

Exercise 1 Determine whether the relations defined below are reflexive, symmetric, antisymmetric, transitive.

1. Let $\mathcal{R}$ be the relation given by $a\mathcal{R}b \iff a = |b|$, for all $a, b \in \mathbb{Z}$.
2. Let S be the relation given by $aSb \iff a + b$ is even, for all $a, b \in \mathbb{Z}$.

Exercise 2 Let $\mathcal{R}$ be the relation on the set of ordered pairs of real numbers, $\mathbb{R} \times \mathbb{R}$, such that

$$(x, y)\mathcal{R}(u, v) \iff x^2 + y^2 = u^2 + v^2.$$

- 1 Show that $\mathcal{R}$ is an equivalence relation.
- 2 Find the equivalence class of $(0, 0)$ and $(0, 1)$, deduce the equivalence class of $(1, 0)$. Interpret these equivalence classes geometrically.
- 3 Find the quotient set $(\mathbb{R}^2)/\mathcal{R}$.

Exercise 3 We define the following relation S on $\mathbb{N}^*$

$$\forall n, m \in \mathbb{N}^*, nSm \iff m \text{ divides } n$$

- 1 Verify that $6S2$ and $5S1$.
- 2 Show that the relation S is a partial order relation on $\mathbb{N}^*$.
- 3 Let $A = \{2, 3, 4\}$ be a subset of $\mathbb{N}^*$
 - (a) What are the minimal elements? least element? maximal elements? the greatest element?
 - (b) What are the bounds of A (upper bound and lower bound)?

Homework

Exercise 1 Determine whether the relation $\mathcal{R}$ on the set of all integers is reflexive, symmetric, antisymmetric and transitive. Where $x\mathcal{R}y$ if and only if

- 1 $x \neq y$
- 2 $xy \geq 1$
- 3 $x = y + 1$ or $x = y - 1$
- 4 $x \equiv y \pmod{7}$
- 5 $x + y = 7$

Exercise 2 Let $\mathcal{R}$ be the relation on the set of ordered pairs of positive integers such that

$$\forall (a, b), (c, d) \in \mathbb{Z}^+ \times \mathbb{Z}^+, (a, b)\mathcal{R}(c, d) \iff ad = bc.$$

Show that $\mathcal{R}$ is an equivalence relation.

Exercise 3 Let $\mathcal{R}$ be a relation on the set $\mathbb{N}^*$

$$\forall a, b \in \mathbb{N}^*, a\mathcal{R}b \iff \exists k \in \mathbb{N}: \frac{b}{a} = 2^k$$

1. Show that $\mathcal{R}$ is an order relation on $\mathbb{N}^*$.
2. Decide whether $\mathcal{R}$ is a totally order relation on $\mathbb{N}^*$. Why?

Remark 1. For the first exercise of homework, choose only two relations.