**Numerical questions**

**Lecture 1**

1. The following table lists foreign exchange rates between U.S. dollars and British pounds during April:

|  |  |  |  |
| --- | --- | --- | --- |
| **Date** | **U.S. Dollars per GBP** | **Date** | **U.S. Dollars per GBP** |
| 4/1 | 1.9564 | 4/18 | 1.7504 |
| 4/4 | 1.9293 | 4/19 | 1.7255 |
| 4/5 | 1.914 | 4/20 | 1.6914 |
| 4/6 | 1.9374 | 4/21 | 1.672 |
| 4/7 | 1.961 | 4/22 | 1.6684 |
| 4/8 | 1.8925 | 4/25 | 1.6674 |
| 4/11 | 1.8822 | 4/26 | 1.6857 |
| 4/12 | 1.8558 | 4/27 | 1.6925 |
| 4/13 | 1.796 | 4/28 | 1.7201 |
| 4/14 | 1.7902 | 4/29 | 1.7512 |
| 4/15 | 1.7785 |  |  |

Which day would have been the best day to convert $200 into British pounds?

Which day would have been the worst day? What would be the difference in pounds?

**Solution:** The best day is 4/25. At a rate of $1.6674/pound, you would have £119.95. The worst  
day is 4/7. At $1.961/pound, you would have £101.99, or a difference of £17.96.

**Lecture 2 &3**

1. Calculate the present value of a $1,000 zero-coupon bond with 5 years to maturity if the required annual interest rate is 6%.

**Solution:** *PV* = *FV*/(1 + *i*)*n*

where *FV* = 1000, *i* = 0.06, *n* = 5

*PV* = 747.25

2. A lottery claims its grand prize is $10 million, payable over 20 years at $500,000 per year. If the first payment is made immediately, what is this grand prize really worth? Use a discount rate of 6%.

**Solution:** This is a simple present value problem. Using a financial calculator,

*N* = 20; *PMT* = 500,000; *FV* = 0; *I* = 6%; *Pmts* in BEGIN mode.

Compute *PV*: *PV* = $6,079,058.25

3. Consider a bond with a 7% annual coupon and a face value of $1,000. Complete the following table:

|  |  |  |
| --- | --- | --- |
| **Years to Maturity** | **Discount Rate** | **Current Price** |
| 3 | 5 |  |
| 3 | 7 |  |
| 6 | 7 |  |
| 9 | 7 |  |
| 9 | 9 |  |

What relationship do you observe between yield to maturity and the current market value?

**Solution:**

|  |  |  |
| --- | --- | --- |
| **Years to Maturity** | **Yield to Maturity** | **Current Price** |
| 3 | 5 | $1,054.46 |
| 3 | 7 | $1,000.00 |
| 6 | 7 | $1,000.00 |
| 9 | 5 | $1,142.16 |
| 9 | 9 | $ 880.10 |

When yield to maturity is above the coupon rate, the band’s current price is below its face value. The opposite holds true when yield to maturity is below the coupon rate. For a given maturity, the bond’s current price falls as yield to maturity rises. For a given yield to maturity, a bond’s value rises as its maturity increases. When yield to maturity equals the coupon rate, a bond’s current price equals its face value regardless of years to maturity.

4. Consider a coupon bond that has a $1,000 par value and a coupon rate of 10%. The bond is currently selling for $1,150 and has 8 years to maturity. What is the bond’s yield to maturity?

**Solution:** To calculate the bond’s yield to maturity using a financial calculator,

*N* = 8; *PMT* = 1000 × 0.10 = 100; *FV* = 1000; *PV* = 1150

Compute *I*: *I* = 7.44

5. Suppose that a commercial bank wants to buy Treasury Bills. These instruments pay $5,000 in one year and are currently selling for $5,012. What is the yield to maturity of these bonds? Is this a typical situation? Why?

**Solution:** The yield to maturity of these bonds solves the following equation: 5,000/(1+i) = 5,012. After some algebra, the yield to maturity happens to be around – 0.24%. This is not a typical situation. In normal times banks will not choose to pay more than the face value of a discount bond, since that implies negative yields to maturity. This example illustrates situations as the ones described in the Global Box in this chapter.

6. What is the price of a perpetuity that has a coupon of $50 per year and a yield to maturity of 2.5%? If the yield to maturity doubles, what will happen to its price?

**Solution:** The price would be $50/.025 = $2000. If the yield to maturity doubles to 5%, the price would fall to half its previous value, to $1000 = $50/.05.

