Suite TD 5

Exo 4

Le champ électrique est radial $\vec{E}(M) = E(r)\vec{e_r}$.

calculer le flux

2) surface de bases (S_1 et S_2) et la surface latérale

$$\phi = \oiint_{(S_G)} \vec{E}\cdot\vec{dS} = \phi_1 + \phi_2 + \phi_2 = \iint_{S_1} \vec{E}\cdot\vec{dS_1} + \iint_{S_2} \vec{E}\cdot\vec{dS_2} \iint_{S_3} \vec{E}\cdot\vec{dS_3}$$

$$= \iint_{S_1} (E\,ds)(\vec{e_1}\cdot\vec{n_1}) + \iint_{S_2} E\,ds(\vec{e_2}\cdot\vec{n_2}) + \iint_{S_3} (E\,ds)(\vec{e_3}\cdot\vec{n_3})$$

$$\boxed{\text{sachant que } \iint_{(S_2)} E\,ds\,(\vec{e_r} \perp \vec{n_{1.2}})}$$

le champ est constant sur la surface latérale.

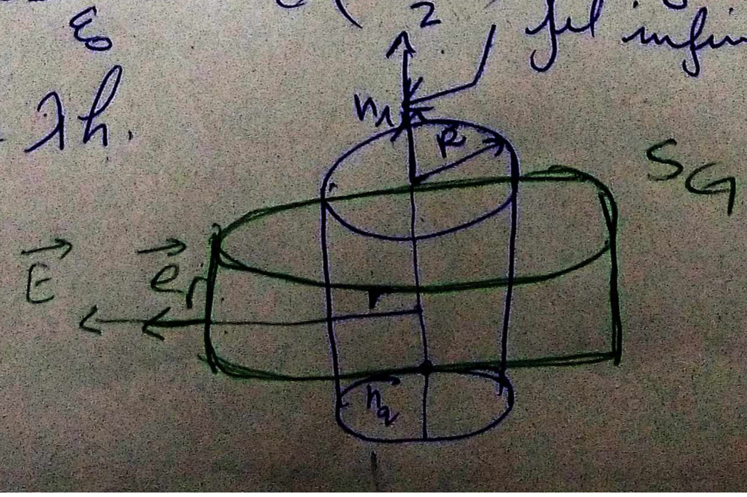
$$\phi = E \iint_{S_3} ds = E S_3 = E(2\pi r h)$$

Théorème de Gauss :

$$\phi = \oiint_{S_G} \vec{E}\,\vec{ds} = \frac{Q_{int}}{\varepsilon_0} \rightarrow E(r)(2\pi r h) = \frac{Q_{int}}{\varepsilon_0} \rightarrow E_r = \frac{Q_{int}}{2\pi\varepsilon_0 r h}$$

si $r < R$: $Q_{int} = \lambda h$.

$$\boxed{E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}}$$



si $r > R$ $= Q_{int} = \lambda h + \sigma 2\pi R h$

$$E_{(r)} = \frac{\lambda + \sigma 2\pi R}{2\pi \varepsilon_0 r}$$

Ex 05

le théorème de Gauss énonce que :

$$\oint\!\!\oint \vec{E}(M) \cdot \vec{ds} = \frac{q_{int}}{\varepsilon_0}$$

le champ est radial :

