EXO1 :

1) $F_1 = a(a+b)$

$= a.a + a.b$

$= a + a.b$

$= a(1+b)$

$= a \cdot 1$

$= a$

2) $F_2 = a + abc + \bar{a}bc + \bar{a}b + ad + a\bar{d}$

$= a + bc(a+\bar{a}) + \bar{a}b + a(d+\bar{d})$

$= a + bc + \bar{a}b + a$

$= a + b + bc$

$= a + b$

3) $F_3 = (a+b)(\bar{a}+\bar{b})$

$= a\bar{a} + a\bar{b} + b\bar{a} + b\bar{b}$

$= a\bar{b} + \bar{a}b$

$= a \oplus b$

4) $F_4 = (a+b)(\bar{a}+b)$

$= a\bar{a} + ab + \bar{a}b + b.b$

$= 0 + ab + \bar{a}b + b$

$= b(1+\bar{a}+a)$

$= b$

5) $F_5 = abc + a\bar{b}c + \bar{a}$

$= ac(b+\bar{b}) + \bar{a}$

$= ac + \bar{a}$

$= \bar{a} + c$

6) $F6 = (\bar a + b)(a + b + d)\bar d$

$= \underbrace{\bar a a \bar d}_{0} + \bar a b \bar d + \underbrace{\bar a d \bar d}_{0} + a b \bar d + b b \bar d + \underbrace{b d \bar d}_{0}$

$= \bar a b \bar d + a b \bar d + a b \bar d + b b \bar d$

$= \bar d (\bar a b + a b + b)$

$= \bar d (b(\underbrace{\bar a + a + 1}_{1}))$

$= \bar d b$

EXO2 : (Les formes Canoniques)

1) $F_1 = ab + bc + ac$

$1^{ère}$ forme : $\Sigma\Pi = \Sigma$ minterme

$2^{ème}$ forme : $\Pi\Sigma = \Pi$ maxterme

$F_1 = ab(c+\bar c) + (a+\bar a)bc + a(b+\bar b)c$

$= \underline{abc} + ab\bar c + \underline{abc} + \bar a bc + \underline{abc} + a\bar b c$

$= \underset{111}{abc} + \underset{110}{ab\bar c} + \underset{011}{\bar a bc} + \underset{101}{a\bar b c}$

$\boxed{F_1 = \Sigma\, 7, 6, 3, 5}$

$\overline{F_1} = \Sigma\, 0, 1, 2, 4 \quad \Rightarrow \quad \boxed{F_1 = \Pi\, 7, 6, 5, 3}$

$F_1 = (a+b+c).(a+b+\bar c).(a+\bar b+c).(\bar a+b+c)$

2) $F_2 = (a+b)(\bar a + b + d)$

$= \underset{111}{(a+b+d)}\underset{110}{(a+b+\bar d)}\underset{011}{(\bar a+b+d)}$

$\boxed{F_2 = \Pi\, 7, 6, 3}$

$\overline{F_2} = \Pi\, 0, 1, 2, 4, 5$

$\boxed{F_2 = \Sigma\, 7, 6, 5, 3, 2}$

3) $F3 = a.(b+c)$

$= \underset{111}{(a+b+c)}\underset{110}{(a+b+\bar c)}\underset{101}{(a+\bar b+c)}\underset{100}{(a+\bar b+\bar c)}\underset{111}{(a+b+c)}\underset{011}{(\bar a+b+c)}$

$\boxed{F_3 = \Pi\, 7, 6, 5, 4, 3}$

$\overline{F_3} = \Pi\ 0, 1, 2$

$F_3 = \Sigma\ 7, 6, 5$

$$\boxed{F_3 = (abc) + (ab\overline{c}) + (a\overline{b}c)}$$

Exo3 :

1) D'après la Table :

$$F(A,B,C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}C + AB\overline{C}$$
$$= \overline{A}\,\overline{B}(\overline{C} + C) + A\overline{B}C + AB\overline{C}$$
$$= \overline{A}\,\overline{B} + \overline{B}C + AB\overline{C}$$

2) On peut utiliser un tableau pour effectuer cette même simplification. Il n'y a qu'un regroupement possible.