7. Property taxes in DeKalb County are roughly 2.66% of the purchase price every year. If you just bought a $100,000 home, what is the *PV* of *all* the future property tax payments? Assume that the house remains worth $100,000 forever, property tax rates never change, and that a 9% discount rate   
is used for discounting.

**Solution:** The taxes on a $100,000 home are roughly 100,000 × 0.0266 = 2,660.

The *PV* of all future payments = 2,660/0.09 = $29,555.55 (a perpetuity).

8. Suppose you want to take out a loan and that your local bank wants to charge you an annual real interest rate equal to 3%. Assuming that the annualized expected rate of inflation over the life of the loan is 1%, determine the nominal interest rate that the bank will charge you. What happens if, over the life of the loan, actual inflation is 0.5%?

**Solution:** The bank will charge you a nominal interest rate equal to 1% + 3% = 4%. However, if actual inflation turns out to be lower than expected, then you will be worse off than originally planned, since the real cost of borrowing (measured by the real interest rate) turned out to be 4% – 0.5% = 3.5%.

9. Lucia just bought two coupon bonds, one with a face value of $1,000 and the other with a face value of $5,000. Both bonds have a coupon rate of 5% and sold at par today. Calculate both bonds´ current yield and both bonds rate of return if Lucia is able to sell these bonds one year later for $100 more of the buying price. Can you estimate what happened to the interest rate over that year?

**Solution:** The current yield (CY) is calculated as the coupon payment over the selling price of the bond. When a coupon bond sells at par, its current yield equals the coupon rate, since the numerator of the CY is: Face Value x Coupon Rate (always) and the denominator is Face Value (in this particular situation only in which Price = FV). Both bonds have a CY = 5%. If Lucia is able to sell the $1,000 FV coupon bond for $1,100, then the rate of return is: 5% + 10% (since the rate of capital gain is 100/1,000 =10%). The same reasoning yields a RET = 5% + 2% (g = 100/5,000) for the other bond. The interest rate must have fallen over that year for bond´s prices to increase. Note, however, that it is unlikely that both bond´s prices increased by the same amount. Other determinants of bond´s prices (see chapters 4 and 5) likely explained this effect.

10. You have paid $980.30 for an 8% coupon bond with a face value of $1,000 that mature in five years. You plan on holding the bond for one year. If you want to earn a 9% rate of return on this investment, what price must you sell the bond for? Is this realistic?

**Solution:** To find the price, solve:



Although this appears possible, the yield to maturity when you purchased the bond was 8.5%. At that yield, you only expect the price to be $983.62 next year. In fact, the yield would have to drop to 8.35% for the price to be $988.53.

11. Calculate the duration of a $1,000 6% coupon bond with three years to maturity. Assume that all market interest rates are 7%.

**Solution:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **1** | **2** | **3** | **Sum** |
| Payments | 60.00 | 60.00 | 1060.00 |  |
| *PV* of Payments | 56.07 | 52.41 | 865.28 | 973.76 |
| Time Weighted *PV* of Payments | 56.07 | 104.81 | 2595.83 |  |
| Time Weighted *PV* of Payments  Divided by Price | 0.06 | 0.11 | 2.67 | 2.83 |

This bond has a duration of 2.83 years. Note that the current price of the bond is $973.76, which is the sum of the individual “*PV* of payments.”

12. Consider the bond in the previous question. Calculate the expected price change if interest rates   
drop to 6.75% using the duration approximation. Calculate the actual price change using discounted cash flow.

**Solution:** Using the duration approximation, the price change would be:



The new price would be $980.20. Using a discounted cash flow approach, the price is 980.23—only $.03 different.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **1** | **2** | **3** | **Sum** |
| Payments | 60.00 | 60.00 | 1060.00 |  |
| *PV* of payments | 56.21 | 52.65 | 871.3 | 980.23 |

13. The duration of a $100 million portfolio is 10 years. $40 million dollars in new securities are added to   
the portfolio, increasing the duration of the portfolio to 12.5 years. What is the duration of the   
$40 million in new securities?

**Solution:** First, note that the portfolio now has $140 million in it. The duration of a portfolio is the weighted average duration of its individual securities. Let *D* equal the duration of the   
$40 million in new securities. Then, this implies:



The new securities have a duration of 18.75 years.