$$\boxed{\vec{E}(M) = E_r \cdot \vec{e_r}}$$

rayon $r = \|\vec{OM}\|$

$$\vec{ds} = ds \cdot \vec{e_r}$$

le flux de champ à travers la surface de Gauss est donc

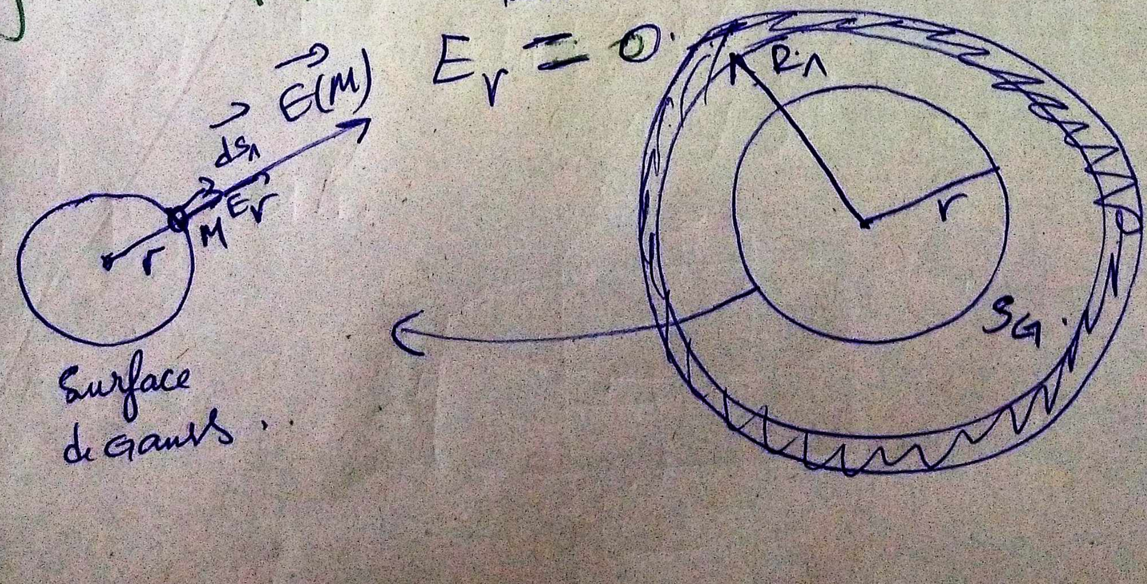
$$\oint\!\!\oint \vec{E}(M) \cdot \vec{ds} = \oint\!\!\oint E_r \cdot ds = E_r \oint\!\!\oint ds = E_r \cdot 4\pi r^2$$

la loi de Gauss nous donne le champ radial.

$$\boxed{E_r = \frac{q_{int}}{4\pi \varepsilon_0 r^2}}$$

Région $r < R_1$ $q_{int} = 0$

$E_r = 0$



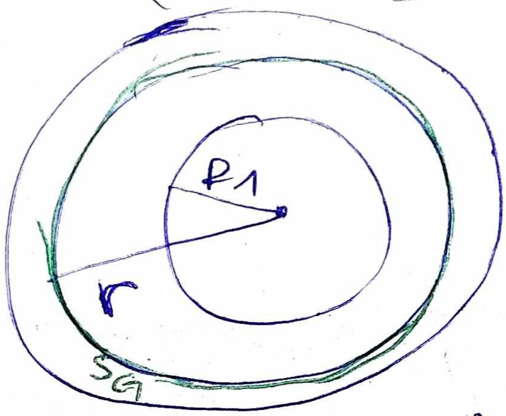
Région 2 $R_1 < r < R_2$

$$q_{int} = \rho(V_G - V_1) = \frac{4\pi}{3}\rho(r^3 - R_1^3).$$

on obtient le champ suivant

$$E_r = \frac{\rho(r^3 - R_1^3)}{3\varepsilon_0 r^2} = \frac{\rho}{3\varepsilon_0}\left(r - \frac{R_1^3}{r^2}\right).$$

$$\vec{E_2}(M) = \frac{\rho}{3\varepsilon_0}\left(r - \frac{R_1^3}{r^2}\right)\vec{e_r}$$

