Tel

(1) $I = \int_{-3}^{0} \frac{x}{\sqrt{1-x}}\, dx$

$u(x) = x \implies u'(x) = 1$

$v'(x) = \frac{1}{\sqrt{1-x}} \implies v(x) = -2\sqrt{1-x}$

01/5

$I = \left[-2x\sqrt{1-x}\right]_{-3}^{0} + 2\int_{-3}^{0} \sqrt{1-x}\, dx$

$= \left[-2x\sqrt{1-x}\right] + 2\int_{-3}^{0} (1-x)^{\frac{1}{2}}\, dx$

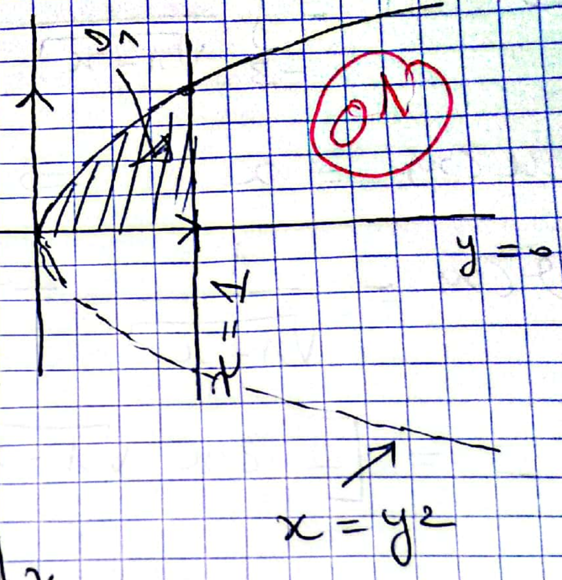
$= \left[-2x\sqrt{1-x} - \frac{4}{3}(1-x)\sqrt{1-x}\right]_{-3}^{0}$

$= \left[\left(-\frac{4}{3}\right) - \left(6\sqrt{4} - \frac{4}{3}(4)\sqrt{4}\right)\right]$

$= -\frac{8}{3}$

② $D_1 = \{(x,y) \in \mathbb{R}^2 \ / \ x \leq 1 \ ; \ y \geq 0 \text{ et } y^2 \leq x\}$

a) tracer D_1



$D_1 = \{(x,y) \in \mathbb{R}^2 \ / \ 0 \leq x \leq 1 \text{ et } 0 \leq y \leq \sqrt{x}\}$

$$I_1 = \int_0^1 \left[\int_0^{\sqrt{x}} y x^2 \, dy \right] dx$$

$$= \int_0^1 \left[\frac{y^2}{2} x^2 \right]_0^{\sqrt{x}} dx$$

$$= \int_0^1 \frac{x^3}{2} \, dx$$

$$= \left[\frac{x^4}{8} \right]_0^1$$

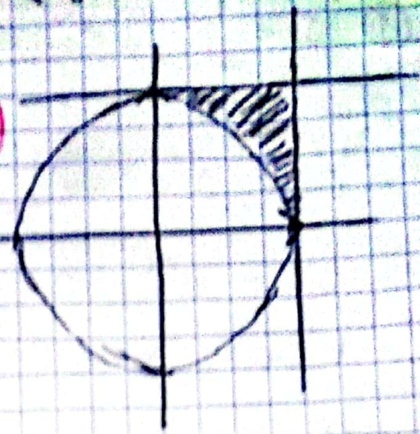
$$= \frac{1}{8}$$

Tel

③ $D_2 = \{(x, y) \in \mathbb{R}^2 / \; 0 \leq x \leq 1, \; 0 \leq y \leq 1 \text{ et } x^2 + y^2 \geq 1\}$

a)

01



b) $l'aire(D_2) = \iint_{D_2} 1 \, dx\, dy$

$D_2 = \{(x, y) \in \mathbb{R}^2 / \; 0 \leq x \leq 1 \text{ et } \sqrt{1-x^2} \leq y \leq 1\}$.

$$l'aire(D_2) = \int_0^1 \left[\int_{\sqrt{1-x^2}}^1 dy \right] dx$$

02

$$= \int_0^1 \left[1 - \sqrt{1-x^2}\right] dx$$

$$= [x]_0^1 - \int_0^1 \sqrt{1-x^2}\, dx = 1 - \frac{\pi}{4}$$

$\int_0^1 \sqrt{1-x^2}\, dx = ?$ $x = \sin t$

$dx = \cos t \, dt$

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = \frac{\pi}{2}$.