14. A bank has two, 3-year commercial loans with a present value of $70 million. The first is a $30 million loan that requires a single payment of $37.8 million in 3 years, with no other payments until then. The second is for $40 million. It requires an annual interest payment of $3.6 million. The principal of   
$40 million is due in 3 years.

a. What is the duration of the bank’s commercial loan portfolio?

b. What will happen to the value of its portfolio if the general level of interest rates increased from 8% to 8.5%?

**Solution:** The duration of the first loan is 3 years since it is a zero-coupon loan. The duration of the second loan is as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **1** | **2** | **3** | **Sum** |
| Payment | 3.60 | 3.60 | 43.60 |  |
| *PV* of Payments | 3.33 | 3.09 | 34.61 | 41.03 |
| Time Weighted *PV* of Payments | 3.33 | 6.18 | 103.83 |  |
| Time Weighted *PV* of Payments   Divided by Price | 0.08 | 0.15 | 2.53 | 2.76 |

The duration of a portfolio is the weighted average duration of its individual securities.   
So, the portfolio’s duration = 3/7 × (3) + 4/7 × (2.76) = 2.86

If rates increased, 

15. Consider a bond that promises the following cash flows. The required discount rate is 12%.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year** | **0** | **1** | **2** | **3** | **4** |
| Promised Payments |  | 160 | 170 | 180 | 230 |

You plan to buy this bond, hold it for 2½ years, and then sell the bond.

a. What total cash will you receive from the bond after the 2½ years? Assume that periodic cash flows are reinvested at 12%.

b. If immediately after buying this bond, all market interest rates drop to 11% (including your reinvestment rate), what will be the impact on your total cash flow after 2½ years? How does   
this compare to part (a)?

c. Assuming all market interest rates are 12%, what is the duration of this bond?

**Solution:**

a. You will receive 160 reinvested for 1.5 years, and 170 reinvested for 0.5 years. Then you will sell the remaining cash flows, discounted at 12%. This gives you:



b. This is the same as part (a), but the rate is now 11%.



Notice that this is only $0.05 different from part (a).

c. The duration is calculated as follows:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year** | **1** | **2** | **3** | **4** | **Sum** |
| Payments | 160.00 | 170.00 | 180.00 | 230.00 |  |
| *PV* of Payments | 142.86 | 135.52 | 128.12 | 146.17 | 552.67 |
| Time Weighted *PV* of Payments | 142.86 | 271.05 | 384.36 | 584.68 |  |
| Time Weighted *PV* of Payments  Divided by Price | 0.26 | 0.49 | 0.70 | 1.06 | 2.50 |

Since the duration and the holding period are the same, you are insulated from immediate changes in interest rates! It doesn’t always work out this perfectly, but the idea is important.

**Lecture 4**

1. You own a $1,000-par zero-coupon bond that has 5 years of remaining maturity. You plan on selling the bond in one year and believe that the required yield next year will have the following probability distribution:

|  |  |
| --- | --- |
| **Probability** | **Required Yield** |
| 0.1 | 6.60% |
| 0.2 | 6.75% |
| 0.4 | 7.00% |
| 0.2 | 7.20% |
| 0.1 | 7.45% |

a. What is your expected price when you sell the bond?

b. What is the standard deviation?

**Solution:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Probability** | **Required Yield** | **Price** | **Prob × Price** | **Prob × (Price  Exp. Price)2** |
| 0.1 | 6.60% | $774.41 | $ 77.44 | 12.84776241 |
| 0.2 | 6.75% | $770.07 | $154.01 | 9.775668131 |
| 0.4 | 7.00% | $762.90 | $305.16 | 0.013017512 |
| 0.2 | 7.20% | $757.22 | $151.44 | 6.862609541 |
| 0.1 | 7.45% | $750.02 | $ 75.02 | 16.5903224 |
|  |  |  | $763.07 | 46.08937999 |

The expected price is $763.07.

The variance is $46.09, or a standard deviation of $6.79.

2. Consider a $1,000-par junk bond paying a 12% annual coupon. The issuing company has 20% chance of defaulting this year; in which case, the bond would not pay anything. If the company survives the first year, paying the annual coupon payment, it then has a 25% chance of defaulting in the second year. If the company defaults in the second year, neither the final coupon payment nor par value of the bond will be paid. What price must investors pay for this bond to expect a 10% yield to maturity? At that price, what is the expected holding period return? Standard deviation of returns? Assume that periodic cash flows are reinvested at 10%.

