

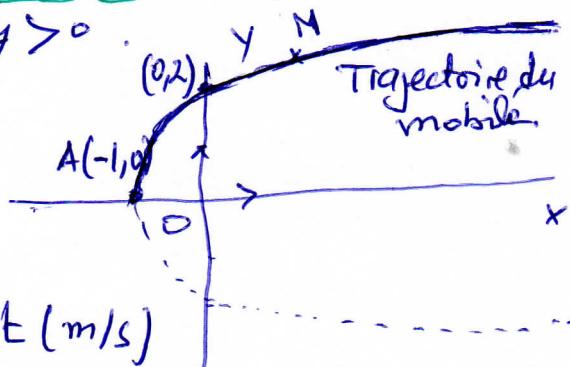
Exercice 2: Plan (XOY).

$$M(t) \begin{cases} x(t) = t^2 - 1 \\ y(t) = 2t \end{cases}$$

(1) La trajectoire et sa nature:

$$x = t^2 - 1 \Rightarrow t = \sqrt{x^2 + 1} \Rightarrow y = 2\sqrt{x^2 + 1}.$$

C'est l'équation d'une partie de la courbe parabolique d'équation $x = \frac{y^2}{4} - 1$ avec $y > 0$.
À $t = 0$, le mobile se trouve au point A(-1, 0).



(2) Vitesse et accélération

$$\ast \vec{v}(t) : \begin{cases} \dot{x}(t) = v_x(t) = \frac{dx}{dt} = 2t \text{ (m/s)} \\ \dot{y}(t) = v_y(t) = \frac{dy}{dt} = 2 \text{ m/s} \end{cases}$$

$$\|\vec{v}(t)\| = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{4t^2 + 4} = 2\sqrt{t^2 + 1} \text{ (m/s)}$$

$$\ast \vec{a}(t) : \begin{cases} \ddot{x}(t) = a_x(t) = \frac{d^2x}{dt^2} = 2 \text{ (m/s}^2) \\ \ddot{y}(t) = a_y(t) = \frac{d^2y}{dt^2} = 0 \text{ (m/s}^2) \end{cases}$$

$$\|\vec{a}(t)\| = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{4 + 0} = 2 \text{ m/s}^2 = \text{cte.}$$

(3) Nature du mouvement: c'est un mouvement curviligne uniformément varié (accéléré) car:

- La trajectoire est une courbe \Rightarrow mouvement curviligne
- $\vec{a} = \text{cte} \Rightarrow$ mouvement uniformément varié
- $\vec{a} \cdot \vec{v} = 4t > 0 \Rightarrow$ mouvement uniformément accéléré.

(4) * Accélération tangentielle:

$$\vec{a} = a_t \vec{u}_t + a_n \vec{u}_n$$

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(2\sqrt{t^2 + 1}) = 2 \times \frac{1}{2} \times 2t (t^2 + 1)^{\frac{1}{2}-1} = \frac{2t}{\sqrt{t^2 + 1}} \text{ (m/s}^2)$$

* Accélération normale:

$$\vec{a} = a_t \vec{u}_t + a_n \vec{u}_n \Rightarrow a^2 = a_t^2 + a_n^2$$

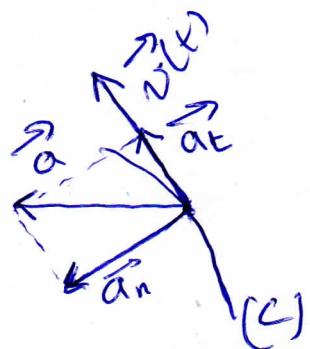
$$\Rightarrow a_n^2 = a^2 - a_t^2 = (2)^2 - \left(\frac{2t}{\sqrt{t^2 + 1}}\right)^2 = \frac{4}{t^2 + 1} \text{ (m/s}^2)$$

$$\Rightarrow a_n = \frac{2}{\sqrt{t^2 + 1}} \text{ (m/s}^2).$$

$$\ast \text{Rayon de courbure:}$$

$$a_n = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_n} = \frac{4(t^2 + 1)}{\left(\frac{2}{\sqrt{t^2 + 1}}\right)} = 2(t^2 + 1)^{3/2}$$