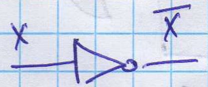
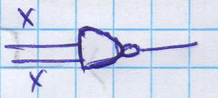
C \ AB	00	01	11	10
0	1	0	1	0
1	1	0	0	1

d'où la fonction :

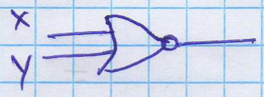
$$F(A,B,C) = \overline{A}\,\overline{B} + \overline{B}C + AB\overline{C}$$

Exo4 :

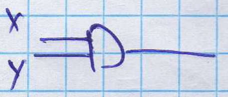
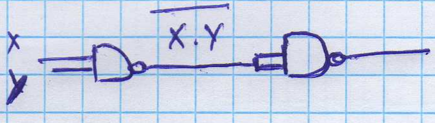
1) - La fonction NOT avec la porte logique NAND :

 $\Longleftrightarrow$  $\overline{X \cdot X} = \overline{X}$

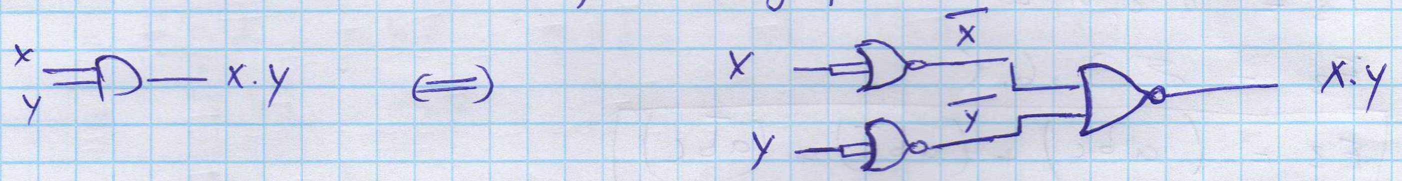
- Avec la porte logique NOR :

 $\Longleftrightarrow$  $\overline{X + X} = \overline{X}$

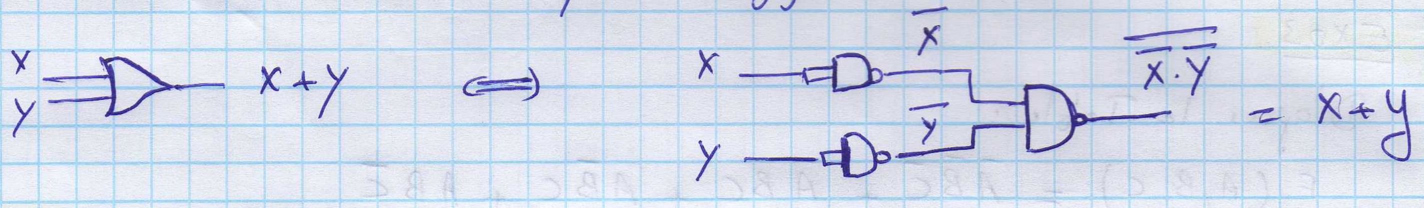
2) - La fonction AND avec la porte logique NAND :

 $\Longleftrightarrow$  $X \cdot Y$

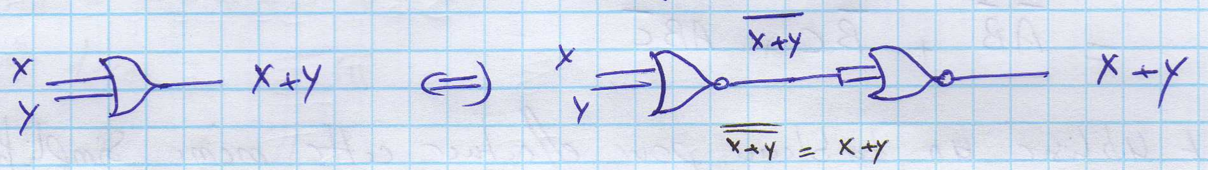
- La fonction AND avec la porte logique NOR :