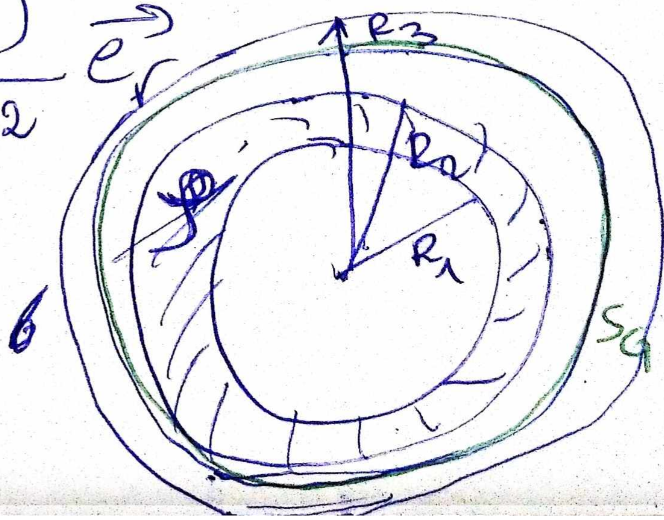


région 3 $(R_2 < r < R_3)$

$$q_{int} = \rho(V_2 - V_1) = \frac{4\pi}{3}\rho(R_2^3 - R_1^3).$$

$$E_r = \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0 r^2}$$

$$\vec{E_3}(M) = \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0 r^2}\vec{e_r}$$



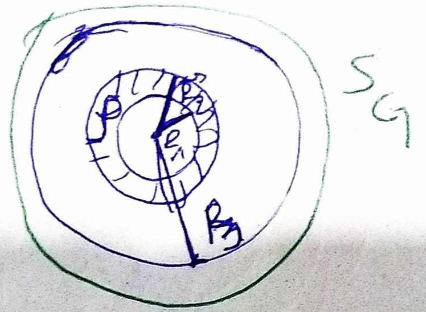
Region $r > R_3$

$$q_{int} = \rho(V_2 - V_1) + \sigma S_3 = \frac{4\pi}{3}\rho(R_2^3 - R_1^3) + 4\pi\sigma R_3^2$$

le champ sera :

$$E_r = \frac{\rho(R_2^3 - R_1^3) + 3\sigma R_3^2}{3\varepsilon_0 r^2}$$

$$\vec{E_4}(M) = \frac{\rho(R_2^3 - R_1^3) + 3\sigma R_3^2}{3\varepsilon_0 r^2}\vec{e_r}.$$



calculs du potentiel:

région 4 $(r > R_3)$.

$$V_4(r) = -\int \vec{E_4}\cdot\vec{dl} = -\int E_r\,dr.$$

$$V_4(r) = -\int \frac{\rho(R_2^3 - R_1^3) + 3\sigma R_3^2}{3\varepsilon_0 r^2}dr.$$

Après l'intégration $\quad V_4(r) = \frac{\rho(R_1^3 - R_1^3) + 3\sigma R_3^2}{3\varepsilon_0 r} + c$

sachant que $V_4(+\infty) = 0 + c = 0$

$$\boxed{V_4(r) = \frac{\rho(R_2^3 - R_1^3) + 3\sigma R_3^2}{3\varepsilon_0 r}}$$

Region 3 $R_2 < r < R_3$

$$V_3(r) = -\int \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0 r^2} \cdot \vec{dl} = -\int E_r dr.$$

$$V_3(r) = -\int \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0 r^2} dr.$$

Après integration

$$V_3(r) = \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0 r^2} + C$$

$$V_3(r = R_3) = V_4(r = R_3).$$

$$\frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0 R_3} + C = \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0 R_3}$$

$$\boxed{C = \frac{\sigma R_3}{\varepsilon_0}}$$

$$\boxed{V_3(r) = \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0 r} + \frac{\sigma R_3}{\varepsilon_0}}$$

Region 2 ($R_1 < r < R_2$)

$$V_2(r) - \int \vec{E_2} \cdot \vec{dl}.$$

$$V_2(r) = -\int \frac{\rho(r^3 - R_1^3)}{3\varepsilon_0 r^2} \cdot dr$$

$$V_2(r) = \frac{\rho}{3\varepsilon_0}\left(-\int r \cdot dr + \int \frac{R_1^3}{r^2} \cdot dr\right).$$

$$V_2(r) = -\frac{\rho r^2}{6\varepsilon_0} - \frac{\rho}{3\varepsilon_0}\frac{R_1^3}{r} + C.$$