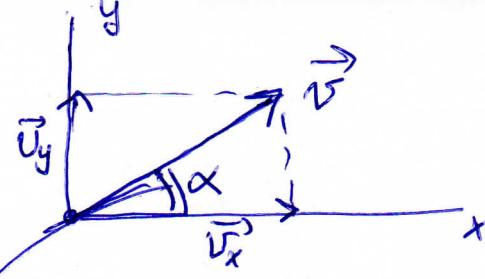
$$R(t) = 2(t^2 + 1)^{3/2}$$



$$\textcircled{5} \quad \alpha = (\vec{v}, \vec{Ox}) =$$

$$\sin \alpha = ?$$

$$\sin \alpha = \frac{v_y}{v} = \frac{2}{2\sqrt{t^2+1}} = \frac{1}{\sqrt{t^2+1}}$$



Exercice 3:

Dans le plan (xoy) on a :

$$M : \begin{cases} s(t) = b \cos \omega t \\ \theta(t) = \omega t \end{cases}$$

$$\text{Donc } \vec{r}(t) = b \cos \theta \cdot \vec{e}_g, \quad b = \text{cte}, \quad \omega = \text{cte}$$

\textcircled{1} Les vecteurs - position, vitesse et accélération Y dans la base $(\vec{e}_g, \vec{e}_\theta)$.



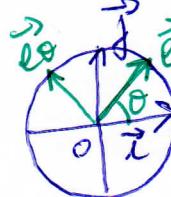
$$\vec{OM} = \vec{r} = s \vec{e}_g = b \cos \omega t \cdot \vec{e}_g$$

$$\|\vec{OM}\| = s = b \cos \omega t.$$

$$\vec{v}(t) = \frac{d\vec{OM}}{dt} = f(b \omega \sin \omega t) \vec{e}_g + (b \cos \omega t) \cdot \frac{d\vec{e}_g}{dt}$$

$$\text{or : } \frac{d\vec{e}_g}{dt} = \frac{d\vec{e}_g}{d\theta} \cdot \frac{d\theta}{dt} = \vec{e}_\theta \cdot \omega$$

$$\frac{d\vec{e}_\theta}{dt} = \frac{d\vec{e}_\theta}{d\theta} \cdot \frac{d\theta}{dt} = -\omega \vec{e}_g$$



$$\begin{cases} \vec{e}_g = \cos \theta \vec{i} + \sin \theta \vec{j} \\ \vec{e}_\theta = -\sin \theta \vec{i} + \cos \theta \vec{j} \end{cases}$$

$$\begin{aligned} \frac{d\vec{e}_g}{d\theta} &= -\sin \theta \vec{i} + \cos \theta \vec{j} = \vec{e}_\theta \\ \frac{d\vec{e}_\theta}{d\theta} &= -\cos \theta \vec{i} - \sin \theta \vec{j} = -\vec{e}_g \end{aligned}$$

$$\text{Donc } \vec{v} = -b \omega \sin \omega t \vec{e}_g + b \omega \cos \omega t \vec{e}_\theta$$

$$\boxed{\vec{v} = b \omega (-\sin \omega t \vec{e}_g + \cos \omega t \vec{e}_\theta)}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = b \omega \left[-\omega \cos \omega t \vec{e}_g - \sin \omega t \frac{d\vec{e}_g}{dt} \right]$$

$$\vec{a} = b \omega \left[-\omega \cos \omega t \vec{e}_g - \omega \sin \omega t \vec{e}_\theta - \omega \sin \omega t \vec{e}_g - \omega \cos \omega t \vec{e}_g \right]$$

$$\boxed{\vec{a} = -2b\omega^2 [\cos \omega t \vec{e}_g + \sin \omega t \vec{e}_\theta]}$$