$$\int_0^1 \sqrt{1-x^2}\, dx = \int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\, \cos t \, dt$$

$$= \int_0^{\frac{\pi}{2}} \cos^2 t \, dt$$

$$= \frac{1}{2}\left[t + \frac{1}{2}\sin(2t)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\frac{\pi}{2}\right] = \frac{\pi}{4}$$

① $I = \int_{\ln 2}^{\ln 3} \sqrt{e^x - 2}\, dx$

$t = \sqrt{e^x - 2}$

$t^2 = e^x - 2$

$t^2 + 2 = e^x$

$2t\, dt = e^x\, dx$

$2t\, dt = (t^2 + 2)\, dx$

$$\boxed{dx = \frac{2t}{t^2+2}\, dt}$$ 0,5

$x = \ln 2 \Rightarrow t = 0$

$x = \ln 3 \Rightarrow t = 1$

0,5

$I = \int_0^1 \frac{2t^2}{t^2+2}\, dt$ 01

$I = 2\int_0^1 \left(1 - \frac{2}{t^2+2}\right) dt$

$= 2\left[t - \sqrt{2}\arctan\left(\frac{t}{\sqrt{2}}\right)\right]_0^1$

$= 2\left[1 - \sqrt{2}\arctan\left(\frac{1}{\sqrt{2}}\right)\right]$

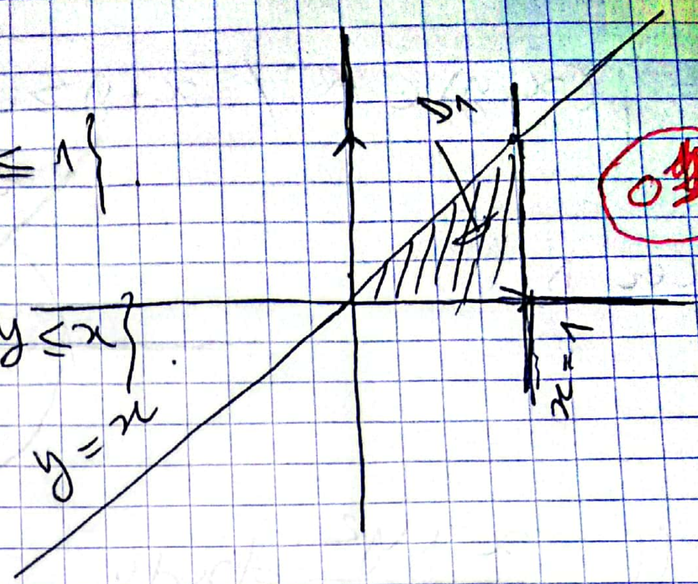
Auto

(2)

$D_1 = \{(x,y) \in \mathbb{R}^2 \,/\, 0 \leq y \leq x \leq 1\}$

$= \{(x,y) \in \mathbb{R}^2 \,/\, 0 \leq x \leq 1 \text{ et } 0 \leq y \leq x\}$

(ou $0 \leq y \leq 1$, $y \leq x \leq 1$)



