Serie N° 3.

Etat plan de contraintes.

Ex 1 : Un état de contraintes est défini par les valeurs des contraintes principales σ_1 et σ_2.
Determiner les valeurs des contraintes Normales et tangentielles σ_x, σ_y, et τ_{xy} sur les deux facettes perpendiculaires lorsqu'on tourne d'un angle θ / à l'axe x.

a/ $\sigma_1 = 20\ daN/mm^2$; $\sigma_2 = 10\ daN/mm^2$; $\theta = 30°$

b/ $\sigma_1 = -20\ daN/mm^2$; $\sigma_2 = -10\ daN/mm^2$; $\theta = 30°$

Ex 2 : Un élément est soumis à l'action des contraintes $\sigma_x = 25\ KN/mm^2$; $\sigma_y = 11\ KN/mm^2$; $\tau_{xy} = 5\ KN/mm^2$
Determiner la grandeur et le sens des contraintes principales.

Ex 3 : Un état de contrainte est défini par :

$\sigma_x = -30\ KN/mm^2$; $\sigma_y = 13\ KN/mm^2$, $\tau_{xy} = 6\ KN/mm^2$

Determiner la grandeur et la direction des contraintes principales.

[redacted] Solution serie N° 3 – ETUDE DES CONTRAINTE RDM.

EX 1

a/ Contraintes principales :

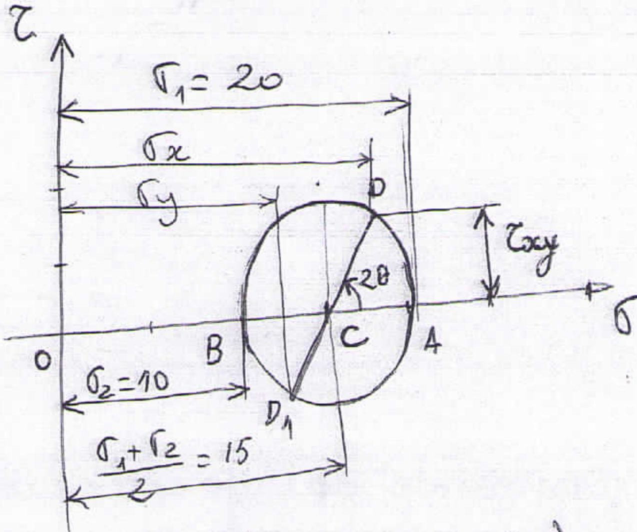
$$\sigma_1 = 20 \qquad \sigma_2 = 10 \qquad \theta = 30°$$

Determinons les valeurs des contraintes normales et tangentielles σ_x, σ_y et τ_{xy} –

Tracé du cercle de Mohr :

$$OC = \frac{\sigma_1 + \sigma_2}{2} = \frac{20+10}{2} = 15$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{20-10}{2} = 5$$



Contrainte sur une facette inclinée de 30° (cercle de Mohr).

$$2\theta = 2 \times 30° = 60°$$

$$\sigma_x = \frac{1}{2}(\sigma_1 + \sigma_2) + \frac{\sigma_1 - \sigma_2}{2}\cos 60° = \frac{1}{2}(20+10) + \frac{1}{2}(20-10)\cos 60$$

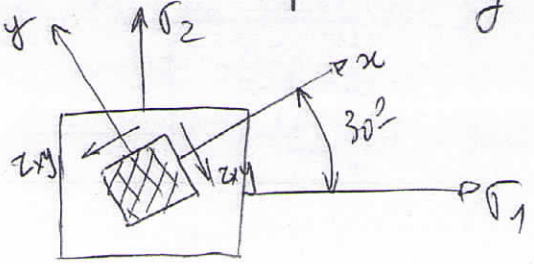
$$= 15 + 5 \cdot \cancel{\tfrac{1}{2}} \cdot \frac{1}{2} = 15 + 2{,}50 = \cancel{16{,}25}\ 17{,}5$$

$$\sigma_y = \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{\sigma_1 - \sigma_2}{2}\cos 60 = \frac{1}{2}(20+10) - \frac{1}{2}(20-10)\cos 60$$

$$= 15 - 10 \cdot \frac{1}{2} \cdot \frac{1}{2} = 15 - 1{,}25 = \cancel{12{,}75}\ 12{,}5$$

$$|\tau_{xy}| = \left|\frac{\sigma_1 - \sigma_2}{2}\sin 60\right| = \frac{20-10}{2} \times \frac{\sqrt{3}}{2} = \frac{10\sqrt{3}}{4} = 4{,}33$$

en D : $\tau_{xy} > 0$ et en D_1 : $\tau_{xy} < 0$.