**Solution:** The expected cash flow at *t*1 = 0.20 (0) + 0.80 (120) = 96

The expected cash flow at *t*2 = 0.25 (0) + 0.75 (1,120) = 840

The price today should be: 

At the end of two years, the following cash flows and probabilities exist:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Probability** | **Final Cash Flow** | **Holding Period Return** | **Prob × HPR** | **Prob × (HPR  Exp. HPR)2** |
| 0.2 | $ 0.00 | −100.00% | −20.00% | 19.80% |
| 0.2 | $ 132.00 | −83.11% | −16.62% | 13.65% |
| 0.6 | $1,252.00 | 60.21% | 36.12% | 22.11% |
|  |  |  | −0.50% | 55.56% |

The expected holding period return is almost zero (−0.5%). The standard deviation is roughly 74.5% (the square root of 55.56%).

3. Last month, corporations supplied $250 billion in bonds to investors at an average market rate of 11.8%. This month, an additional $25 billion in bonds became available, and market rates increased to 12.2%. Assuming a Loanable Funds Framework for interest rates, and that the demand curve remained constant, derive a linear equation for the demand for bonds, using prices instead of interest rates.

**Solution:** First, translate the interest rates into prices.





We know two points on the demand curve:



So, the slope  

Using the point-slope form of the line, Price = 0.12755 × Quantity + Constant. We can substitute in either point to determine the constant. Let’s use the first point:



Finally, we have:



4. An economist has estimated that, near the point of equilibrium, the demand curve and supply curve for bonds can be estimated using the following equations:



a. What is the expected equilibrium price and quantity of bonds in this market?

b. Given your answer to part (a), which is the expected interest rate in this market?

**Solution:**

a. Solve the equations simultaneously:



This implies that *P* = 814.2857.

b. 

5. As in Question 6, the demand curve and supply curve for bonds are estimated using the following equations:



Following a dramatic increase in the value of the stock market, many retirees started moving money out of the stock market and into bonds. This resulted in a parallel shift in the demand for bonds, such that the price of bonds at all quantities increased $50. Assuming no change in the supply equation for bonds, what is the new equilibrium price and quantity? What is the new market interest rate?

**Solution:**

The new demand equation is as follows:



Now, solve the equations simultaneously:



This implies that *P* = 850.00.



6. Following Question 5, the demand curve and supply curve for bonds are estimated using the following equations:

*Bd:* Price  

*Bs*: Price  Quantity  500

As the stock market continued to rise, the Federal Reserve felt the need to increase the interest rates. As a result, the new market interest rate increased to 19.65%, but the equilibrium quantity remained unchanged. What are the new demand and supply equations? Assume parallel shifts in the equations.

**Solution:** Prior to the change in inflation, the equilibrium was *Q* = 350.00 and *P* = 850.00. The new equilibrium price can be found as follows:



This point (350, 835.771) will be common to both equations. Further since the shift was a parallel shift, the slope of the equations remains unchanged. So, we use the equilibrium point and the slope to solve for the constant in each equation:

*Bd*: 835.771  

*Bd*: Price  

and

*Bs:* 835.771 = 350 + constant, or constant = 485.771

*Bs*: Price = Quantity + 485.771

**Lecture 5**

1. What would be the underestimation of your earnings as an investor if you use the discount rate instead of the investment rate to measure the return on your investment if you buy a $5,000 T-bill that matures in 91 days for $4,999.55?

**Solution:**

Annualized discount rate = [(5,000 – 4,999.55)/5,000] × (360/91) = 0.035604%.

Annualized investment rate = [(5,000 – 4,999.55)/4,999.55] × (365/91) = 0.036102%.

0.036102% – 0.035604% = 0.000498%

Therefore, you would underestimate your return by 0.000498% if you use the discount rate.

2. What is the annualized discount rate % and your annualized investment rate % on a Treasury bill that you purchase for $9,940 that will mature in 91 days for $10,000?

**Solution:** Discount Rate   

Investment Rate   

3. If you want to earn an annualized discount rate of 3.5%, what is the most you can pay for a 91-day Treasury bill that pays $5,000 at maturity?

**Solution:**  



4. What is the minimum discount rate you will accept if you want to earn at least a 0.25% annualized investment rate on a 182 day $1,000 T-bill?

**Solution:** In order to obtain at least a 0.25% annualized investment rate, you will have to pay at most 998.754975 for the $1.000 T-bill maturing in 182 days. This price comes from solving the following equation, where P is the price of the T-bill:

0.0025 < [($1,000 ‒ *P*) / *P*] × 365/182 → *P* < 998.754975

Thereby, you should bid a minimum discount rate of 0.246269% (the minimum discount rate that guarantees a price of 998.754975). This discount rate satisfies the following equation:

(($1,000 – 998.754975) / $1,000) × 360/182 = 0.00246269

5. The price of 182-day commercial paper is $7,840. If the annualized investment rate is 4.093%, what will the paper pay at maturity?