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la continuité de potentiel

$$V_2(r=R_2) = V_3(r=R_2)$$

$$-\frac{\rho R_2^2}{6\varepsilon_0} - \frac{\rho}{3\varepsilon_0}\frac{R_1^3}{R_2} + C = \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0 R_2} + \frac{\sigma R_3}{\varepsilon_0}$$

$$C = \frac{\rho R_2^2}{2\varepsilon_0} + \frac{\sigma R_3}{\varepsilon_0}.$$

$$V_2(r) = -\frac{\rho r^2}{6\varepsilon_0} - \frac{\rho}{3\varepsilon_0}\frac{R_1^3}{r} + \frac{\rho R_2^2}{2\varepsilon_0} + \frac{\sigma R_3}{\varepsilon_0}.$$

$$\boxed{V_2(r) = \frac{\rho(3R_2^2 - r^2)}{6\varepsilon_0} - \frac{\rho}{3\varepsilon_0}\frac{R_1^3}{r} + \frac{\sigma R_3}{\varepsilon_0}}$$

Région 1 $(r < R_1)$

$$V_1(r) = -\int \vec{E_1}\cdot\vec{dl} = -\int E_r\,dr.$$

~~On sait~~

on obtient $V_1(r) = C.$

la continuité de potentiel $V_1(r=R_1) = V_2(r=R_1).$

$$C = \frac{\rho(3R_2^2 - R_1^2)}{6\varepsilon_0} - \frac{\rho}{3\varepsilon_0}\frac{R_1^3}{R_1} + \frac{\sigma R_3}{\varepsilon_0}.$$

$$C = \frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0} + \frac{\sigma R_3}{\varepsilon_0}.$$

$$\boxed{V_1(r) = \frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0} + \frac{\sigma R_3}{\varepsilon_0}}$$

Exo 6 :

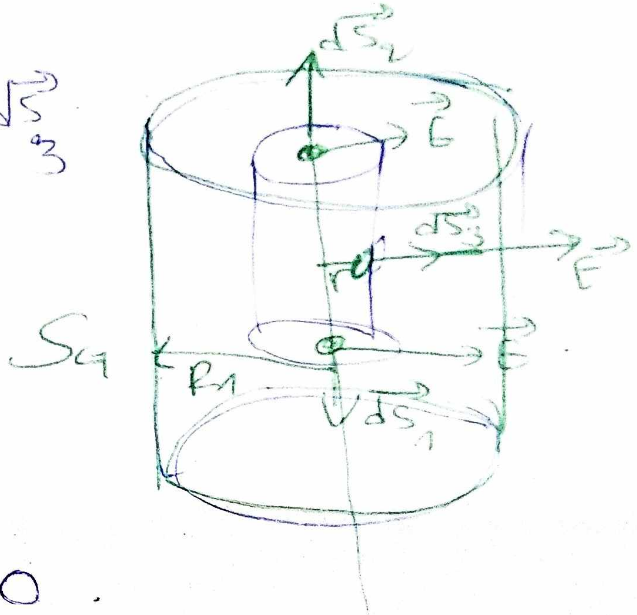
Le théorème de Gauss $\oiint \vec{E}(M).\vec{dS} = \frac{q_{int}}{\varepsilon_0}$

le champ est radial $\vec{E}(M) = E_r . \vec{e_r}$

le rayon $r = \|\vec{OM}\|$, la hauteur h.

$$\oiint \vec{E}.\vec{dS} = \iint_{S_1} \vec{E}\vec{dS_1} + \iint_{S_2} \vec{E}\vec{dS_2} + \iint \vec{E}\vec{dS_3}$$

$$= E_r . 4\pi r^2$$



puisque $\vec{E} \perp \vec{dS_1}$ et $\vec{E} \perp \vec{dS_2}$

donc $\iint_{S_1} \vec{E}\vec{dS_1} = \iint_{S_2} \vec{E}\vec{dS_2} = 0$.

$$\oiint \vec{E}(M)\vec{dS} = \iint_{S_3} \vec{E}\vec{dS_3} = \iint_{S_3} \vec{E}\, dS_L$$ puisque $\vec{E} \parallel \vec{dS_L}$

r est cst donc le champ E constant.