Calculons l'angle (\vec{a}, \vec{v}) :

$$\text{on a : } \vec{a} \cdot \vec{v} = \|\vec{a}\| \cdot \|\vec{v}\| \cos(\vec{a}, \vec{v}) = 2b\omega^2 \sin \omega t \cos \omega t - 2b\omega^2 \sin \omega t \cos \omega t$$

$$\vec{a} \cdot \vec{v} = 0 \Rightarrow (\vec{a}, \vec{v}) = \frac{\pi}{2} \quad \forall t \quad (\vec{a} \text{ est toujours } \perp \vec{a} \vec{v})$$

$$\|\vec{a}\| = 2b\omega^2, \quad \|\vec{v}\| = b\omega.$$

Q) Représentation à $t_1 = 0$ s et à $t_2 = \frac{\pi}{4\omega}$ s.

* Au temps $t_1 = 0$ s on a: $\theta_1 = 0^\circ$, $f_1 = b$

Position: $\vec{OM}_1 = f \vec{e}_g = b \vec{e}_g$

Vitesse: $\vec{v}_1 = b\omega \vec{e}_\theta$

Accélération: $\vec{a}_1 = -2b\omega^2 \vec{e}_g$

$$\|\vec{OM}_1\| = b, \|\vec{v}_1\| = b\omega, \|\vec{a}_1\| = 2b\omega^2$$

$\theta = 0^\circ \Rightarrow$ le point M se trouve sur l'axe (Ox):

* Au temps $t_2 = \frac{\pi}{4\omega}$ s $\Rightarrow \theta_2 = \omega \cdot \frac{\pi}{4\omega} = \frac{\pi}{4}$ $\cos \frac{\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$f_2 = b \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} b.$$

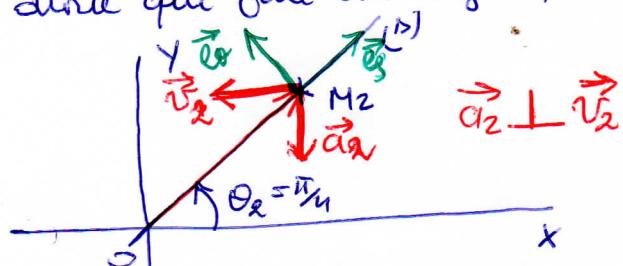
Position: $\vec{OM}_2 = f_2 \vec{e}_g = b \cos \frac{\pi}{4} \vec{e}_g = \frac{b\sqrt{2}}{2} \vec{e}_g$

Vitesse: $\vec{v}_2 = b\omega (-\sin \frac{\pi}{4} \vec{e}_g + \cos \frac{\pi}{4} \vec{e}_\theta) = \frac{b\sqrt{2}}{2} \omega (-\vec{e}_g + \vec{e}_\theta)$

Accélération: $\vec{a}_2 = -2b\omega^2 (\cos \frac{\pi}{4} \vec{e}_g + \sin \frac{\pi}{4} \vec{e}_\theta) = -\sqrt{2} b\omega^2 (\vec{e}_g + \vec{e}_\theta)$

$$\|\vec{OM}_2\| = \frac{b\sqrt{2}}{2}, \|\vec{v}_2\| = b\omega, \|\vec{a}_2\| = 2b\omega^2.$$

$\theta = \frac{\pi}{4} \Rightarrow M_2$ se trouve sur la droite qui fait un angle de $\pi/4$ avec l'axe (Ox).



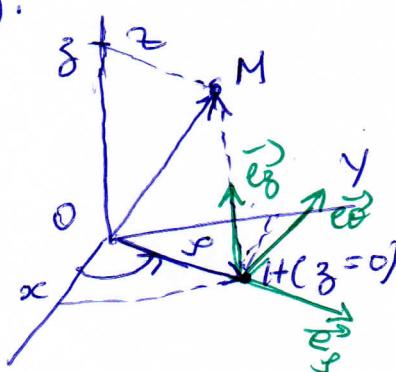
Exercice 4:

* Les coordonnées cylindriques d'un point M sont (ρ, θ, z)

H = projection de M sur (XOY).

