433.500 \\ 166.667 \\ 1078.833 \\ 158.278 \end{pmatrix}$$

3. LA MATRICE DES DONNÉES CENTRÉES Y :

$$Y = X - \mathbf{1}_n \cdot g^T = \begin{pmatrix} -281.66667 & -5.61111 & -40.50000 & -5.66667 & -208.83333 & 6.72222 \\ -43.66667 & 0.38889 & 34.50000 & 10.33333 & 31.16667 & 1.72222 \\ -337.66667 & -16.61111 & -9.50000 & 1.33333 & -28.83333 & -6.27778 \\ -409.66667 & -25.61111 & -21.50000 & -5.66667 & -148.83333 & -7.27778 \\ -46.66667 & 13.38889 & 5.50000 & -2.66667 & 26.16667 & 6.72222 \\ -334.66667 & -2.61111 & -4.50000 & 2.33333 & 1.16667 & 1.72222 \\ 164.33333 & -5.61111 & 15.50000 & 2.33333 & 81.16667 & -4.27778 \\ -66.66667 & -29.61111 & -9.50000 & -3.66667 & -68.83333 & -18.27778 \\ 1032.33333 & 43.38889 & 18.50000 & 6.33333 & 241.16667 & 21.72222 \\ -465.66667 & -29.61111 & -34.50000 & -9.66667 & -263.83333 & -18.27778 \\ -61.66667 & 24.38889 & -5.50000 & -4.66667 & -18.83333 & 16.72222 \\ 166.33333 & -2.61111 & 11.50000 & 5.33333 & 81.16667 & -0.27778 \\ 366.33333 & 30.38889 & 35.50000 & 2.33333 & 291.16667 & 1.72222 \\ 361.33333 & 13.38889 & 4.50000 & 3.33333 & 1.16667 & 8.72222 \\ -189.66667 & -4.61111 & -2.50000 & -0.66667 & 50.16667 & -14.27778 \\ 137.33333 & -1.61111 & 6.50000 & -1.66667 & 16.16667 & 6.72222 \\ 347.33333 & 15.38889 & 25.50000 & 6.33333 & 41.16667 & 14.72222 \\ -337.66667 & -16.61111 & -29.50000 & -5.66667 & -123.83333 & -18.27778 \end{pmatrix}$$

4. LA MATRICE DES VARIANCES-COVARIANCES (V)

$$V = Y^T \cdot D \cdot Y = \begin{pmatrix} 132055.556 & 5732.537 & 5476.556 & 1181.778 & 38159.611 & 2850.870 \\ 5732.537 & 392.127 & 272.861 & 53.259 & 2017.046 & 197.275 \\ 5476.556 & 272.861 & 461.583 & 94.222 & 2482.361 & 120.639 \\ 1181.778 & 53.259 & 94.222 & 26.667 & 492.722 & 28.815 \\ 38159.611 & 2017.046 & 2482.361 & 492.722 & 17715.361 & 749.991 \\ 2850.870 & 197.275 & 120.639 & 28.815 & 749.991 & 139.201 \end{pmatrix}$$

5. LA MATRICE DES DONNÉES CENTRÉES-RÉDUITES (Z) :

$$Z = Y \cdot S = \begin{pmatrix} -0.7750989 & -0.2833582 & -1.8850808 & -1.0973453 & -1.5690068 & 0.5697604 \\ -0.1201633 & 0.0196387 & 1.6058095 & 2.0010414 & 0.2341614 & 0.1459717 \\ -0.9292014 & -0.8388524 & -0.4421794 & 0.2581989 & -0.2166306 & -0.5320903 \\ -1.1273332 & -1.2933477 & -1.0007219 & -1.0973453 & -1.1182147 & -0.6168481 \\ -0.1284188 & 0.6761319 & 0.2559986 & -0.5163978 & 0.1965954 & 0.5697604 \\ -0.9209459 & -0.1318597 & -0.2094534 & 0.4518481 & 0.0087654 & 0.1459717 \\ 0.4522175 & -0.2833582 & 0.7214507 & 0.4518481 & 0.6098215 & -0.3625748 \\ -0.1834554 & -1.4953456 & -0.4421794 & -0.7100469 & -0.5171587 & -1.5491833 \\ 2.8408062 & 2.1911162 & 0.8610863 & 1.2264447 & 1.8119336 & 1.8411267 \\ -1.2814357 & -1.4953456 & -1.6058095 & -1.8719420 & -1.9822328 & -1.5491833 \\ -0.1696962 & 1.2316261 & -0.2559986 & -0.9036961 & -0.1414986 & 1.4173379 \\ 0.4577211 & -0.1318597 & 0.5352698 & 1.0327956 & 0.6098215 & -0.0235438 \\ 1.0080872 & 1.5346230 & 1.6523548 & 0.4518481 & 2.1875936 & 0.1459717 \\ 0.9943280 & 0.6761319 & 0.2094534 & 0.6454972 & 0.0087654 & 0.7392759 \\ -0.5219305 & -0.2328587 & -0.1163630 & -0.1290994 & 0.3769122 & -1.2101523 \\ 0.3779180 & -0.0813603 & 0.3025438 & -0.3227486 & 0.1214634 & 0.5697604 \\ 0.9558024 & 0.7771308 & 1.1869027 & 1.2264447 & 0.3092934 & 1.2478224 \\ -0.9292014 & -0.8388524 & -1.3730835 & -1.0973453 & -0.9303847 & -1.5491833 \end{pmatrix}$$

