— INT01 — 2025/2026

① calculer $I = \int_{-1}^{1} \frac{x+2}{x^2+2x+5}\,dx$ 3

②

$D = \{ (x,y) \in \mathbb{R}^2 ;\ y \leq 0 \text{ et } x^2+y^2 \leq 4 \}$

1 ① tracer D

1 ② calculer l'aire de (D) (En utilisant les intégrales double)

2,5 ③ calculer $I = \iint_D y^2\,dx\,dy$

— Bon courage —

Tel

~ corrigé ~

(1)

$$x^2 + 2x + 5 = (x+1)^2 + 4$$

on pose $t = x + 1 \Rightarrow x = t - 1$

$\Rightarrow dx = dt$

$x = -1 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 2$

$$I = \int_{-1}^{1} \frac{x+2}{x^2+2x+5}\,dx$$

$$= \int_{0}^{2} \frac{t+1}{t^2+4}\,dt$$

$$= \int_{0}^{2} \frac{t}{t^2+4}\,dt + \int_{0}^{2} \frac{1}{t^2+4}\,dt$$

$$= \left[\frac{1}{2}\ln(t^2+4) + \frac{1}{2}\arctan\left(\frac{t}{2}\right)\right]_0^2$$

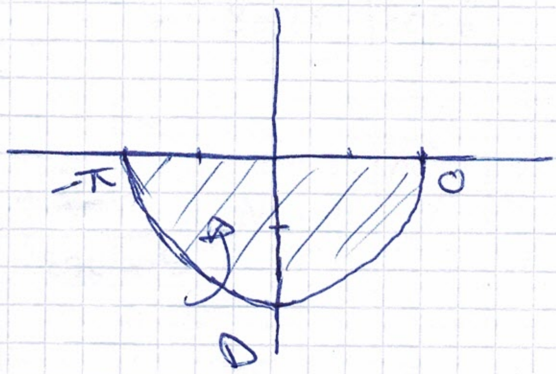
$$= \frac{1}{2}\ln(8) - \frac{1}{2}\ln(4) + \frac{1}{2}\arctan(1) - \frac{1}{2}\arctan(0)$$

$$= \frac{1}{2}\ln(2) + \frac{\pi}{8}$$

$$= \frac{4\ln(2) + \pi}{8}$$

ⓔ

①



②

$$\text{Aire}(D) = \iint_D dx\,dy$$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad |J| = r$$

$$\Delta = \{(r,\theta) \in \mathbb{R}^2 \,/\, 0 \leq r \leq 2 \; ; -\pi \leq \theta \leq 0\}.$$

$$\text{Aire}(D) = \iint_\Delta r\,dr\,d\theta$$

$$= \int_0^2 r\,dr \times \int_{-\pi}^0 d\theta = 2\pi.$$

③

$$I = \iint_D y^2\,dx\,dy$$

$$= \iint_\Delta r^2 \sin^2\theta \; r\,dr\,d\theta$$

$$= \int_0^2 r^3\,dr \times \int_{-\pi}^0 \sin^2\theta\,d\theta$$

$$= \left[\frac{1}{4} r^4\right]_0^2 \times \frac{1}{2} \int_{-\pi}^0 (1 - \cos(2\theta))\,d\theta$$

$$= 2\left[\theta - \frac{1}{2}\sin(2\theta)\right]_{-\pi}^0$$

$$= 2\pi$$