$$\begin{cases} \rho = \|\vec{OH}\| \\ \theta = (\vec{Ox}, \vec{OH}) \\ z = HM \end{cases}$$

* La base locale associée à ces coordonnées est $(\vec{e}_g, \vec{e}_\theta, \vec{e}_z = \vec{k})$. \vec{e}_g est \perp à \vec{e}_θ et se trouvent dans le plan (XOY)



* Relations entre les coordonnées cylindriques et cartésiennes:

$$\begin{cases} x = \rho \cos \theta \\ y = \rho \sin \theta \\ z = z \end{cases} \Leftrightarrow \begin{cases} \rho = \sqrt{x^2 + y^2} \\ \theta = \arctan \frac{y}{x} \\ z = z \end{cases}$$

Données : $M : \begin{cases} x(t) = R \cos \omega t \\ y(t) = R \sin \omega t \\ z(t) = \alpha t \end{cases}$

(1) Coordonnées cylindriques de M :

* $f = \sqrt{x^2 + y^2} = \sqrt{R^2 \cos^2 \omega t + R^2 \sin^2 \omega t}$

$$f = R \sqrt{\cos^2 \omega t + \sin^2 \omega t} = R$$

* $\tan \theta = \frac{y}{x} = \frac{R \sin \omega t}{R \cos \omega t} = \tan \omega t$
 $\Rightarrow \theta = \omega t$

* $z = \alpha t$

(2) les vecteurs :

* position : $\vec{OM} = \vec{OH} + \vec{HM} = f \hat{e}_s + z \hat{k} = R \hat{e}_s + \alpha t \hat{k}$

* vitesse : $\vec{v} = \frac{d\vec{OM}}{dt} = R \frac{d\hat{e}_s}{dt} + \alpha \hat{k} = R \omega \hat{e}_\theta + \alpha \hat{k}$

* accélération : $\vec{a} = \frac{d\vec{v}}{dt} = R \omega \frac{d\hat{e}_\theta}{dt} = -R \omega^2 \hat{e}_s$ (R, ω, α et \hat{k} sont constants)

(3) L'angle entre \vec{a} et \vec{v} :

$$\|\vec{a}\| = R \omega^2, \|\vec{v}\| = \sqrt{R^2 \omega^2 + \alpha^2}$$

$$\vec{a} \cdot \vec{v} = \|\vec{a}\| \cdot \|\vec{v}\| \cdot \cos(\vec{a}, \vec{v}) = (-R \omega, 0, 0) \cdot (0, R \omega, \alpha) = 0$$

$$\Rightarrow \cos(\vec{a}, \vec{v}) = 0 \Rightarrow (\vec{a}, \vec{v}) = \frac{\pi}{2} (\vec{a} \perp \vec{v}).$$

(4) L'abscisse curviligne $s(t)$:

$$s(t) = \int v(t) \cdot dt = \int \sqrt{R^2 \omega^2 + \alpha^2} \cdot dt = \sqrt{R^2 \omega^2 + \alpha^2} \cdot t + C$$

$$s(0) = 0 \Rightarrow C = 0 \Rightarrow s(t) = \sqrt{R^2 \omega^2 + \alpha^2} \cdot t$$

(5) les accélérations tangentielle et normale :

$$\vec{a} = \vec{a}_t + \vec{a}_n = a_t \vec{u}_t + a_n \vec{u}_n$$

avec $a_t = \frac{dv}{dt}$

$$a_n = \frac{v^2}{R}$$

$$v = \sqrt{R^2 \omega^2 + \alpha^2}$$

Donc $a_t = \frac{dv}{dt} = 0 \text{ m/s}^2$

$$\boxed{a_t = 0}$$

$$a^2 = a_n^2 + a_t^2$$

$$a_n^2 = a^2 - a_t^2 = a^2 = R^2 \omega^4$$

$$\Rightarrow \boxed{a_n = R \omega^2}$$

(5)

$$\textcircled{6} \quad \text{Le rayon de courbure } R_c : R_c = \frac{R^2 w^2 + \alpha^2}{R w^2} = R + \frac{\alpha^2}{R w^2}$$