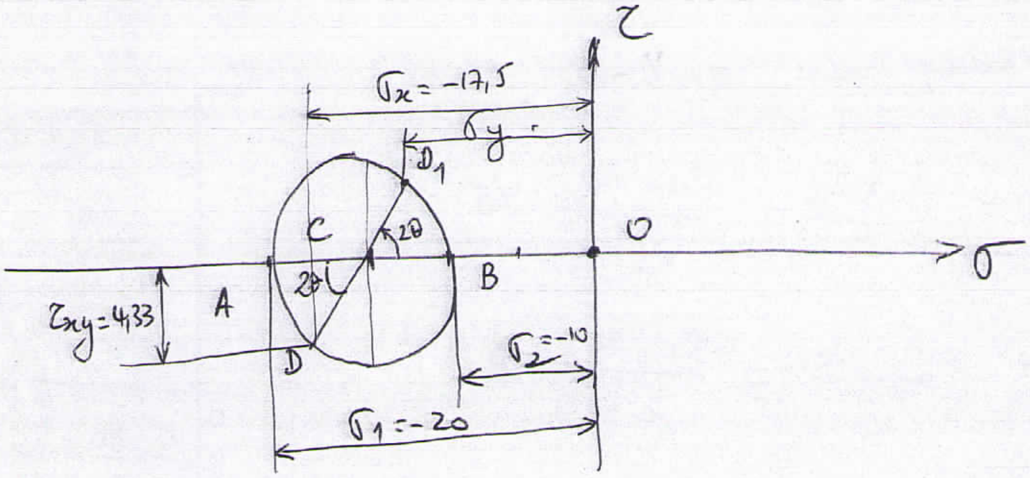


Exo 1 b) $\sigma_1 = -20$ $\sigma_2 = -10$ $\theta = 30°$

Tracé du cercle de Mohr :

$$OC = \frac{\sigma_1 + \sigma_2}{2} = \frac{-20-10}{2} = -15$$

$$|R| = \left|\frac{\sigma_1 - \sigma_2}{2}\right| = \left|\frac{-20+10}{2}\right| = 5$$



Contrainte sur facette inclinée de $\theta = 30°$

$$\widehat{ACD} = 60° = 2\theta.$$

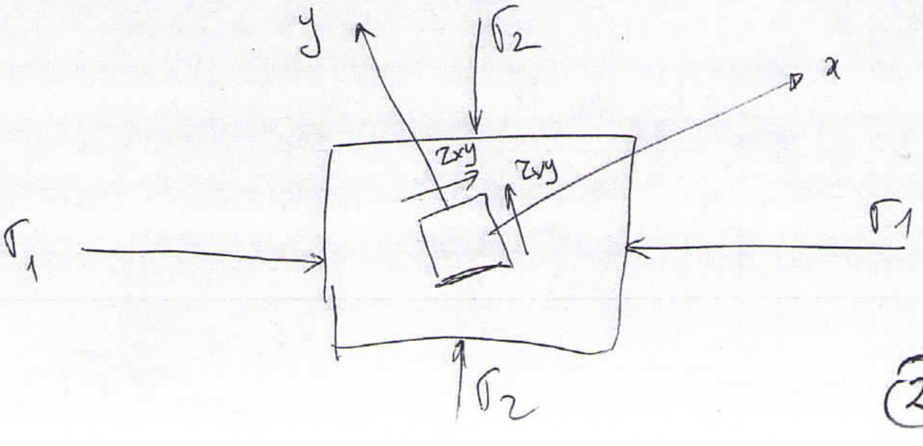
$$\sigma_x = \frac{1}{2}(\sigma_1 + \sigma_2) + \frac{1}{2}(\sigma_1 - \sigma_2)\cos 2\theta = \frac{1}{2}(-20-10) + \frac{1}{2}(-20+10)\cos 60$$

$$= -\frac{30}{2} - \frac{10}{2}\cos 60 = -15 - 5 \cdot \frac{1}{2} = -17,5$$

$$\tau_{xy} = \left|\frac{\sigma_1 - \sigma_2}{2}\right| \sin 60 = \left|-\frac{20+10}{2}\right| \sin 60 = \left|-\frac{10}{2} \cdot \frac{\sqrt{3}}{2}\right| = 4,33$$