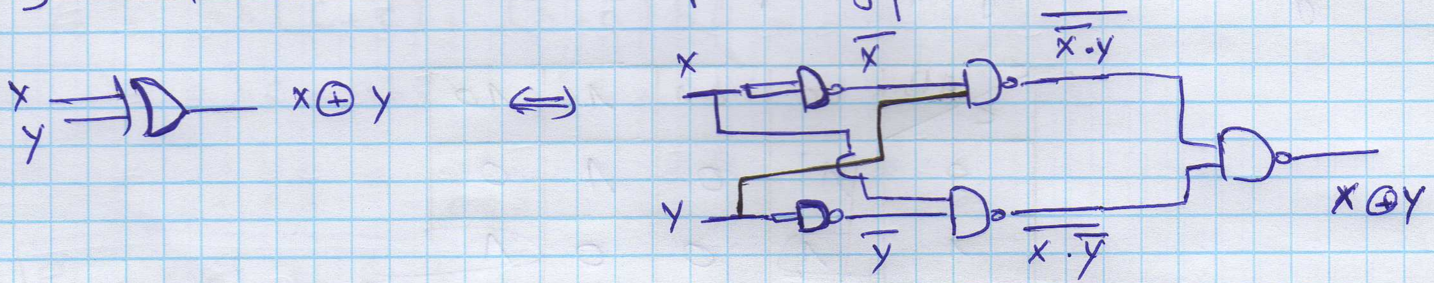
3) La fonction OR avec la porte logique NAND :



- La fonction OR avec la porte logique NOR :

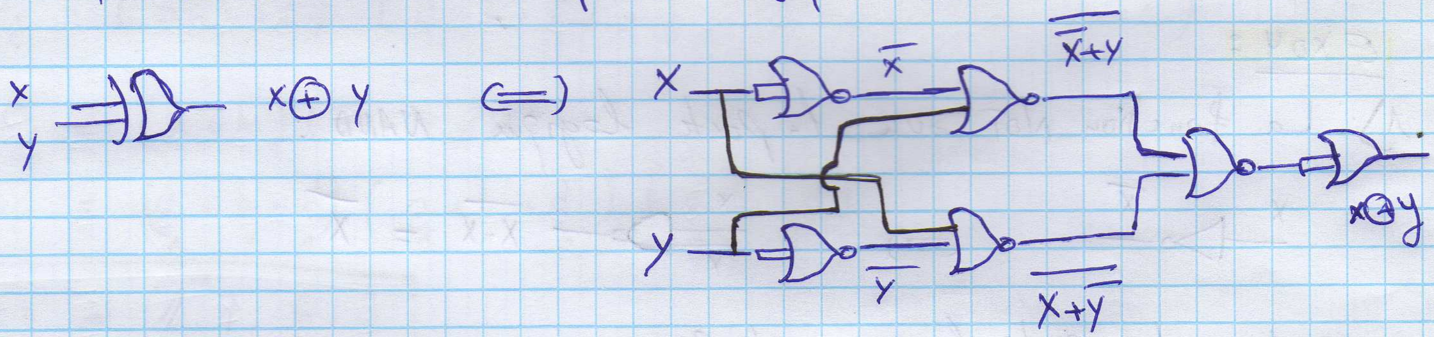


4) La fonction XOR avec la porte logique NAND :



$$x \oplus y = \bar{x}y + x\bar{y}$$
$$= \overline{\overline{\bar{x}y + x\bar{y}}} = \overline{\overline{\bar{x}y} \cdot \overline{x\bar{y}}}$$