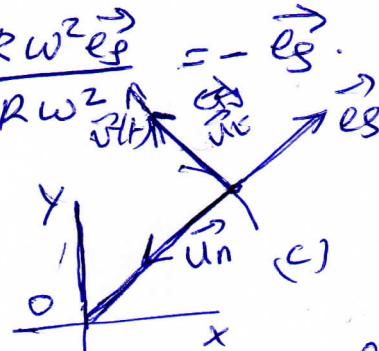
$$a_n = \frac{v^2}{R_c} \Rightarrow R_c = \frac{v^2}{a_n} = \frac{R^2 w^2 + \alpha^2}{R w^2}$$

$$\textcircled{7} \quad \text{des vecteurs unitaires } \vec{u}_t \text{ et } \vec{u}_n.$$

On a : $\vec{v} = v \cdot \vec{u}_t \Rightarrow \vec{u}_t = \frac{\vec{v}}{v} = \frac{R w \vec{e}_\theta + \alpha \vec{k}}{\sqrt{R^2 w^2 + \alpha^2}}$

$$\Rightarrow \vec{u}_t = \left(\frac{R w}{\sqrt{R^2 w^2 + \alpha^2}} \right) \vec{e}_\theta + \left(\frac{\alpha}{\sqrt{R^2 w^2 + \alpha^2}} \right) \vec{k}$$

$$\vec{u}_n = a_n \vec{u}_n \Rightarrow \vec{u}_n = \frac{\vec{a}_n}{a_n} = \frac{-R w^2 \vec{e}_r}{R w^2 \cancel{a_n}} = -\vec{e}_r.$$



Exercice 5 :

Les équations horaires en coordonnées polaires

ont données par :

$$r(t) = r_0 e^{-t/b}, \quad \theta(t) = \theta_0 + \frac{t}{b}$$

$$b = \text{cte}, \quad \theta_0 = \text{cte}.$$

\textcircled{1} Le vecteur-vitesse de la particule :

$$\vec{OM}(t) = \vec{r}(t) \vec{e}_r = r_0 e^{-t/b} \vec{e}_r \quad (\text{avec } \vec{e}_r = \vec{e}_r).$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{dr}{dt} \vec{e}_r = -\frac{r_0}{b} e^{-t/b} \vec{e}_r + (r_0 e^{-t/b}) \frac{d\vec{e}_r}{dt}$$

$$\text{or } \frac{d\vec{e}_r}{dt} = \frac{d\vec{e}_r}{d\theta} \cdot \frac{d\theta}{dt} = \dot{\theta} \frac{d\vec{e}_r}{d\theta} = \frac{1}{b} \cdot \vec{e}_\theta$$

$$\text{Donc : } \vec{v}(t) = -\frac{r_0}{b} e^{-t/b} \vec{e}_r + r_0 e^{-t/b} \cdot \frac{1}{b} \vec{e}_\theta$$

$$\boxed{\vec{v}(t) = \frac{r_0}{b} e^{-t/b} (-\vec{e}_r + \vec{e}_\theta)}$$

$$\textcircled{2} \quad \alpha = (\vec{v}, \vec{e}_\theta) = \text{cte. ?}$$

$$\vec{v} \cdot \vec{e}_\theta = \|\vec{v}\| \cdot \|\vec{e}_\theta\| \cdot \cos(\vec{v}, \vec{e}_\theta) = \vec{v} \cdot \vec{e}_\theta$$

$$\text{avec : } \vec{v} \left(\begin{array}{c} -\frac{r_0}{b} e^{-t/b} \\ r_0 e^{-t/b} \end{array} \right) \text{ et } \vec{e}_\theta \left(\begin{array}{c} 0 \\ 1 \end{array} \right)$$