0,5

$$I_1 = \int_0^1 \left[\int_0^x \frac{1}{(1+x^2)(1+y^2)} \, dy \right] dx$$

0,75

$$= \int_0^1 \left[\frac{1}{1+x^2} \left[\arctan(y) \right]_0^x \right] dx$$

$$= \int_0^1 \frac{1}{1+x^2} \arctan(x) \, dx$$

$$= \left[\frac{1}{2} \arctan^2(x) \right]_0^1$$

$$= \frac{1}{2} \cdot \frac{\pi^2}{16}$$

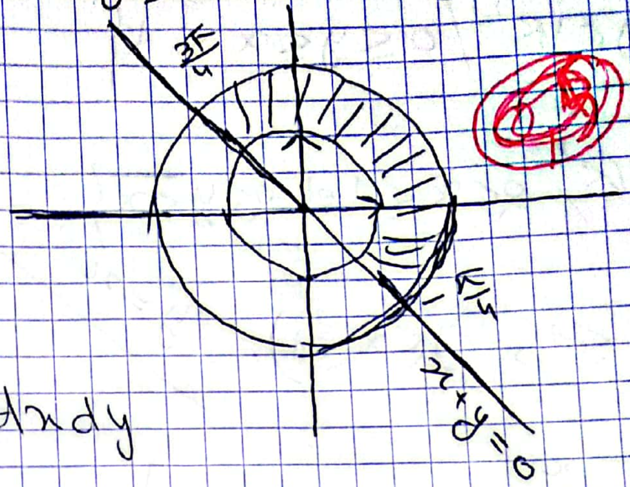
$$= \frac{\pi^2}{32}$$

Anbo

③

$$D_2 = \{(x,y) \in \mathbb{R}^2 \,/\, x+y \geq 0 \text{ et } 1 \leq x^2+y^2 \leq 4\}$$

a) hacer D_2



0,5 0,1

b) $$I_2 = \iint_{D_2} \frac{x^2+y^2}{1+\sqrt{x^2+y^2}} \, dx\,dy$$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \text{tq} \quad \begin{matrix} 1 \leq r \leq 2 \\ -\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4} \end{matrix} \qquad |J| = r$$

0,5

$$I_2 = \int_1^2 \frac{r^2}{1+r} \cdot r\,dr \cdot \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} d\theta$$

$$= \int_1^2 \frac{r^3}{1+r}\,dr \cdot [\theta]_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$\begin{array}{r|l} r^3 & 1+r \\ \hline -r^3-r^2 & r^2-r+1 \\ -r^2 & \\ r^2+r & \\ r & \\ -r-1 & \\ -1 & \end{array}$$

0,2

$$= \int_1^2 \left(r^2-r+1-\frac{1}{r+1}\right) dr \cdot \pi$$

0,5

$$= \pi \left[\frac{1}{3}r^3 - \frac{1}{2}r^2 + r - \ln(r+1)\right]_1^2$$

$$= \pi \left[\frac{8}{3} - \cancel{2} + \cancel{2} - \ln(3) - \frac{1}{3} + \frac{1}{2} - 1 + \ln 2\right]$$

$$= \pi \left[\frac{11}{6} + \ln\left(\frac{2}{3}\right)\right]$$

INT 02 TD 01+02

① Déterminer la nature de l'intégrale impropre suivante :

$$I = \int_{4}^{+\infty} \frac{3x^2+1}{2x^5+\sin x}\,dx \quad (PS = +\infty)$$

la fct $\frac{3x^2+1}{2x^5+\sin x}$; continue et positive sur $[4, +\infty[$

(1)

$\forall x \in [4, +\infty[: -1 \leq \sin x \leq +1$

$$2x^5 - 1 \leq 2x^5 + \sin x \leq 2x^5 + 1$$

$$\frac{1}{2x^5+1} \leq \frac{1}{2x^5+\sin x} \leq \frac{1}{2x^5-1}$$

$$\frac{3x^2+1}{2x^5+\cancel{\sin}\,1} \leq \frac{3x^2+1}{2x^5+\sin x} \leq \frac{3x^2+1}{2x^5-1}$$

(1)

$$\frac{3x^2+1}{2x^5-1} \underset{+\infty}{\sim} \frac{3}{2x^3}$$

$$\int_{4}^{+\infty} \frac{3}{2x^3}\,dx = \frac{3}{2}\int_{4}^{+\infty} \frac{1}{x^3}\,dx \text{ converge}$$

(Riemann pour $\alpha = 3 > 1$) (1)

D'après le critère d'équivalence $\int_{4}^{+\infty} \frac{3x^2+1}{2x^5-1}\,dx$ CV (1)

et " " " de comparaison $\int_{4}^{+\infty} \frac{3x^2+1}{2x^5+\sin x}\,dx$ CV

(1)

(2) $I = \iiint_V 4x^2 \, e^{(x^2+y^2)^2} \, dx\,dy\,dz$

$V = \{ (x,y,z) \in \mathbb{R}^3 / 0 \leq z \leq 2 \text{ et } x^2+y^2 \leq 1 \}$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \quad |J| = r \quad \begin{matrix} 0 \leq r \leq 1 \\ 0 \leq \theta \leq 2\pi \\ 0 \leq z \leq 2 \end{matrix}$$ (01)

