- INt 01 - 2025|2026

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1) calculer $I = \int_{1}^{e^2} \frac{1}{x(4+(\ln x)^2)}\,dx$

2) Soit $D = \{(x,y) \in \mathbb{R}^2 ;\ y \geq 0 \text{ et } x^2+y^2 \leq 1\}$

1 ① tracer D

1 ② calculer l'aire (D)

2,5 ③ calculer $I = \iint_D \frac{y^2}{9+x^2+y^2}\,dx\,dy$

- Bon courage -

Auto

①

$$I = \int_1^{e^2} \frac{1}{x(4+(\ln x)^2)}\, dx$$

on pose $t = \ln x \Rightarrow dt = \frac{1}{x}\, dx$

$\Rightarrow \boxed{dx = x\, dt}$

$x = 1 \Rightarrow t = 0$

$x = e \Rightarrow t = 2$

$$I = \int_0^{2} \frac{1}{\cancel{x}(4+t^2)}\, \cancel{x}\, dt$$

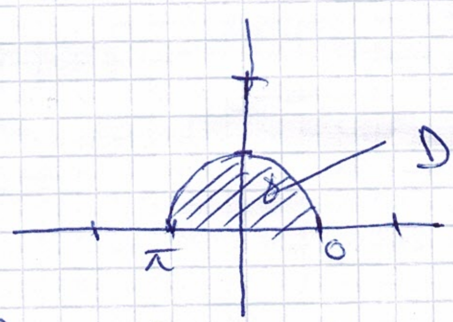
$$= \int_0^2 \frac{1}{4+t^2}\, dt$$

$$= \left[\frac{1}{2}\arctan\left(\frac{t}{2}\right)\right]_0^2$$

$$= \frac{\pi}{8}$$

②

①



② $\text{Aire}(D) = \iint_D dx\,dy$

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\quad |J| = r$

①

$$\Delta = \{ (r, \theta) \in \mathbb{R}^2 \mid 0 \leq r \leq 1 \ ; \ 0 \leq \theta \leq \pi \}$$

$$\text{Aire}(D) = \iint_\Delta r \, dr \, d\theta$$

$$= \int_0^1 r \, dr \times \int_0^\pi d\theta$$

$$= \frac{\pi}{2}$$

③ $$I = \iint_D \frac{y^2}{9 + x^2 + y^2} \, dx \, dy$$

$$= \iint_\Delta \frac{r^2 \sin^2\theta}{9 + r^2} \times r \, dr \, d\theta$$

$$= \int_0^1 \frac{r^3}{9 + r^2} \, dr \times \int_0^\pi \sin^2\theta \, d\theta$$

$$= \int_0^1 \left(r - \frac{9r}{r^2 + 9} \right) dr \times \int_0^\pi \frac{1 - \cos(2\theta)}{2} \, d\theta$$

$$= \left[\frac{1}{2} r^2 - \frac{9}{2} \ln(r^2 + 9) \right]_0^1 \times \frac{1}{2} \left[\theta - \frac{1}{2} \sin(2\theta) \right]_0^\pi$$

$$= \frac{\pi \left[1 - 9 \ln\left(\frac{10}{9}\right) \right]}{4}$$