\textcircled{6}

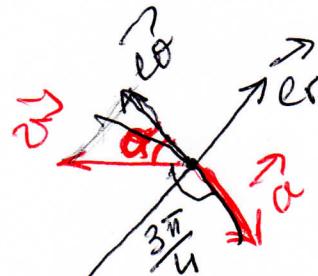
$$\|\vec{v}\| = \frac{\sqrt{2} r_0}{b} e^{-t/b}$$

$$\|\vec{e}_\theta\| = 1$$

$$\text{Donc } \vec{v} \cdot \vec{e}_\theta = \frac{\sqrt{2} r_0}{b} e^{-t/b} \cdot \cos(\vec{v}, \vec{e}_\theta) = \frac{r_0}{b} e^{-t/b}$$

$$\Rightarrow \cos(\vec{v}, \vec{e}_\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \text{cte}$$

$$\text{Donc } (\vec{v}, \vec{e}_\theta) = \pm \pi/4 = \text{cte.}$$



$$\vec{v} = \frac{r_0 e^{-t/b}}{b} (-\vec{e}_r + \vec{e}_\theta)$$

$$\tan \alpha = \frac{v_r}{v_\theta} = -1 \Rightarrow (\vec{v}, \vec{e}_\theta) = -\frac{\pi}{4}$$

Vecteur-acceleration:

$$\vec{a}_{ff} = \frac{d\vec{v}}{dt} = \frac{r_0}{b^2} e^{-t/b} \vec{e}_r - \frac{r_0}{b} e^{-t/b} \frac{d\vec{e}_r}{dt} - \frac{r_0}{b^2} e^{-t/b} \vec{e}_\theta + \frac{r_0}{b} e^{-t/b} \frac{d\vec{e}_\theta}{dt}$$

$$\text{or} \left\{ \begin{array}{l} \frac{d\vec{e}_r}{dt} = \frac{1}{b} \vec{e}_\theta \\ \frac{d\vec{e}_\theta}{dt} = -\frac{1}{b} \vec{e}_r \end{array} \right.$$

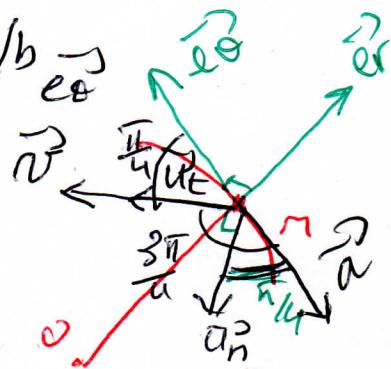
$$\left\{ \begin{array}{l} \frac{d\vec{e}_r}{dt} = \frac{1}{b} \vec{e}_\theta \\ \frac{d\vec{e}_\theta}{dt} = -\frac{1}{b} \vec{e}_r \end{array} \right.$$

$$\text{Donc } \vec{a}(t) = \frac{r_0}{b^2} e^{-t/b} \vec{e}_r - \frac{r_0}{b^2} e^{-t/b} \vec{e}_\theta - \frac{r_0}{b^2} e^{-t/b} \vec{e}_\theta - \frac{r_0}{b^2} e^{-t/b} \vec{e}_r$$

$$\boxed{\vec{a}(t) = -\frac{2r_0}{b^2} e^{-t/b} \vec{e}_\theta}$$

$$\textcircled{1} \quad \beta = (\vec{a}, \vec{u}_n) = \text{cte.}?$$

$$\text{on remarque que } \left\{ \begin{array}{l} \vec{a} = -\frac{2r_0}{b^2} e^{-t/b} \vec{e}_\theta \\ (\vec{v}, \vec{a}_\theta) = -\frac{\pi}{4} \end{array} \right.$$



$$\text{Donc } (\vec{a}, \vec{v}) = 3\pi/4$$

$$\text{D'autre part: } \vec{v} = v \cdot \vec{u}_r \Rightarrow (\vec{a}, \vec{u}_r) = \frac{3\pi}{4}$$