- La fonction XOR avec la porte logique NOR :



$$x \oplus y = \overline{\overline{\bar{x}y}} + \overline{\overline{x\bar{y}}}$$
$$= \overline{\overline{\overline{x\bar{y}} + \overline{\bar{x}y}}}$$

Exo 1 :

$F_1(a,b,c,d) = \bar{a}\bar{c}d + \bar{a}cb + \bar{b}\bar{c}d + \bar{a}\bar{b}cd$

1ère étape $\longrightarrow$ Écrire F_1 sous la forme canonique.

$$F_1 = \bar{a}(b+\bar{b})\bar{c}d + \bar{a}bc(d+\bar{d}) + (a+\bar{a})\bar{b}\bar{c}d + \bar{a}\bar{b}cd$$

$$= \underset{0101}{\bar{a}b\bar{c}d} + \underset{0001}{\bar{a}\bar{b}\bar{c}d} + \underset{0111}{\bar{a}bcd} + \underset{0110}{\bar{a}bc\bar{d}} + \underset{1001}{a\bar{b}\bar{c}d} + \underset{0001}{\bar{a}\bar{b}\bar{c}d} + \underset{0011}{\bar{a}\bar{b}cd}$$

$F_1 = \Sigma\ 5, 1, 7, 6, 9, 3$

cd \ ab	00	01	11	10
00	(0)	(4)	(12)	(8)
01	1 (1)	1 (5)	(13)	1 (9)
11	1 (3)	1 (7)	(15)	(11)
10	(2)	1 (6)	(14)	(10)

$$\boxed{F_1 = \bar{a}d + \bar{a}bc + \bar{b}\bar{c}d}$$

2) $F_2(a,b,c) = \Sigma\ 0, 1, 3.$

c \ ab	00	01	11	10
0	1 (0)	(2)	(6)	(4)
1	1 (1)	1 (3)	(7)	(5)

$F_2 = \bar{a}\bar{b} + \bar{a}c$

$$\boxed{F_2 = \bar{a}(b+c)}$$

3) $F_3(a,b,c) = \Sigma\ 0, 3, 4, 6, 7$

c \ ab	00	01	11	10
0	1 (0)	(2)	1 (6)	1 (4)
1	(1)	1 (3)	1 (7)	(5)

$F_3 = \bar{b}\bar{c} + ab + bc$

$$\boxed{F_3 = \bar{b}\bar{c}\ b(a+c)}$$

4) $F_4(a,b,c,d) = \sum 5, 7, 13, 15$

cd \ ab	00	01	11	10
00	(0)	(4)	(12)	(8)
01	(1)	1 (5)	1 (13)	(9)
11	(3)	1 (7)	1 (15)	(11)
10	(2)	(6)	(14)	(10)

$$\boxed{F_4 = bd}$$

5) $F_5(a,b,c,d) = \sum 0, 5, 9, 10$ et ϕ pour $2, 3, 8, 15$

cd \ ab	00	01	11	10
00	1 (0)	(4)	(12)	ϕ (8)
01	(1)	1 (5)	(13)	1 (9)
11	ϕ (3)	(7)	ϕ (15)	(11)
10	ϕ (2)	(6)	(14)	1 (10)