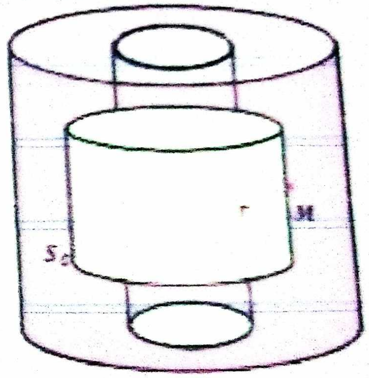
~~$\oiint \vec{E}(M).\vec{dS} = \iint_S \vec{E}\vec{dS}$~~

$$S_L = 4\pi r h.$$

$$\boxed{\oiint \vec{E}(M).\vec{dS} = E S_L = E\, 4\pi r h}$$

1- **Région I : Si $r < R_1$**

$$\oiint \vec{E}_I(M).\overrightarrow{dS} = \frac{q_{int}}{\varepsilon_0} \rightarrow E_I S_L = \frac{q_{int}}{\varepsilon_0}$$



$$q_{int} = 0$$
$$E_I = 0$$

2- **Région II : Si $R_1 \leq r < R_2$**

$$E_{II} S_L = \frac{q_{int}}{\varepsilon_0} \quad (S_L = 2\pi r h)$$

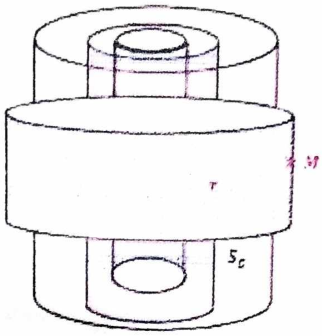
q_{int} est la charge comprise dans la couche entre le cylindre de Gauss et le cylindre de rayon R_1

$$q_{int} = \rho(V_G - V_1)$$

V_G Le volume du cylindre de Gauss
V_1 Le volume du cylindre de de rayon R_1

$$q_{int} = \rho(\pi r^2 h - \pi R_1^2 h)$$

$$E_{II}.2\pi rh = \frac{\rho(\pi r^2 h - \pi R_1^2 h)}{\varepsilon_0} \rightarrow E_{II} = \frac{\rho(r^2 - R_1^2)}{2\varepsilon_0 r} = \frac{\rho}{2\varepsilon_0}\left(r - \frac{R_1^2}{r}\right)$$



$$E_{IV}S_L = \frac{q_{int}}{\varepsilon_0} \quad (S_L = 2\pi rh)$$

q_{int} est la charge $q_1$comprise dans la couche entre le cylindre de rayon R_2et le cylindre de rayon R_1plus la charge $q_2$qui se trouve sur le cylindre de rayon R_3

$$q_{int} = q_1 - q_2$$

$$q_1 = \rho(V_2 - V_1)$$

V_2 Le volume du cylindre de de rayon R_2
V_1 Le volume du cylindre de de rayon R_1

$$q_1 = \rho(\pi R_2^2 h - \pi R_1^2 h)$$

$$q_2 = \sigma S_3$$

S_3 La surface latérale du cylindre de rayon R_3

$$q_2 = \sigma S_3 = 2\pi\sigma R_3 h$$

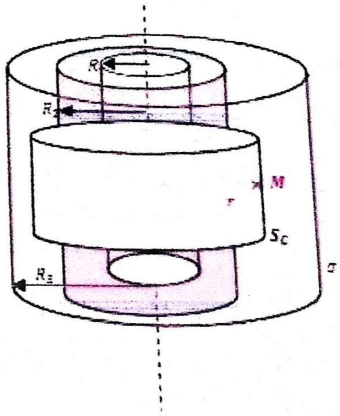
$$q_{int} = q_1 + q_2 = \rho(\pi R_2^2 h - \pi R_1^2 h) + 2\pi\sigma R_3 h$$

$$E_{IV}.2\pi rh = \frac{\rho(\pi R_2^2 h - \pi R_1^2 h) + 2\pi\sigma R_3 h}{\varepsilon_0} \rightarrow E_{IV} = \frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0 r} + \frac{\sigma R_3}{\varepsilon_0 r}$$

$$\vec{\mathbf{E}}_{IV}(M) = \left(\frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0 r} + \frac{\sigma R_3}{\varepsilon_0 r}\right)\vec{\boldsymbol{e}}_r$$

$$\vec{\mathbf{E}}_{II}(M) = \frac{\rho}{2\varepsilon_0}\left(r - \frac{R_1^2}{r}\right)\vec{\boldsymbol{e}}_r$$

