EL N° 1

INT ~~Analyse~~ et Auto y

ok

① $V = \left\{ (x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 \leq z \leq 1 \right\}$

$$I = \iiint x^2 z \, dx\,dy\,dz$$ 2,5

② $$I = \int_1^{+\infty} e^{-x^2} \, dx$$ 2,5

③ $$u_n = \frac{1}{2 + n^2 + \cos(n)}$$ 2,5

① $V = \{(x,y,z) \in \mathbb{R}^3, x^2+y^2 \leq z \leq 1\}$

$$I = \iiint_V x^2 z \, dx\,dy\,dz$$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \qquad |J| = r$$

$\Delta = \{(r,\theta,z) \in \mathbb{R}^3 ; 0 \leq r \leq 1, 0 \leq \theta \leq 2\pi$
$r^2 \leq z \leq 1\}$.

$$I = \iiint_\Delta r^2 \cos^2\theta \, \cancel{z} \cdot r \, dr\,d\theta\,dz$$

$$= \int_0^{2\pi} \cos^2\theta \, d\theta \times \int_0^1 \left[\int_{r^2}^1 r^3 z \, dz\right] dr$$

$$= \int_0^{2\pi} \frac{1+\cos(2\theta)}{2} d\theta \times \int_0^1 \left[\frac{1}{2} r^3 z^2\right]_{r^2}^1 dr$$

$$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin(2\theta)\right]_0^{2\pi} \times \int_0^1 \frac{1}{2} r^3 (1-r^4)\, dr$$

$$= \frac{1}{2}[2\pi] \times \int_0^1 -\frac{1}{8}(-4r^3)(1-r^4)\, dr$$

$$= \pi \times \left[-\frac{1}{16}(1-r^4)^2\right]_0^1.$$

$$= -\frac{\pi}{16}\left[(0) - (1)\right] = +\frac{\pi}{16}.$$

② .

②

$$I = \int_1^{+\infty} e^{-x^2} dx$$

La fct $x \mapsto e^{-x^2}$ continue et positive sur $[1, +\infty[$.

$\forall x \in [1, +\infty[$.

$$x^2 \geq x$$
$$-x^2 \leq -x$$
$$e^{-x^2} \leq e^{-x}$$

$$\int_1^{+\infty} e^{-x} dx = \lim_{x \to +\infty} \int_1^x e^{-t} dt$$

$$= \lim_{x \to +\infty} \left[-e^{-t}\right]_1^x$$

$$= \lim_{x \to +\infty} -e^{-x} + e^{-1} = e^{-1}$$

CV

D'après le critère de comparaison $\int_1^{+\infty} e^{-x^2} dx$ CV

③

$$-1 \leq \cos(n) \leq 1$$

$$1+n^2 \leq 2+n^2+\cos(n) \leq 3+n^2$$

$$\frac{1}{3+n^2} \leq \frac{1}{2+n^2+\cos(n)} \leq \frac{1}{1+n^2}$$

$$\frac{1}{3+n^2} \leq u_n \leq \frac{1}{1+n^2}$$

$$u_n \leq \frac{1}{1+n^2} \underset{+\infty}{\sim} \frac{1}{n^2}$$

CV Riemann $\alpha = 2 > 1$

Alors $\sum u_n$ CV.