— INt01 — 2025/2026

①

$$I = \int_0^2 \frac{x\sqrt{x}}{x+2}\,dx$$ ③

②

Soit $D = \{(x,y) \in \mathbb{R}^2 \,/\, 1 \leq x+y \leq 2 \,,\ 0 \leq x-2y \leq 1\}$.

1 ① tracer D

1 ② calculer l'aire (D) (en utilisant les intégrals doubles)

③ calculer $I = \iint_D (x^2+4y^2-4xy)(x+y)\,dx\,dy$

2

— Bon courage —

Auto

(1)

$$I = \int_0^2 \frac{x\sqrt{x}}{x+2}\,dx$$

on pose $t = \sqrt{x} \Rightarrow dt = \frac{1}{2\sqrt{x}}\,dx$

$$\Rightarrow dt = \frac{1}{2t}\,dx$$

$$\Rightarrow \boxed{dx = 2t\,dt}$$

$x = 0 \Rightarrow t = 0$; $x = 2 \Rightarrow t = \sqrt{2}$

$$I = \int_0^{\sqrt{2}} \frac{2t^4}{t^2+2}\,dt$$

$$= 2\int_0^{\sqrt{2}} \frac{t^4 - 4 + 4}{t^2+2}\,dt$$

$$= 2\int_0^{\sqrt{2}} \left(t^2 - 2 + \frac{4}{t^2+2}\right)dt$$

$$= 2\left[\frac{1}{3}t^3 - 2t + \frac{4}{\sqrt{2}}\arctan\left(\frac{t}{\sqrt{2}}\right)\right]_0^{\sqrt{2}}$$

$$= 2\left[\frac{2\sqrt{2}}{3} - 2\sqrt{2} + \frac{4}{\sqrt{2}}\cdot\frac{\pi}{4}\right]$$

$$= 2\times\left[\frac{-4\sqrt{2}}{3} + \frac{\pi\sqrt{2}}{2}\right]$$

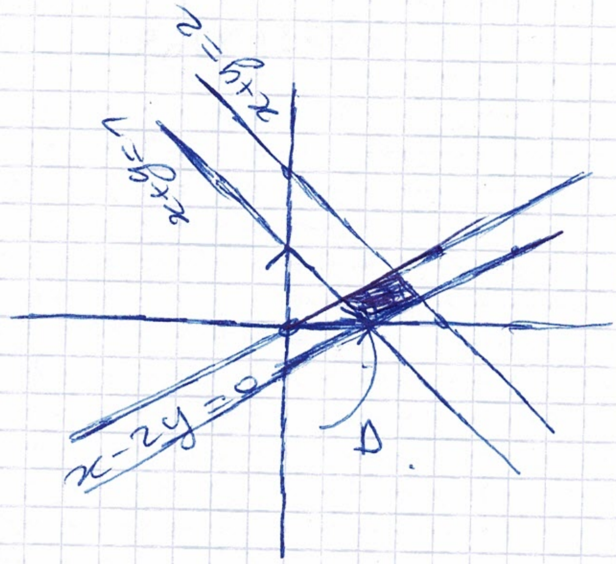
$$= -\frac{8\sqrt{2}}{3} + \pi\sqrt{2}$$

$$= \frac{(3\pi - 8)\sqrt{2}}{3}$$

②

~~$x - 2y = 1$~~

~~$x - 2y$~~



$$\begin{cases} u = x + y & \text{--- ①} \\ v = x - 2y & \text{--- ②} \end{cases}$$

$$\Delta = \{(u,v) \in \mathbb{R}^2 \mid 1 \leq u \leq 2 \,;\, 0 \leq v \leq 1\}.$$

① − ② $\Rightarrow u - v = 3y$ | on replace ds ① :

$\Rightarrow \boxed{y = \frac{u-v}{3}}$ | $x = u - y = u - \frac{u-v}{3} = \boxed{\frac{2u+v}{3}}$

$$J = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{pmatrix}$$

$$\det(J) = -\frac{2}{9} - \frac{1}{9} = -\frac{3}{9} = -\frac{1}{3}.$$

$$|J| = \frac{1}{3}.$$

$$I = \iint_{D} (x-2y)^2 (x+y)\, dx\, dy$$

$$= \iint_{\Delta} v^2 \cdot u \cdot \frac{1}{3}\, du\, dv.$$

$$= \frac{1}{3} \int_0^1 v^2 dv \times \int_1^2 u\, du = \frac{1}{3}\left[\frac{1}{3} v^3\right]_0^1 \times \left[\frac{1}{2} u^2\right]_1^2$$

$$= \frac{1}{3}\left[\frac{1}{\cancel{3}}\right] \times \left[\frac{\cancel{3}}{2}\right]$$

③ $= \frac{1}{6}$