3- **Région *III*:Si $R_1 \leq r < R_2$**



$$E_{III}S_L = \frac{q_{int}}{\varepsilon_0} \quad (S_L = 2\pi rh)$$

q_{int} est la charge comprise dans la couche entre le cylindre de rayon R_2et le cylindre de rayon R_1

$$q_{int} = \rho(V_2 - V_1)$$

V_2 Le volume du cylindre de de rayon R_2
V_1 Le volume du cylindre de de rayon R_1

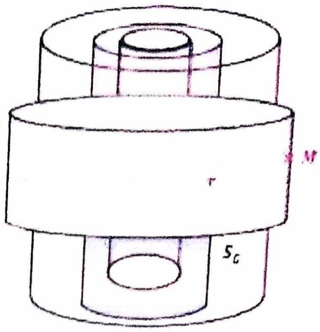
$$q_{int} = \rho(\pi R_2^2 h - \pi R_1^2 h)$$

$$E_{II}.2\pi rh = \frac{\rho(\pi R_2^2 h - \pi R_1^2 h)}{\varepsilon_0} \rightarrow E_{II} = \frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0 r}$$

$$\vec{\mathbf{E}}_{III}(M) = \frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0 r}\vec{\boldsymbol{e}}_r$$

4- **Région *IV*: Si $r \geq R_3$**

$$E_{IV}S_L = \frac{q_{int}}{\varepsilon_0} \quad (S_L = 2\pi r h)$$



q_{int} est la charge q_1 comprise dans la couche entre le cylindre de rayon R_2 et le cylindre de rayon R_1 plus la charge q_2 qui se trouve sur le cylindre de rayon R_3

$$q_{int} = q_1 - q_2$$

$$q_1 = \rho(V_2 - V_1)$$

V_2 Le volume du cylindre de de rayon R_2
V_1 Le volume du cylindre de de rayon R_1

$$q_1 = \rho(\pi R_2^2 h - \pi R_1^2 h)$$

$$q_2 = \sigma S_3$$

S_3 La surface latérale du cylindre de rayon R_3

$$q_2 = \sigma S_3 = 2\pi\sigma R_3 h$$

$$q_{int} = q_1 + q_2 = \rho(\pi R_2^2 h - \pi R_1^2 h) + 2\pi\sigma R_3 h$$

$$E_{IV}.\,2\pi r h = \frac{\rho(\pi R_2^2 h - \pi R_1^2 h) - 2\pi\sigma R_3 h}{\varepsilon_0} \rightarrow E_{IV} = \frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0 r} + \frac{\sigma R_3}{\varepsilon_0 r}$$

$$\vec{E}_{IV}(M) = \left(\frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0 r} - \frac{\sigma R_3}{\varepsilon_0 r}\right)\vec{e}_r$$

2. Calcul du potentiel

- **région I ($r < R_1$)**

En coordonnées cylindriques, le potentiel est donné par:

$$V_I(r) = -\int \vec{E}_I.\vec{dl} = -\int E_r dr$$

On obtient:

$$V_I(r) = C$$

On a :

$$V_I(r = 0) = 0 \rightarrow C = 0$$

On trouve :

$$V_I(r) = 0$$

- **région II ($R_1 \leq r < R_2$)**

$$V_{II}(r) = -\int \vec{E}_{II}.\vec{dl} = -\int E_r dr$$

On obtient:

$$V_{II}(r) = -\int \frac{\rho(r^2 - R_1^2)}{2\varepsilon_0 r} dr$$

Apres intégration, nous obtenons :

$$V_{II}(r) = -\frac{\rho}{4\varepsilon_0}r^2 + \frac{\rho R_1^2}{2\varepsilon_0}\ln(r) + C$$