**Solution:** Let *B*  what will be paid at maturity



6. Your minimum discount rate bid of 0.35% for a $10,000 T-bill that matures in 91 days has been accepted. Calculate your annualized investment rate.

**Solution:** Since your bid was accepted, you will pay for this T-bill $9,991.16. We can calculate this price using the formula for the discount rate:

0.0035 = [($10,000 – *P*) / *P*] × 360/91 → *P* = $9,991.16

Therefore, your annualized investment rate will be 0.3548611%

[($10,000 - $9,991.160) / $9,991.16] × 365/91 = 0.003548611

7. The price of $8,000 face value commercial paper is $7,930. If the annualized discount rate is 4%, when will the paper mature? If the annualized investment rate % is 4%, when will the paper mature?

**Solution:** Let *N*  when the paper matures

Discount Rate:







8. A $5,000 commercial paper issued by CMF Corp. matures in 182 days and sells for $4,995. The investment rate on same maturity T-bills is currently 0.1%. Would you state that commercial paper issued by CMF Corp. is default free?

**Solution:** Commercial paper issued by CMF Corp. has an investment rate equal to 0.20075%

[($5,000 - $4,995) / 4,995] × 365/182 = 0.0020075%

This investment rate is more than double the investment rate of the same maturity T-bill. Therefore, we cannot state that this security is default free (assuming both instruments have the same liquidity).

9. The annualized discount rate on a particular money market instrument is 3.75%. The face value is $200,000 and it matures in 51 days. What is its price? What would be the price if it had 71 days to maturity?

**Solution:** Let *B*  the price with 51 days to maturity

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If 71 days to maturity, then *B*  $198,520.83.

10. The annualized yield is 3% for 91-day commercial paper and 3.5% for 182-day commercial paper. What is the expected 91-day commercial paper rate 91 days from now?

**Solution:** Let *A*  the expected 91-day rate, 91 days from now.

Assume that the 182-day rate is the average of the current 91-day rate and the expected   
91-day rate.



11. In a Treasury auction of $2.1 billion par value 91-day T-bills, the following bids were submitted:

|  |  |  |
| --- | --- | --- |
| **Bidder** | **Bid Amount** | **Price per**  **$100** |
| 1 | $500 million | $99.40 |
| 2 | $750 million | $99.01 |
| 3 | $1.5 billion | $99.25 |
| 4 | $1 billion | $99.36 |
| 5 | $600 million | $99.39 |

If only these competitive bids are received, who will receive T-bills, in what quantity, and at what price?

**Solution:** Bidders 1, 4, and 5 will receive T-bills in the amount requested all at $99.36.

12. If the Treasury also received $750 million in non-competitive bids, who will receive T-bills, in what quantity, and at what price?

**Solution**: All competitive bids are accepted at the highest yield paid to competitive bids. Thus, all $750 million will be accepted at $99.36.

**Lecture 6**

1. A bond pays $80 per year in interest (8% coupon). The bond has 5 years before it matures at which time it will pay $1,000. Assuming a discount rate of 10%, what should be the price of the bond (Review Chapter 3)?

**Solution:** $924.18

2. Calculate the price of a bond that matures in 8 years, has a face value of $5.000, a coupon rate of 2% (paid semiannually) if the market interest rate is 1%. What is the price of the bond if the market interest rate drops to 0.5%?

**Solution:** In this case the semiannual coupon payment is (5,000 x 0.02)/2 = $50.

The price of the bond when market interest rate is 1% solves the following equation:



Using a financial calculator (or an excel spreadsheet), P = $5,383.50

The price of the bond when market interest rate is 0.5% solves the following equation:



Using a financial calculator (or an excel spreadsheet), P = $5,587.44

3. Consider the two bonds described below:

|  |  |  |
| --- | --- | --- |
|  | **Bond A** | **Bond B** |
|  |  |  |
| Maturity | 15 yrs | 20 yrs |
| Coupon Rate  (Paid semiannually) | 10% | 6% |
| Par Value | $1,000 | $1,000 |

a. If both bonds had a required return of 8%, what would the bonds’ prices be?

b. Describe what it means if a bond sells at a discount, a premium, and at its face amount   
(par value). Are these two bonds selling at a discount, premium, or par?

c. If the required return on the two bonds rose to 10%, what would the bonds’ prices be?