6. LA MATRICE DES COEFFICIENTS DE CORRÉLATION (R) :

$$R = S \cdot V \cdot S = \begin{pmatrix} 1.00000 & 0.79663 & 0.70146 & 0.62976 & 0.78895 & 0.66493 \\ 0.79663 & 1.00000 & 0.64136 & 0.52083 & 0.76529 & 0.84438 \\ 0.70146 & 0.64136 & 1.00000 & 0.84927 & 0.86809 & 0.47593 \\ 0.62976 & 0.52083 & 0.84927 & 1.00000 & 0.71687 & 0.47295 \\ 0.78895 & 0.76529 & 0.86809 & 0.71687 & 1.00000 & 0.47760 \\ 0.66493 & 0.84438 & 0.47593 & 0.47295 & 0.47760 & 1.00000 \end{pmatrix}$$

VALEURS PROPRES & VECTEURS PROPRES DE LA MATRICE (R) :

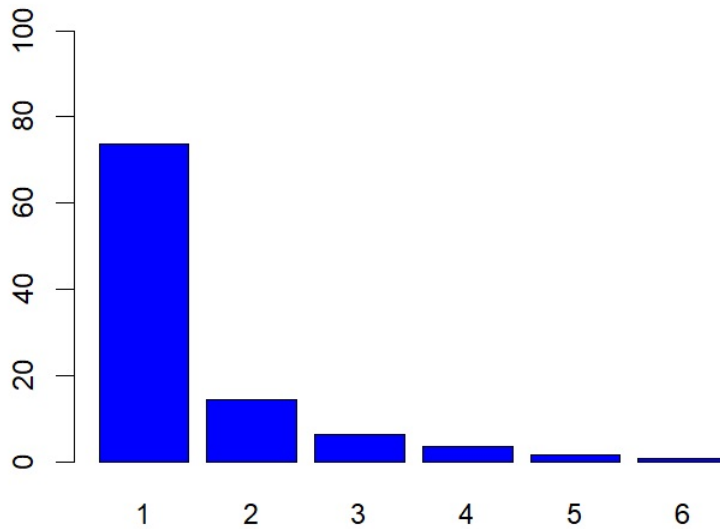
$$\lambda = \begin{pmatrix} \lambda_1 = 4.42085806 \\ \lambda_2 = 0.85606229 \\ \lambda_3 = 0.37306608 \\ \lambda_4 = 0.21392209 \\ \lambda_5 = 0.09280121 \\ \lambda_6 = 0.04329027 \end{pmatrix}$$

$$\mathbf{u} = \begin{pmatrix} -0.424936 & 0.124191 & -0.353613 & 0.807786 & -0.151580 & -0.058895 \\ -0.421794 & 0.415774 & -0.184920 & -0.357792 & 0.293735 & -0.633033 \\ -0.421460 & -0.411818 & 0.067634 & -0.279752 & -0.730569 & -0.190292 \\ -0.386922 & -0.446087 & 0.604868 & 0.211569 & 0.478190 & -0.109566 \\ -0.430512 & -0.242676 & -0.484396 & -0.301711 & 0.304558 & 0.580812 \\ -0.358944 & 0.619863 & 0.485472 & -0.073574 & -0.188655 & 0.458522 \end{pmatrix}$$

DIAGRAMME DES VALEURS PROPRES :

| | λ | percent | cuml |
|---|-----------|---------|--------|
| 1 | 4.42 | 73.68 | 73.68 |
| 2 | 0.86 | 14.27 | 87.95 |
| 3 | 0.37 | 6.22 | 94.17 |
| 4 | 0.21 | 3.57 | 97.73 |
| 5 | 0.09 | 1.55 | 99.28 |
| 6 | 0.04 | 0.72 | 100.00 |

diagramme des valeurs propres



LES DONNÉES PROJÉTÉES :