$I = \iiint_\Delta 4r^2\cos^2\theta \, e^{r^4} \cdot r \, dr\,d\theta\,dz$ (01)

$= \int_0^1 4r^3 e^{r^4} dr \times \int_0^{2\pi} \cos^2\theta \, d\theta \times \int_0^2 dz$

$= \left[e^{r^4} \right]_0^1 \times \int_0^{2\pi} \frac{1+\cos(2\theta)}{2} d\theta \times \left[z \right]_0^2$ (01)

$= [e-1] \times \cancel{\frac{1}{2}} \int_0^{2\pi} (1+\cos(2\theta)) \, d\theta \times [\cancel{2}]$

$= (e-1)\left[\theta + \frac{1}{2}\sin(2\theta) \right]_0^{2\pi}$ (01)

$= \cancel{2\pi(e-1)}$

$= 2\pi(e-1)$

INT02 Auto03+04

① Determiner la nature de l'intégrale impropre suivante : $I = \int_0^{\frac{1}{2}} \frac{\ln x}{x^2-1}\,dx$ (PS = 0)

La fct $x \longmapsto \frac{\ln x}{x^2-1}$; continue et positive sur $]0, \frac{1}{2}]$ 01

$\frac{\ln x}{x^2-1} = \frac{1}{x^2-1}\ln x \sim_0 \frac{1}{-1}\ln x = -\ln x$

$\int_0^{\frac{1}{2}} -\ln x\,dx = \lim_{x \to 0} \int_x^{\frac{1}{2}} -\ln t\,dt$ 01

$= \lim_{x \to 0} \left[-t\ln t + t\right]_x^{\frac{1}{2}}$ (par partie) 01

$= \lim_{x \to 0} -\frac{1}{2}\ln\frac{1}{2} + \frac{1}{2} + \underbrace{\cancel{x\ln x}}_{0} - \underbrace{\cancel{x}}_{0}$

$= \frac{1}{2}(1 + \ln 2)$ Converge.

D'après le critère d'équivalence $\int_0^{\frac{1}{2}} \frac{\ln x}{x^2-1}\,dx$ converge. 01

②

$$I = \iiint_V z^2 \, dx\, dy\, dz$$

$$V = \{ (x, y, z) \in \mathbb{R}^3 / \; x^2 + y^2 + z^2 \leq 9 \}.$$

$$\begin{cases} x = r\cos\theta\cos\varphi \\ y = r\sin\theta\cos\varphi \\ z = r\sin\varphi \end{cases} \qquad \begin{aligned} 0 \leq r &\leq 3 \\ 0 \leq \theta &\leq 2\pi \\ -\tfrac{\pi}{2} \leq \varphi &\leq \tfrac{\pi}{2} \end{aligned} \qquad |J| = r^2\cos\varphi$$

01

$$I = \iiint_\Delta r^2\sin^2\varphi \cdot r^2\cos\varphi \, dr\, d\theta\, d\varphi$$

$$= \int_0^3 r^4\, dr \times \int_0^{2\pi} d\theta \times \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\varphi\sin^2\varphi\, d\varphi$$ 01

$$= \left[\frac{1}{5} r^5\right]_0^3 \times \left[\theta\right]_0^{2\pi} \times \left[\frac{1}{3}\cos^3\varphi\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$ 01

$$= \frac{243}{5} \times 2\pi \times \frac{2}{3} = \frac{972}{15}\pi = \frac{324\pi}{5}$$ 1

INt02 Tcl 03t 04 + ELN02

① Determiner la nature de l'intégrale impropre suivante :

$$I = \int_0^1 \ln(1-x)\,dx \qquad (PS = 1)$$

on pose $t = 1 - x \Rightarrow dt = -dx$

$x = 0 \Rightarrow t = 1$

$x = 1 \Rightarrow t = 0$

03,5

$$I = \int_1^0 \ln t\,(-dt)$$

$$= \int_0^1 \ln t\,dt$$

$$= \lim_{x \overset{>}{\to} 0} \left[t \ln t - t \right]_x^1$$

$$= \lim_{x \overset{>}{\to} 0} -1 - \underbrace{x \ln x}_{\to 0} + \underbrace{x}_{\to 0} = -1 \quad \text{(converge)}$$