**Solution:**

a. Bond A  $1,172.92

Bond B  $802.07

b. Bond A is selling at a premium

Bond B is selling at a discount

c. Bond A  $1,000

Bond B  $656.82

4. A 2-year $1,000 par zero-coupon bond is currently priced at $819.00. A 2-year $1,000 annuity is currently priced at $1,712.52. If you want to invest $10,000 in one of the two securities, which is a better buy? You can assume

a. the pure expectations theory of interest rates holds,

b. neither bond has any default risk, maturity premium, or liquidity premium, and

c. you can purchase partial bonds.

**Solution:** With *PV*  $819; *FV*  $1,000; *PMT*  0; and *N*  2, the yield to maturity on the two-year zero-coupon bonds is 10.5%. For the two-year annuity, *PV*  $1,712.52; *PMT*  $1000;   
*FV*  0; and *N*  2 gives a yield to maturity of 11.00%. The two year annuities are   
the better buy.

5. Consider the following cash flows. All market interest rates are 12%.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year** | **0** | **1** | **2** | **3** | **4** |
| Cash Flow |  | 160 | 170 | 180 | 230 |

a. What price would you pay for these cash flows? What total wealth do you expect after 2½ years if you sell the rights to the remaining cash flows? Assume interest rates remain constant.

b. Immediately after buying these cash flows, all market interest rates drop to 11%. What is the impact on your total wealth after 2½ years?

**Solution:**

a. 



b. Expected Wealth  

Since you are holding the cash flows for their duration, you are essentially immunized from interest rate changes (in this simplistic example).

6. The yield on a corporate bond is 10% and it is currently selling at par. The marginal tax rate is 20%. A par value municipal bond with a coupon rate of 8.50% is available. Which security is a better buy?

**Solution:** The equivalent tax-free rate  taxable interest rate × (1  marginal tax rate). In this case, 0.10 × (1  0.20)  8%. The corporate bond offers a lower after-tax yield given the marginal tax rate, so the municipal bond is a better buy.

7. Suppose a municipal bond offers a yield to maturity of 5% and a same maturity corporate bond offers a 4% yield. For which values of the marginal tax rate an investor would prefer to buy the corporate bond?

**Solution:** The marginal tax rate for which an investor would choose to buy the corporate bond satisfies the following equation:

0.05 × (1 – *t*) > 0.04 → 1- *t* > 0.04 / 0.05 → 1 – (0.04/0.05) > *t* → 0.2 > *t*

Therefore, the investor would prefer to buy the corporate bond if she faces a marginal tax rate lower than 20%.

8. M&E Inc. has an outstanding convertible bond. The bond can be converted into 20 shares of common equity (currently trading at $52/share). The bond has 5 years of remaining maturity, a $1,000 par value, and a 6% annual coupon. M&E’s straight debt is currently trading to yield 5%. What is the minimum price of the bond?

**Solution:** The price must exceed the straight bond value or the value of conversion (you will see why in the next question).

If converted, the debt is worth $52 × 20  $1,040.

Assuming a 5% YTM is correct, the price of straight debt is computed as:

*PMT*  60; *N*  5; *FV*  1000; *I*  5

Compute *PV*; *PV*  1,043.29

The bond must be trading for at least $1,043.29.

9. Assume the debt in the previous question is trading at 1,035. How can you earn a riskless profit from this situation (arbitrage)?

**Solution:**

a. Short 20 shares of M&E at $52/share. 

b. Purchase a convertible bond. 

c. Convert the bond to shares, and use to close short position.

Assuming these transactions are completed simultaneously, you make a riskless profit of $5.

\*Typically, small investors cannot short stock and have use of the proceeds—the broker retains it as collateral. So, this doesn’t quite work. But the idea is correct.

10. Consider two $10,000 face value corporate bonds. One is currently selling for $9,980 and matures in 15 years. The other bond sells for $9,350 and matures in 3 years. Calculate the current yield for both bonds if both have a coupon rate equal to 5%. Which current yield is a better approximation of the yield to maturity? (Assume a yearly coupon payment).

**Solution:**  Current yield for bond that sells for $9,980 equals 5.01% = 500/9,980. Current yield for bond that sells for $9,350 equals 5.35% = 500/9,350. The first current yield is a better approximation of the yield to maturity of that bond, since the time to maturity is longer and the price of the bond is closer to $10,000.