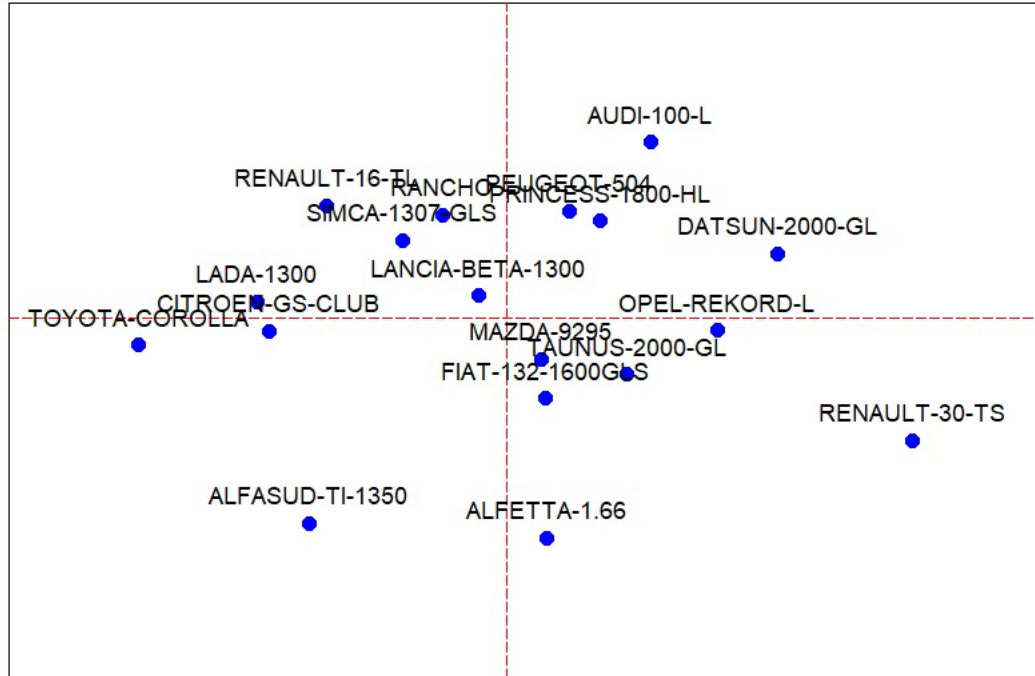
$$F_i = Z.u^T = \begin{pmatrix} -2.138924 & -1.785681 & -0.571862 & -0.201927 & -0.301357 & -0.053921 \\ 1.561459 & 1.527040 & -1.315271 & 0.211352 & 0.148516 & 0.327238 \\ -1.119385 & 0.674505 & -0.456588 & 0.167626 & -0.375364 & -0.271803 \\ -2.573742 & -0.112884 & -0.148570 & 0.017343 & 0.226853 & -0.263476 \\ 0.427855 & -0.695567 & 0.193286 & 0.627754 & 0.263505 & 0.037153 \\ -0.304238 & 0.196149 & -0.675803 & 0.555941 & -0.445086 & -0.200083 \\ 0.683928 & 0.933057 & 0.256823 & -0.203134 & 0.208652 & -0.153892 \\ -1.948493 & 0.980448 & 0.619575 & -0.630319 & 0.293165 & -0.108642 \\ 4.409735 & -1.063633 & 0.593528 & -0.847248 & -0.374890 & -0.043997 \\ -3.985782 & -0.236240 & 0.303133 & -0.265122 & 0.278428 & 0.328892 \\ 0.437658 & -1.912448 & -0.024942 & 0.758910 & 0.168103 & 0.054241 \\ 1.018175 & 0.841712 & -0.216610 & -0.303426 & -0.184876 & -0.184894 \\ 2.941080 & 0.559175 & 1.243988 & 0.772171 & 0.054412 & 0.057263 \\ 1.314880 & -0.486522 & -0.282621 & -0.582226 & -0.066735 & 0.253092 \\ -0.691111 & 0.897721 & 0.628407 & 0.357737 & -0.377086 & 0.121532 \\ 0.385709 & -0.356185 & 0.075583 & -0.102899 & 0.527043 & -0.338832 \\ 2.289768 & -0.104345 & -0.796384 & -0.236347 & 0.338461 & 0.156681 \\ -2.708574 & 0.143699 & 0.574329 & -0.096185 & -0.381746 & 0.283448 \end{pmatrix}$$

LES COORDONNÉES FACTORIELLES DES INDIVIDUS :

| Modèle | F1 | F2 |
|------------------|-------|-------|
| ALFASUD-TI-1350 | -2.14 | -1.79 |
| AUDI-100-L | 1.56 | 1.53 |
| SIMCA-1307-GLS | -1.12 | 0.67 |
| CITROEN-GS-CLUB | -2.57 | -0.11 |
| FIAT-132-1600GLS | 0.43 | -0.70 |
| LANCIA-BETA-1300 | -0.30 | 0.20 |
| PEUGEOT-504 | 0.68 | 0.93 |
| RENAULT-16-TL | -1.95 | 0.98 |
| RENAULT-30-TS | 4.41 | -1.06 |
| TOYOTA-COROLLA | -3.99 | -0.24 |
| ALFETTA-1.66 | 0.44 | -1.91 |
| PRINCESS-1800-HL | 1.02 | 0.84 |
| DATSUN-2000-GL | 2.94 | 0.56 |
| TAUNUS-2000-GL | 1.31 | -0.49 |
| RANCHO | -0.69 | 0.90 |
| MAZDA-9295 | 0.39 | -0.36 |
| OPEL-REKORD-L | 2.29 | -0.10 |
| LADA-1300 | -2.71 | 0.14 |

graphe des individus

F1 (14.27 %)



F2 (73.68 %)