②

$$I = \iiint_V \cos(x^2+y^2)\,dx\,dy\,dz$$

$$V = \{(x,y,z) \in \mathbb{R}^3 ;\ 0 \le z \le 1 \text{ et } x^2+y^2 \le 4\}.$$

$$\begin{cases} x = r\cos\theta & 0 \le r \le 2 \\ y = r\sin\theta & 0 \le \theta \le 2\pi \\ z = z & 0 \le z \le 1 \end{cases} \quad |J| = r$$

01

$$I = \iiint_\Delta \cos(r^2)\,.\,r\,dr\,d\theta\,dz$$

01

$$= \int_0^2 r\cos(r^2)\,dr \times \int_0^{2\pi} d\theta \times \int_0^1 dz$$

$$= \left[\frac{1}{2}\sin(r^2)\right]_0^2 \times [\theta]_0^{2\pi} \times [z]_0^1$$

$$= \frac{\sin(4)}{\cancel{2}} \times \cancel{2}\pi \times 1.$$

$$= \pi\sin(4).$$

01,5

01,5

②

INt02 Auto 1 + 2 + ELNO1.

① Déterminer la nature de l'intégrale impropre suivante : $\int_0^{+\infty} \frac{x^2+x+1}{x+2} e^{-x} dx$ (PS = $+\infty$)

la fct $x \longmapsto \frac{x^2+x+1}{x+2} e^{-x}$, continue et positive sur $[0, +\infty[$ et :

$$\frac{x^2+x+1}{x+1} \sim_{+\infty} x$$

01

$$\frac{x^2+x+1}{x+1} e^{-x} \sim x e^{-x}$$

$$\int_0^{+\infty} x e^{-x} dx = \lim_{x \to +\infty} \int_0^x t e^{-t} dt \quad \text{(par partie)}$$

01

$$= \lim_{x \to +\infty} \left[-(t+1)e^{-t}\right]_0^x$$

$$= \lim \underbrace{-(x+1)e^{-x}}_{0} + 1 = 1$$

Donc $\int_0^{+\infty} x e^{-x} dx$ converge

0,15

et d'après le critère d'équivalence

$\int_0^{+\infty} \frac{x^2+x+1}{x+2} e^{-x} dx$ converge.

①

② $I = \iiint_V xy \, dx\,dy\,dz$

$V = \{ (x, y, z) \in \mathbb{R}^3 \mid x \geq 0,\ y \geq 0,\ z \geq 0 \text{ et } x^2 + y^2 + z^2 \leq 1 \}$

$$\begin{cases} x = r\cos\theta\cos\varphi \\ y = r\sin\theta\cos\varphi \\ z = r\sin\varphi \end{cases} \qquad \begin{array}{l} 0 \leq r \leq 1 \\ 0 \leq \theta \leq \frac{\pi}{2} \\ 0 \leq \varphi \leq \frac{\pi}{2} \end{array}$$

01

$$|J| = r^2\cos\varphi$$

$$I = \iiint_\Delta r\cos\theta\cos\varphi \cdot r\sin\theta\cos\varphi \cdot r^2\cos\varphi \, dr\,d\theta\,d\varphi$$

$$= \int_0^1 r^4\,dr \times \int_0^{\frac{\pi}{2}} \cos\theta\sin\theta\,d\theta \times \int_0^{\frac{\pi}{2}} \cos^3\varphi\,d\varphi$$

01

$$= \left[\frac{1}{5}r^5\right]_0^1 \times \left[\frac{1}{2}\sin^2\theta\right]_0^{\frac{\pi}{2}} \times \int_0^{\frac{\pi}{2}} (1 - \sin^2\varphi)\cos\varphi\,d\varphi$$

01

$$t = \sin\varphi \Rightarrow dt = \cos\varphi\,d\varphi$$

$$\varphi = 0 \Rightarrow t = 0 \ ; \ \varphi = \frac{\pi}{2} \Rightarrow t = 1$$

$$I = \left[\frac{1}{5}\right] \times \left[\frac{1}{2}\right] \times \int_0^1 (1 - t^2)\,dt$$

$$= \frac{1}{10}\left[t - \frac{1}{3}t^3\right]_0^1$$

1

$$= \frac{1}{10} \times \left[\frac{2}{3}\right] = \frac{1}{15}$$