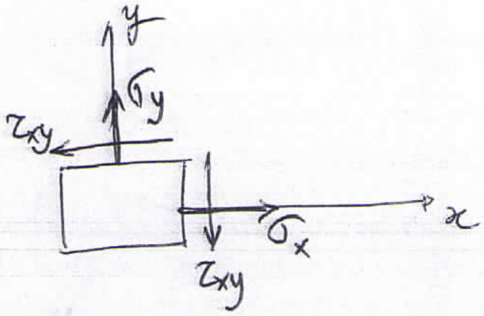
$$\sigma_y = \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{1}{2}(\sigma_1 - \sigma_2)\cos 60 = \frac{1}{2}(-20-10) - \frac{1}{2}(-20+10)\cos 60.$$

$$= -15 + 5 \cdot \frac{1}{2} = -15 + 2,5 = -12,5$$



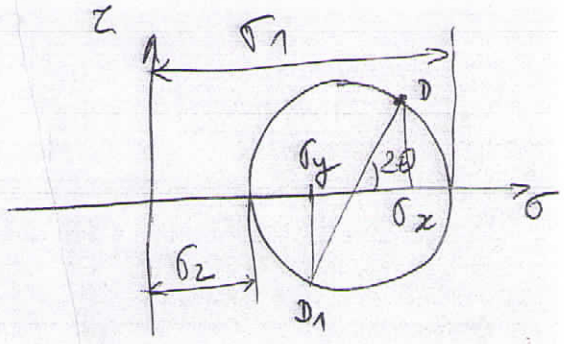
Ex 2 : $\sigma_x = 25 \qquad \sigma_y = 11 \qquad \tau_{xy} = 5$

Grandeur et sens des Contraintes principales σ_1 et σ_2.



$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



d'où :

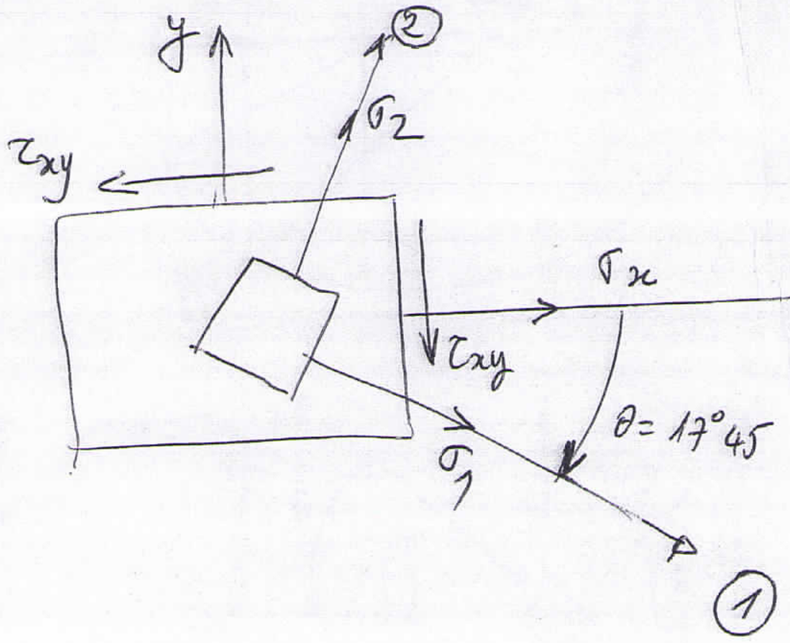
$$\sigma_1 = \frac{25+11}{2} + \sqrt{\left(\frac{25-11}{2}\right)^2 + 5^2}$$

$$= 18 + \sqrt{74} = 26{,}7$$

$$\sigma_2 = \frac{25+11}{2} - \sqrt{\left(\frac{25-11}{2}\right)^2 + 5^2} = 18 - \sqrt{74} = 18 - 8{,}6 = 9{,}4$$

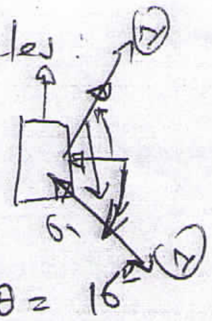
$$tg\,2\theta = -\frac{2\tau_{xy}}{\sigma_x-\sigma_y} = -\frac{2\times 5}{25-11} = -\frac{10}{14} = -0{,}714 \Rightarrow 2\theta = -35°30'$$

$$\theta = -17°45'$$



EX 3 : $\sigma_x = -30 \qquad \sigma_y = 13 \qquad \tau_{xy} = 6$.

Grandeur et direction des contraintes principales :



$$\overline{OC} = \frac{\sigma_x+\sigma_y}{2} = \frac{-30+13}{2} = -8,5$$

$$tg\,2\theta = \left|\frac{\tau_{xy}}{\sigma_x - \overline{OC}}\right| = \left|\frac{6}{-30+8,5}\right| = 0,279 \Rightarrow 2\theta = 16^\circ$$

$$\theta = 8^\circ$$

$$\sigma_1 = -|R+OC| = = -\left|\sqrt{\tau_{xy}^2 + (\sigma_x - OC)^2} + OC\right|$$

$$\sigma_1 = -\left|\sqrt{36+(30-8,5)^2} + 8,5\right| = \cancel{-30,82} \; 13,82$$

$$\sigma_2 = R - OC = \sqrt{36+(30-8,5)^2} - 8,5 = \cancel{13,82} \; -30,82$$