11. A $5,000 corporate bond with an 8.625% coupon rate sells above par. Is the current yield higher, lower, or equal to the coupon rate? Is the current yield higher, lower, or equal to the yield to maturity?

**Solution:** Note that you cannot calculate the current yield, since you do not have data on the price of the bond. However, you know that the bond sells for more than $5,000, which means that the current yield will be lower than the coupon rate. Note that you cannot calculate the yield to maturity as you do not have data on the maturity of the bond. However, you know that the yield to maturity equals the coupon rate and the current yield when the bond sells at par. Since this bond sells above par, you can state that the coupon rate is higher than the current yield and higher than the yield to maturity: CR>CY>YTM (if the bond sells above par).

12. A 1-year discount bond with a face value of $1,000 was purchased for $900. What is the yield to maturity? What is the yield on a discount basis?

**Solution:** 900  1000/(1  YTM), or YTM  11.11%

YDB  (1000 – 900)/1000 × (360/365)  9.86%

13. A corporate bond sells at par and has a current yield equal to 5.625%. Another bond sells for $10,075, has a face value of $10,000 and has a coupon rate equal to 5.5%. Which bond has a higher yield to maturity?

**Solution:** The first bond has a yield to maturity equal to its current yield (and to its coupon rate), since it is selling at par. The second bond sells above par and has therefore a yield to maturity lower than its coupon rate. Therefore, the first bond has a higher yield to maturity.

14. Your company owns the following bonds:

|  |  |  |
| --- | --- | --- |
| **Bond** | **Market Value** | **Duration** |
| A | $13 million | 2 |
| B | $18 million | 4 |
| C | $20 million | 3 |

If general interest rates rise from 8% to 8.5%, what is the approximate change in the value of the portfolio?

**Solution:** Portfolio duration  2 × (13/51)  4 × (18/51)  3 × (20/51)  3.09

Δ Value  Duration × (Δ*i*/(1  *i*)) × Original Value

Δ Value  3.09 × (0.005/1.08) × $51 million  $729,583

**Lecture 7**

Ebay, Inc. went public in September of 1998. The following information on shares outstanding was listed in the final prospectus filed with the SEC[[1]](#footnote-1).

In the IPO, the Ebay issued 3,500,000 new shares. The initial price to the public was $18.00 per share.   
The final first-day closing price was $44.88.

1. Ife investment bankers retained $1.26 per share as fees, what was the net proceeds to Ebay? What was the market capitalization of new shares of Ebay?

**Solution:** Net Proceeds to Ebay  (18.00  1.26) × 3,500,000  $58,590,000.00

Market Cap  44.88 × 3,500,000  $157,080,000

2. Two common statistics in IPOs are *underpricing* and *money left on the table.* Underpricing is defined as percentage change between the offering price and the first day closing price. Money left on the table is the difference between the first day closing price and the offering price, multiplied by the number of shares offered. Calculate the underpricing and money left on the table for Ebay. What does this suggest about the efficiency of the IPO process?

**Solution:** Underpricing  ((44.88  18.00)/18.00)  149.33%

MLOT  (44.88  18.00) × 3,500,000  $94,080,000

3. The shares of Misheak, Inc. are expected to generate the following possible returns over the next 12 months:

|  |  |
| --- | --- |
| **Return** | **Probability** |
| 5% | 0.10 |
| 5% | 0.25 |
| 10% | 0.30 |
| 15% | 0.25 |
| 25% | 0.10 |

If the stock is currently trading at $25/share, what is the expected price in one year. Assume that the stock pays no dividends.

**Solution:** The expected return over the next 12 months is calculated as:

0.05 × 0.10  0.05 × 0.25  0.10 × 0.30  0.15 × 0.25  0.25 × 0.10  0.10

This suggests that the expected price is $25 × (1.10)  $27.50

4. Sine Corp. stock sells for $75.00 and currently pays an annual dividend of $2.50 per share. Calculate analysts´ projected price one year from today if the expected one year return is currently 3%.

**Solution:** If the expected one year return on Sine stock is 3%, we can write:

(X+$2.50 – $75)/$75 = 0.03, where X is the expected sales price of Sine Stock one year from today. Solving yields X = $74.75.

5. Suppose Microsoft, Inc. was trading at $27.29 per share. At that time, it pays an annual dividend of $0.32 per share, and analysts have set a 1-year target price around $33.30 per share. What is the expected return on this stock?