$$\boxed{F_5 = \bar{b}\bar{d} + \bar{a}b\bar{c}d + a\bar{b}\bar{c}}$$

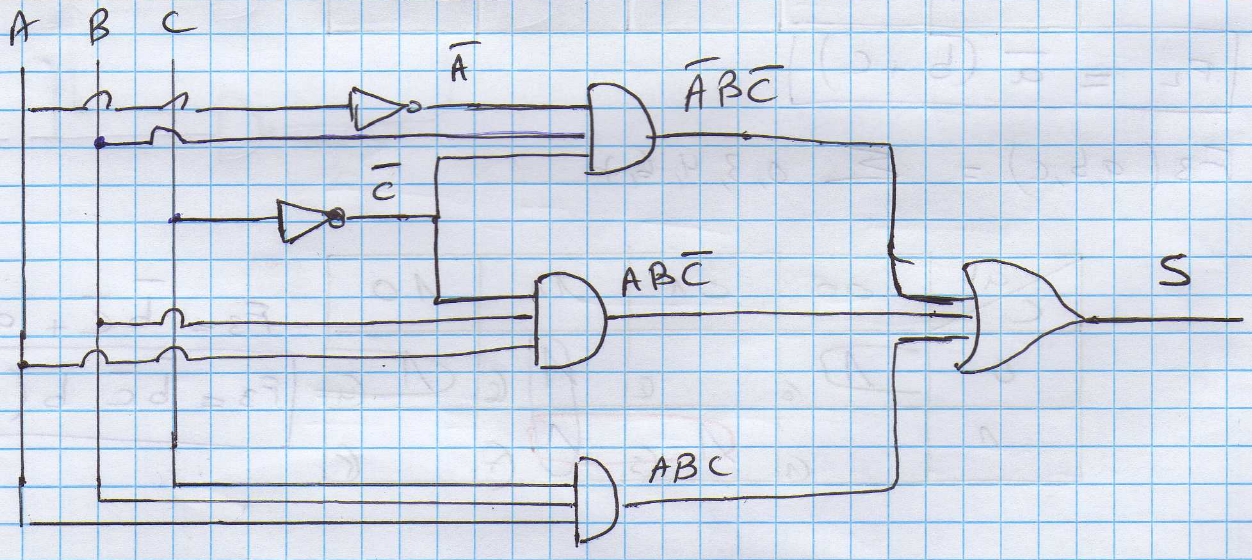
EXO6 :

a) L'expression logique de la Sortie S en fonction des entrées A, B et C

→ La fonction possède 3 variable ⇒ 2^3 combinaison

$$S = \bar{A}B\bar{C} + AB\bar{C} + ABC$$

b) Représentation de logigramme du système logique :



c) Simplification algébrique de l'expression S :

$$S = \overline{A}B\overline{C} + AB\overline{C} + ABC = B(\overline{A}\overline{C} + A\overline{C} + AC)$$
$$= B(\overline{C}(A + \overline{A}) + AC)$$
$$= B(\overline{C} + AC)$$
$$= B(\overline{C} + A)$$
$$= B\overline{C} + AB$$

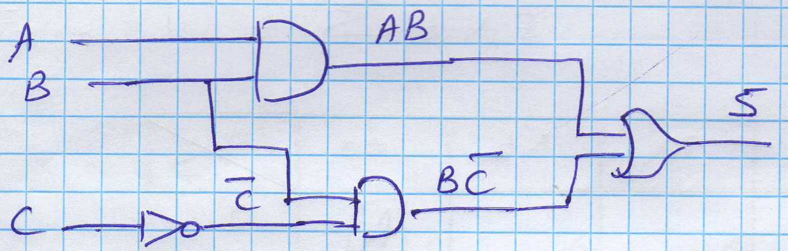
$$\boxed{S = B\overline{C} + AB}$$

d) Simplification de S en utilisant la méthode de Karnaugh.

