

Exercise 1

Which of the following expressions are propositions? In the case of a proposition, say whether it is true or false :

- ❶ $\sqrt{3}$ is an irrational number.
- ❷ The integer n divides 12.
- ❹ $\exists n \in \mathbb{N}, n + 1 = 5$.
- ❺ 25 is a multiple of 5 and 2 divides 7.
- ❻ 25 is a multiple of 5 or 2 divides 7.

Exercise 2

Let P, Q be propositions. Give the truth table of these propositions.

- ❶ $(P \Rightarrow Q) \wedge (\bar{P} \Rightarrow Q)$.
- ❷ $(\bar{P} \Rightarrow \bar{Q}) \Leftrightarrow (P \wedge \bar{Q})$.
- ❸ $(P \Rightarrow Q) \Leftrightarrow (\bar{Q} \Rightarrow \bar{P})$.

Exercise 3

Let f and g be two functions of $\mathbb{R}$ in $\mathbb{R}$, write in terms of quantifiers the following expressions :

- ❶ f never equals zero.
- ❷ f is even.
- ❹ For every real number x , the value of the function f in x is greater than or equal to 1.
- ❺ f is strictly decreasing function.
- ❻ f less than g .

Exercise 4

Show which of the following propositions are true and which are false, then give their negation :

- ❶ $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x + y > 0$.
- ❷ $(\exists x \in \mathbb{R}, x + 1 = 0) \wedge (\exists x \in \mathbb{R}, x + 2 = 0)$.
- ❸ $\exists x \in \mathbb{R}, (x + 1 = 0 \wedge x + 2 = 0)$.
- ❹ $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y > 0$.
- ❺ $\forall x \in \mathbb{R}, x < 2 \Rightarrow x^2 < 4$.

Exercise 5

- ❶ Using the proof by contradiction prove that $\sqrt{2}$ is not a rational number.
- ❷ Prove by induction : $\forall n \in \mathbb{N}^*, 1 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$.
- ❸ Let a be an integer. Prove by cases : 2 divides $a(a + 1)$.
- ❹ By contrapositive, prove that $[(n^2 - 1) \text{ is not divisible by } 8] \Rightarrow (n \text{ is even})$.