**Solution:** 

6. LaserAce is selling at $22.00 per share. The most recent annual dividend paid was $0.80. Using the Gordon Growth model, if the market requires a return of 11%, what is the expected dividend growth rate for LaserAcer?

**Solution:**



7. ANCAP Oil Co. stock has a PE ratio equal to 22, whereas VRC Oil has a PE ratio equal to 20. Both firms´ earnings per share are $1.12. Calculate both firms´ stock prices.

**Solution:** Using the information provided, ANCAP Oil Co. shares sell for 22 × $1.12 = $24.64 per share. VRC Oil sells for 20 × $1.12 = $22.4

8. Refer to the previous exercise. Calculate the premium paid by investors to hold ANCAP Oil Co. shares instead of VRC Oil. Which company do you think is a safer investment?

**Solution:** The premium paid is ($24.64/$22.4) – 1 = 10%. Based on this information only, one can argue that investors feel that ANCAP Oil Co.´s dividends represent a safer stream of payments than that of VRC Oil.

9. Gordon & Co.’s stock has just paid its annual dividend $1.10 per share. Analysts believe that Gordon will maintain its historic dividend growth rate of 3%. If the required return is 8%, what is the expected price of the stock next year?

**Solution:**



10. Macro Systems just paid an annual dividend of $0.32 per share. Its dividend is expected to double for the next four years (*D*1 through *D*4), after which it will grow at a more modest pace of 1% per year.   
If the required return is 13%, what is the current price?

**Solution:**



11. Nat-T-Cat Industries just went public. As a growing firm, it is not expected to pay a dividend for the first five years. After that, investors expect Nat-T-Cat to pay an annual dividend $1.00 per share  
(i.e., *D*6  1.00), with no growth. If the required return is 10%, what is the current stock price?

**Solution:**



so,

*P*0  *P*5/(1.10)5  $6.21

12. Calculate the stock price of OSE Water Co. if the difference between the required rate of return on this investment and the expected growth rate of dividends is 3.6% and dividends per share are $1.728.

**Solution:** According to the Gordon valuation model, OSE Water Co. stocks sells for 1.728/0.036 = $48

13. Refer to the previous exercise. What is the implicit required rate of return if dividends are expected to grow at a 5% annual rate?

**Solution:** According to the Gordon valuation model, the implicit required rate of return is: 5% + 3.6% = 8.6%.

14. Consider the following security information for four securities making up an index:

|  |  |  |  |
| --- | --- | --- | --- |
| **Security** | **Price** | | **Shares Outstanding** |
| **time  0** | **time  1** |
| 1 | 8 | 13 | 20 million |
| 2 | 22 | 25 | 50 million |
| 3 | 35 | 30 | 120 million |
| 4 | 50 | 55 | 75 million |

What is the change in the value of the index from time  0 to time  1 if the index is calculated using a value-weighted arithmetic mean?

**Solution:** For a value-weighted arithmetic mean, the change is calculated as follows:

First, the market value at time  0 is calculated as:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Security** | **Price** | | **Shares Outstanding** | **Market Value** |
| **time  0** | **time  1** |
| 1 | 8 | 13 | 20 million | $ 160 |
| 2 | 22 | 25 | 50 million | $1,100 |
| 3 | 35 | 30 | 120 million | $4,200 |
| 4 | 50 | 55 | 75 million | $3,750 |
|  |  |  |  | $9,210 |

The change is then calculated as:



Index1  Index0 × 1.0027

15. An index had an average (geometric) mean return over 20 years of 3.8861%. If the beginning index value was 100, what was the final index value after 20 years?

**Solution:**

The actual return over the 20 years is (1.038861)20  2.143625

So, the final index value is 214.3625

16. Calculate the dividend paid by a share of stock that you buy today for $55 and you expect to sell one year from today at $56 if you want to earn a one year return equal to 3.5%.

**Solution:** According to the information given, the one year dividend D solves the following equation: ($56 + D – $55)/$55 = 0.035. Solving this equation yields D = $0.925.

17. The projected earnings per share for Risky Ventures, Inc. is $3.50. The average *PE* ratio for the industry composed of Risky Ventures closest competitors is 21. After careful analysis, you decide that Risky Ventures is a little more risky than average, so you decide a *PE* ratio of 23 better reflects the market’s perception of the firm. Estimate the current price of the firm’s stock.

**Solution:**

23 × $3.5  $80.50

1. This information is summarized from http://www.sec.gov/Archieves/edgar/data/1065088/0001012870-98-002475.txt [↑](#footnote-ref-1)