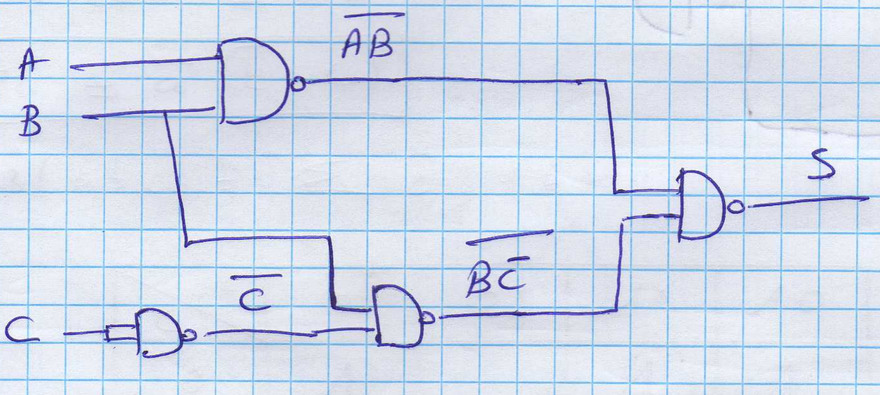
C \ AB	00	01	11	10
0	(0)	1 (2)	1 (6)	(4)
1	(1)	(3)	1 (7)	(5)

$$\boxed{S = B\overline{C} + AB}$$

Le système logique simplifié :



e) Le système logique simplifié en utilisant que des portes NAND



$$AB + B\overline{C} = \overline{\overline{AB + B\overline{C}}} = \overline{\overline{AB}\,.\,\overline{B\overline{C}}}$$

Exo 7 : (Commande de lamps)

1)

a	b	c	R	S
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

$\overline{R} = \overline{a}\,\overline{b}\,\overline{c} \implies R = a + b + c$

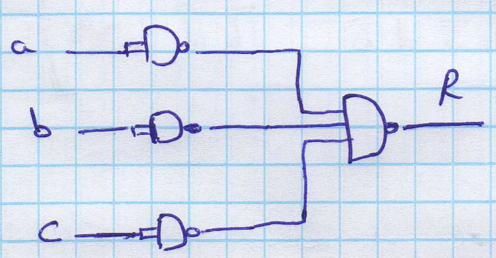
$S = bc + ac + ab$

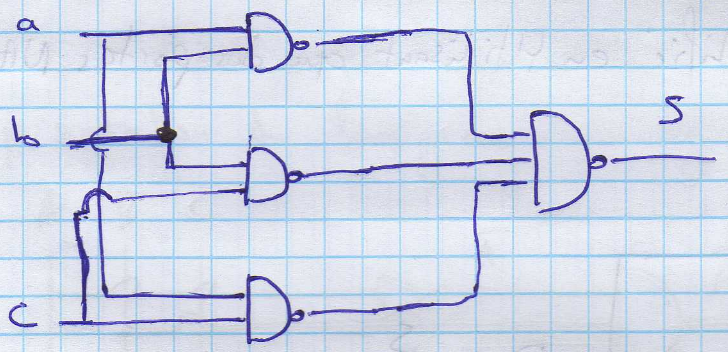
c \ ab	00	01	11	10
0			1	
1		1	1	1

2)

$R = \overline{\overline{R}} = \overline{\overline{a+b+c}} = \overline{\overline{a}.\overline{b}.\overline{c}}$

$S = \overline{\overline{S}} = \overline{\overline{ab+ac+bc}} = \overline{\overline{ab}\,.\,\overline{ac}\,.\,\overline{bc}}$





Ex08 : (commande d'une serrure)

1)

A	B	C	D	S
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

2)

$$S = \underset{0111}{\bar{A}BCD} + \underset{1011}{A\bar{B}CD} + \underset{1100}{AB\bar{C}\bar{D}} + \underset{1101}{AB\bar{C}D} + \underset{1110}{ABC\bar{D}} + \underset{1111}{ABCD}$$

$$S = \Sigma\, 15, 14, 13, 12, 11, 7$$

3)

CD \ AB	00	01	11	10
00	(0)	(4)	1 (12)	(8)
01	(1)	(5)	1 (13)	(9)
11	(3)	1 (7)	1 (15)	1 (11)
10	(2)	(6)	1 (14)	(10)

$$S = AB + BCD + ACD$$

$$S = \overline{\overline{AB + BCD + ACD}}$$

$$= \overline{\overline{AB} \cdot \overline{BCD} \cdot \overline{ACD}}$$

4)