

Exercise 1

Let E be a set E defined by $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

- ❶ Let A, B be two subsets of E , such that : $A = \{3, 4, 6, 7, 1, 9\}$, $B = \{5, 6, 8, 4\}$, $C = \{5, 8, 4\}$. Determine

$$A \cap B, A \cup B, A \setminus B, B \setminus A, A \Delta B, C_E(A), C_E(B).$$

- ❷ Suppose that $F = \{0, 1\}$, $G = \{1, 2\}$. Find

$$F \times G, G \times F, F \times F, (F \cap G) \times F, \mathcal{P}(F), \mathcal{P}(G), \mathcal{P}(F \cap G), \mathcal{P}(F) \cap \mathcal{P}(G).$$

- ❸ Suppose $A_1 = \{a, b, d, e, g, f\}$, $A_2 = \{a, b, c, d\}$, $A_3 = \{b, d, a\}$ and $A_4 = \{a, b, h\}$. Find A and B such that :

$$A = \bigcup_{i=1}^4 A_i, \quad B = \bigcap_{i=1}^4 A_i.$$

Exercise 2

- ❶ Let $E = \{a, b, c\}$. Determine whether the following statements are true

$$a \in E, a \subset E, \emptyset \in E, \{\emptyset\} \subset E, E \cup \emptyset = E, \emptyset \subset E.$$

- ❷ Let $A = \{1, 2, \{1\}, \{1, 2\}\}$. Are the following statements true or false?

$$\begin{array}{llllll} \text{(a)} \quad \{1\} \in A & \text{(b)} \quad \{\{1\}\} \in A & \text{(c)} \quad 2 \in A & \text{(d)} \quad \{2\} \in A & \text{(e)} \quad \{2\} \subset A & \text{(f)} \quad \{1\} \subset A \\ \text{(g)} \quad \{\{1\}\} \subset A & \text{(h)} \quad 2 \subset A & \text{(i)} \quad \{2, \{1, 2\}, 1\} = A & \text{(j)} \quad \{1, 2\} = A \end{array}$$

Exercise 3

- ❶ List all the subsets of the following sets

$$\{1, 2, \emptyset\}, \quad \{\emptyset\}, \quad \{\{\mathbb{R}\}\}.$$

- ❷ Find the cardinality of each of the following sets

$$A = \{1, 2, \{1\}, \{1, 2\}\}, B = \{1, \{2\}\}, C = \{1, 2, 2, 2\}, D = \{\emptyset, \{\emptyset\}\} \text{ and } E = \{5n : n \in \mathbb{N}\}.$$

Exercise 4

Let A, B, C be subsets of a set E . Show that

- ❶ $A \cap B = \emptyset \implies A \subset C_E(B).$
- ❷ $A \subset B \iff C_E(B) \subset C_E(A).$
- ❸ $B \subset A \subset C \implies A \cup B = A \cap C.$
- ❹ $C_E(A \cap B) = C_E(A) \cup C_E(B).$
- ❺ $(A \times C) \cup (B \times C) = (A \cup B) \times C.$
- ❻ $C_E(A \cup B) = C_E(A) \cap C_E(B)$ (**homework**).

