

Exercise 1 Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $g: \mathbb{R} \rightarrow \mathbb{R}$ be the functions defined by $f(x) = 4x - 3$, $g(x) = 4x^2 - 7x + 3$. Find $(f + g)(x)$, $(f + g)(2)$, $(fg)(x)$, $(fg)(0)$ and $(f \circ g)(x)$, $(g \circ f)(x)$.

Exercise 2

- ❶ Let $f: E \rightarrow F$ be a constant function $f(x) = y_0$, for all $x \in E$. Determine $f^{-1}(B)$ for $B \subset F$?
- ❷ Let $f: \mathbb{R} \rightarrow \mathbb{R}$, and $g: \mathbb{R} \rightarrow \mathbb{R}$ be the functions given by $f(x) = x^2$ and $g(x) = (x - 1)^2$.
 - (a) Determine $f(\{-1, 1\})$, $f([-1, 1])$, $g([0, 2])$.
 - (b) Determine $f^{-1}(\{1\})$, $f^{-1}([0, 1])$, $f^{-1}([-1, 1])$, $g^{-1}(\{-1\})$, $g^{-1}([-1, 1])$.
- ❸ Let $f:]0, +\infty[\rightarrow]0, +\infty[$ be the function given by $f(x) = \frac{1}{x}$. Determine $f^{-1}([0, 1])$, and $f^{-1}([1, +\infty[)$?

Exercise 3 Let $f: E \rightarrow F$ be a function. Let A and B be two subsets of E , and let C and D be two subsets of F . Prove that

- ❶ $f(A \cup B) = f(A) \cup f(B)$.
- ❷ $f^{-1}(C_F(C)) = C_E(f^{-1}(C))$
- ❸ If $C \subset D$, then $f^{-1}(C) \subset f^{-1}(D)$.

Exercise 4 Let $f: E \rightarrow F$ be a function. Show that

- ❶ $\forall A \in \mathcal{P}(E)$, $A \subset f^{-1}(f(A))$. Give an example of a function f and a subset $A \subset E$, such that $f^{-1}(f(A)) \subsetneq A$
- ❷ $\forall B \in \mathcal{P}(F)$, $f(f^{-1}(B)) \subset B$. Give an example of a function f and a subset $B \subset F$, such that $B \subsetneq f(f^{-1}(B))$.

Exercise 5 Let f be a function E defined by

$$f: \mathbb{R} \rightarrow \mathbb{R}$$
$$x \mapsto f(x) = \frac{2x}{1 + x^2}.$$

- ❶ Determine $f^{-1}\left(\left\{\frac{1}{2}\right\}\right)$, $f^{-1}\left(\left\{2\right\}\right)$. Is f injective? Surjective?
- ❷ For which $y \in \mathbb{R}$ the equation $f(x) = y$ has solutions in $\mathbb{R}$? Show that $f(\mathbb{R}) = [-1, 1]$.
- ❸ Show that the function g defined by

$$g: [-1, 1] \rightarrow [-1, 1]$$
$$x \mapsto g(x) = f(x).$$

is bijective and find its inverse g^{-1} .

Exercise 1 (Homework) Let $f(x) = \frac{1}{x^2}$, $x \neq 0$, $x \in \mathbb{R}$

- ❶ Determine the direct image $f(E)$ where $E = \{x \in \mathbb{R} : 1 \leq x \leq 2\}$.
- ❷ Determine the inverse image $f^{-1}(G)$, where $G = \{x \in \mathbb{R} : 1 \leq x \leq 4\}$.

Exercise 2 (Homework) Let $f: E \rightarrow F$ and $g: F \rightarrow G$ be functions. Let $H \subset G$. Show that

$$(g \circ f)^{-1}(H) = f^{-1}(g^{-1}(H)).$$