⑦

la base (\vec{u}_t, \vec{u}_n) est orthogonale

$$\Rightarrow (\vec{u}_t, \vec{u}_n) = \frac{\pi}{2}.$$

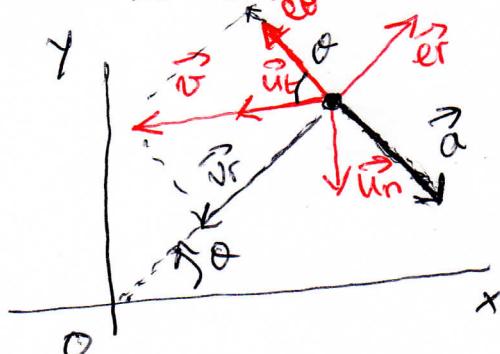
Donc $\boxed{(\vec{a}, \vec{u}_n) = \frac{\pi}{4}}.$

⑤ Rayon d'onde combinée?

On a: $a_n = \frac{v^2}{R_c} \Rightarrow R_c = \frac{a_n}{v^2} = \frac{e^{r_0}}{b^2} e^{-\frac{ct}{b}} / a_n$

$$R_c = a \cos \beta = \frac{e^{r_0}}{b^2} e^{-\frac{ct}{b}} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} e^{r_0}}{b^2} e^{-\frac{ct}{b}}$$

$$R_c = \frac{2 r_0 e^{-\frac{ct}{b}}}{b^2 \times \frac{\sqrt{2} e^{r_0}}{b^2} e^{-\frac{ct}{b}}} = \frac{2 r_0}{\sqrt{2}} e^{-\frac{ct}{b}} = \sqrt{2} r_0 e^{-\frac{ct}{b}}$$



Exercice 6:

① * Le bord de la rivière = Repère absolu

* Eau (courant) = Repère relatif.

* Nageur = mobile

* La vitesse du nageur par rapport à l'eau et la vitesse relative $\vec{v}_r = \vec{v}_1$

* La vitesse du nageur par rapport au bord de la rivière et la vitesse absolue $\vec{v}_a = \vec{v}_3$

* La vitesse de l'eau dans la rivière par rapport à la rive et la vitesse d'entraînement $\vec{v}_e = \vec{v}_2$

⑧

(2)

$$\vec{v}_a = \vec{v}_r + \vec{v}_e$$

$$v_3 = v_1 + v_2$$

$$\Rightarrow v_3 = v_a = \sqrt{v_1^2 + v_2^2}$$

$$v_3 = \sqrt{16 + 9}$$

$$\boxed{v_3 = 5 \text{ m/s}}$$

$$\tan \theta = \frac{v_e}{v_r} = \frac{3}{4} \Rightarrow \theta = \arctan(3/4) = 37^\circ.$$

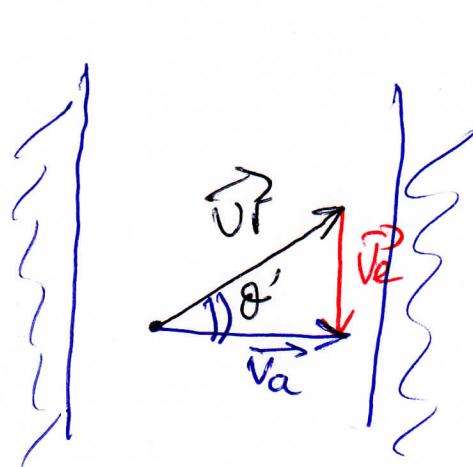
(3) On a trapezoid $\vec{v}_a = \vec{v}_e + \vec{v}_r$

$$v_r^2 = v_a^2 + v_e^2$$

$$\Rightarrow v_a^2 = v_r^2 - v_e^2$$

$$\Rightarrow v_a = \sqrt{(4)^2 - (3)^2}$$

$$\boxed{v_a = \sqrt{7} = 2,64 \text{ m}}$$



Direction

$$\tan \theta' = \frac{v_e}{v_r} = \frac{3}{4,64} = \cancel{\frac{3}{8,64}}$$

$$\Rightarrow \theta' = \arctan \cancel{\frac{3}{8,64}}(1,136) = 48,6^\circ$$

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