Corrigé de la série 4

Exo 1 : $\vec{F}(x,y) = 2xy\,\vec{i} + x^2\,\vec{j}$.

① $\vec{F}$ conservative $\Rightarrow \vec{F} = -\overrightarrow{grad}\,E_p = -\vec{\nabla} E_p$

$\overrightarrow{rot}\,\vec{F} = \vec{\nabla} \wedge \vec{F} = -\vec{\nabla} \wedge \vec{\nabla} E_p = \vec{0}$.

Calculons $\overrightarrow{rot}\,\vec{F}$: $\overrightarrow{rot}\,\vec{F} = \vec{\nabla} \wedge \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & x^2 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2x-2x \end{pmatrix} = \vec{0}$

$\overrightarrow{rot}\,\vec{F} = \vec{0} \Rightarrow \vec{F}$ dérive de E_p

$\Rightarrow \vec{F}$ conservative.

② $\vec{F}$ conservative $\Rightarrow W_{OA}(\vec{F}) = -\Delta E_p = -(E_p(B) - E_p(A))$

Calculons $E_p(x,y)$: $\vec{F} = -\overrightarrow{grad}\,E_p = -\frac{dE_p}{dx}\vec{i} - \frac{dE_p}{dy}\vec{j}$.

$-\frac{dE_p}{dx} = 2xy \Rightarrow E_p = \int -2xy\,dx = -x^2y + E_p(y) + C$.

$-\frac{dE_p}{dy} = x^2 \Rightarrow \frac{dE_p}{dy} = -x^2 = -x^2 + \frac{dE_p(y)}{dy} \Rightarrow \frac{dE_p(y)}{dy} = 0$

Donc $\boxed{E_p(x,y) = -x^2y + C}$

$W_{OA}(\vec{F}) = -(E_p(B) - E_p(A)) = -(0-0) = 0\,J$

③ $A(2,0) \longrightarrow B(2,4)$.

$W_{AB}(\vec{F}) = -(\Delta E_p) = -(E_p(B) - E_p(A))$

$= -(-16 + C - C) = +16\,J$

~~④ $\vec{F}$ suivant $y = x^2$~~

$W_{OB}(\vec{F}) = W_{OA}(\vec{F}) + W_{AB}(\vec{F}) = 0 + 16 = +16\,J$.

④ suivant la courbe $y = x^2$

$y = x^2 \Rightarrow dy = 2x\,dx$.

$W_{OB}(\vec{F}) = \int_O^B \vec{F}\cdot\vec{dl} = \int_O^B [2xy\,dx + x^2\,dy] = \int_O^B (2x^3dx + 2x^3dx)$

$$\Rightarrow W_{OB}(\vec{F}) = \int_0^2 4x^3 dx = [\cancel{x}x^4]_0^2 = 16 J$$

(6) $W_{OB}(\vec{F})$ ne dépend pas du chemin suivi, donc $\vec{F}$ est une force ~~fo~~ conservative.

<u>Exo2 :</u> $E_p = k(x^2+y^2+z^2)$, $k = cte$.

$$\vec{F} = -\vec{grad} E_p = -\frac{dE_p}{dx}\vec{i} - \frac{dE_p}{dy}\vec{j} - \frac{dE_p}{dz}\vec{k}.$$

$$\vec{F} = -2kx\vec{i} - 2ky\vec{j} - 2kz\vec{k} = -2k(x\vec{i}+y\vec{j}+z\vec{k})$$

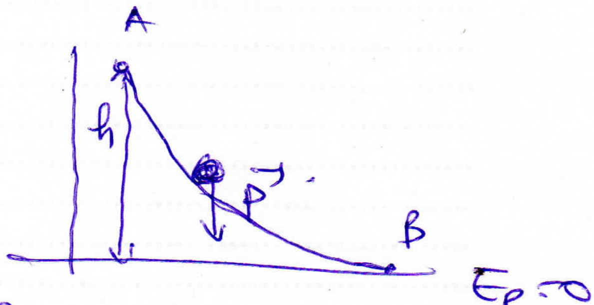
<u>Exercice 3 :</u>

$v_A = 0 \; m/s$

$v_B = 6 \; m/s$

$h = 2m$

$g = 10 m/s^2$



(1) La bille est-elle soumise aux forces de frottement ?

$\vec{P} = m\vec{g}$ est une force conservative $\Rightarrow W_{AB}(\vec{P}) = \Delta E_c$

$W_{AB}(\vec{P}) = mgh = 20m.$

$\Delta E_c = E_c(B) - E_c(A) = \frac{1}{2} m v_B^2 = 18 m.$

$mgh \neq \Delta E_c \Rightarrow$ En plus de $\vec{P} = m\vec{g}$, il y'a aussi des forces de frottement.

(3) Le travail des forces de frottement :

$W_{AB}(\vec{F_{NC}}) = \Delta E_m = E_m(B) - E_m(A)$

$E_m(B) = mgh_B + \frac{1}{2} m v_B^2 = 0 + \frac{1}{2} m v_B^2 = 54 J$

$E_m(A) = mgh_A + \frac{1}{2} m v_A^2 = mgh + 0 = 60 J$

donc $W_{AB}(\vec{F_{NC}}) = E_m(B) - E_m(A) = 54 - 60 = -6 J$

$W_{AB}(\vec{F_{NC}}) < 0 \Rightarrow \vec{F_{NC}}$ est une force de frottement. (s'oppose au mvt)