La continuité de potentiel $V_{II}(r = R_1) = V_I(r = R_1)$

$$-\frac{\rho}{4\varepsilon_0}R_1^2 + \frac{\rho R_1^2}{2\varepsilon_0}\ln(R_1) + C = 0$$

$$C = \frac{\rho}{4\varepsilon_0}R_1^2 - \frac{\rho R_1^2}{2\varepsilon_0}\ln(R_1)$$

$$V_{II}(r) = -\frac{\rho}{4\varepsilon_0}r^2 + \frac{\rho R_1^2}{2\varepsilon_0}\ln(r) + \frac{\rho}{4\varepsilon_0}R_1^2 - \frac{\rho R_1^2}{2\varepsilon_0}\ln(R_1)$$

$$V_{II}(r) = -\frac{\rho}{4\varepsilon_0}(r^2 - R_1^2) + \frac{\rho R_1^2}{2\varepsilon_0}\ln\left(\frac{r}{R_1}\right)$$

- **région III ($R_2 \leq r < R_3$)**

$$V_{III}(r) = -\int \vec{E}_{III}.\vec{dl}$$

$$V_{III}(r) = -\int \frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0 r}.dr$$

$$V_{III}(r) = -\frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0}\ln(r) + C$$

La continuité de potentiel $V_{II}(r = R_2) = V_{III}(r = R_2)$

$$-\frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0}\ln(R_2) + C = -\frac{\rho}{4\varepsilon_0}(R_2^2 - R_1^2) + \frac{\rho R_1^2}{2\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$$

$$C = -\frac{\rho}{4\varepsilon_0}(R_2^2 - R_1^2) + \frac{\rho R_1^2}{2\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right) + \frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0}\ln(R_2)$$

$$C = -\frac{\rho}{4\varepsilon_0}(R_2^2 - R_1^2) + \frac{\rho}{2\varepsilon_0}(R_2^2\ln(R_2) - R_1^2\ln(R_1))$$

$$V_{III}(r) = -\frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0}\ln(r) - \frac{\rho}{4\varepsilon_0}(R_2^2 - R_1^2) + \frac{\rho}{2\varepsilon_0}(R_2^2\ln(R_2) - R_1^2\ln(R_1))$$

- **région IV ($r > R_3$)**

$$V_{IV}(r) = -\int \vec{E}_{IV}.\vec{dl} = -\int E_r dr$$

$$V_{IV}(r) = -\int\left(\frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0 r} + \frac{\sigma R_3}{\varepsilon_0 r}\right).dr$$

$$V_{IV}(r) = -\left(\frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0} + \frac{\sigma R_3}{\varepsilon_0}\right)\ln(r) + C$$

La continuité de potentiel $V_{IV}(r = R_3) = V_{III}(r = R_3)$

$$-\left(\frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0} + \frac{\sigma R_3}{\varepsilon_0}\right)\ln(R_3) + C$$

$$= -\frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0}\ln(R_3) - \frac{\rho}{4\varepsilon_0}(R_2^2 - R_1^2) + \frac{\rho}{2\varepsilon_0}(R_2^2\ln(R_2) - R_1^2\ln(R_1))$$

$$C = \frac{\sigma R_3}{\varepsilon_0}\ln(R_3) - \frac{\rho}{4\varepsilon_0}(R_2^2 - R_1^2) + \frac{\rho}{2\varepsilon_0}(R_2^2\ln(R_2) - R_1^2\ln(R_1))$$

$$V_{IV}(r) = -\left(\frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0} + \frac{\sigma R_3}{\varepsilon_0}\right)\ln(r) + \frac{\sigma R_3}{\varepsilon_0}\ln(R_3) - \frac{\rho}{4\varepsilon_0}(R_2^2 - R_1^2)$$
$$+ \frac{\rho}{2\varepsilon_0}(R_2^2\ln(R_2) - R_1^2\ln(R_1))$$