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# Corrigé de Série N1 Maths II
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# 1 Série N1 Maths II

**Exercice 1.1.** *Calculer les primitives suivantes :*

1. $\int \frac{4x-4}{x^2-2x+3}dx$.
   *On a,*
   $$\begin{aligned}\int \frac{4x-4}{x^2-2x+3}dx &= 2\int \frac{2x-2}{x^2-2x+3}dx \\ &= 2\ln \mid x^2-2x+3 \mid +c, \ \ c \in \mathbb{R}.\end{aligned}$$
   *L'utilisation de* $\int \frac{f'(x)}{f(x)}dx = \ln \mid f(x) \mid +c$.

2. $\int \frac{1+\ln x}{x}dx$.
   *On a,*
   $$\begin{aligned}\int \frac{1+\ln x}{x}dx &= \int (\frac{1}{x}+\frac{\ln x}{x})dx \\ &= \int \frac{1}{x}dx + \int \frac{\ln x}{x}dx \\ &= \int \frac{1}{x}dx + \frac{1}{2}\int 2\frac{1}{x}\ln x dx \\ &= \ln \mid x \mid + \frac{1}{2}(\ln x)^2 + c, \ \ c \in \mathbb{R}.\end{aligned}$$
   *L'utilisation de* $\int \frac{f'(x)}{f(x)}dx = \ln \mid f(x) \mid +c$ *et* $\int f'(x)f^n(x)dx = \frac{f^{n+1}(x)}{n+1} + c$.

3. $\int \cos(5x)dx$.
   *On a,*
   $$\begin{aligned}\int \cos(5x)dx &= \frac{1}{5}\int 5\cos(5x)dx \\ &= \frac{1}{5}\sin(5x) + c, \ \ c \in \mathbb{R}.\end{aligned}$$
   *L'utilisation de* $\int f'(x)g(f(x))dx = G(f(x)) + c$, *où* $G$ *est la primitive de* $g$.

4. $\int (2x-2)\sqrt{x^2-2x+3}dx$.
   *On a,*
   $$\begin{aligned}\int (2x-2)\sqrt{x^2-2x+3}dx &= \int (2x-2)(x^2-2x+3)^{\frac{1}{2}}dx \\ &= \frac{(x^2-2x+3)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c, \ \ c \in \mathbb{R} \\ &= \frac{2}{3}(x^2-2x+3)^{\frac{3}{2}} + c, \ \ c \in \mathbb{R}.\end{aligned}$$

*L'utilisation de* $\int f'(x)f^n(x)dx = \frac{f^{n+1}(x)}{n+1} + c$.

**Exercice 1.2.** *Trouver les primitives suivantes :*

1. ***Intégration par parties***

   (a) $\int x^3 \ln(3x)dx$.
   *On pose*

$$\begin{aligned} u(x) = \ln(3x) \quad &\Rightarrow \quad u'(x) = \frac{(3x)'}{3x} = \frac{3}{3x} = \frac{1}{x}, \\ v'(x) = x^3 \quad &\Rightarrow \quad v(x) = \frac{x^4}{4}, \end{aligned}$$

   *et l'on intègre par parties, ce qui donne :*

$$\begin{aligned} \int x^3 \ln(3x)dx &= \frac{x^4}{4}\ln(3x) - \int \frac{x^4}{4}\frac{1}{x}dx \\ &= \frac{x^4}{4}\ln(3x) - \frac{1}{4}\int x^3 dx \\ &= \frac{x^4}{4}\ln(3x) - \frac{1}{16}x^4 + c, \ \ c \in \mathbb{R}. \end{aligned}$$

   (b) $\int(2x+1)e^x dx$.
   *On pose*

$$\begin{aligned} u(x) = (2x+1) \quad &\Rightarrow \quad u'(x) = 2, \\ v'(x) = e^x \quad &\Rightarrow \quad v(x) = e^x, \end{aligned}$$

   *et l'on intègre par parties, ce qui donne :*

$$\begin{aligned} \int (2x+1)e^x dx &= (2x+1)e^x - \int e^x 2dx \\ &= (2x+1)e^x - 2e^x + c, \ \ c \in \mathbb{R} \\ &= (2x-1)e^x + c, \ \ c \in \mathbb{R}. \end{aligned}$$

   (c) $\int(x+1)\sqrt{2x+1}dx$.
   *On pose*

$$\begin{aligned} u(x) = (x+1) \quad &\Rightarrow \quad u'(x) = 1, \\ v'(x) = \sqrt{2x+1} \quad &\Rightarrow \quad v(x) = \frac{1}{3}(2x+1)^{\frac{3}{2}}, \end{aligned}$$

*et l'on intègre par parties, ce qui donne :*

$$\begin{aligned}
\int (x+1)\sqrt{2x+1}dx &= (x+1)\frac{1}{3}(2x+1)^{\frac{3}{2}} - \int \frac{1}{3}(2x+1)^{\frac{3}{2}}dx \\
&= \frac{(x+1)}{3}(2x+1)^{\frac{3}{2}} - \frac{1}{3}\int (2x+1)^{\frac{3}{2}}dx \\
&= \frac{(x+1)}{3}(2x+1)^{\frac{3}{2}} - \frac{1}{6}\int 2(2x+1)^{\frac{3}{2}}dx \\
&= \frac{(x+1)}{3}(2x+1)^{\frac{3}{2}} - \frac{1}{6}\frac{2}{5}(2x+1)^{\frac{5}{2}} + c,\ \ c \in \mathbb{R} \\
&= \frac{(x+1)}{3}(2x+1)^{\frac{3}{2}} - \frac{1}{15}(2x+1)^{\frac{5}{2}} + c,\ \ c \in \mathbb{R}.
\end{aligned}$$

2. ***Intégration par changement de variables***

   (a) $\int \frac{e^{(2x+1)}}{2+5e^{(2x+1)}}dx.$
   *En posant*

   $$t = e^{(2x+1)} \Rightarrow dt = 2e^{(2x+1)}dx \Rightarrow dx = \frac{1}{2e^{(2x+1)}}dt \Rightarrow dx = \frac{1}{2t}dt,$$

   *on obtient*

   $$\begin{aligned}
   \int \frac{e^{(2x+1)}}{2+5e^{(2x+1)}}dx &= \int \frac{t}{2+5t}\frac{1}{2t}dt \\
   &= \frac{1}{2}\int \frac{1}{2+5t}dt \\
   &= \frac{1}{10}\int \frac{5}{2+5t}dt \\
   &= \frac{1}{10}\ln \mid 2+5t \mid +c,\ c \in \mathbb{R} \\
   &= \frac{1}{10}\ln \mid 2+5e^{(2x+1)} \mid +c,\ c \in \mathbb{R}.
   \end{aligned}$$

   *(à la fin on a remplacé $t = e^{(2x+1)}$).*

   (b) $\int \frac{1}{1+\sqrt{x+1}}dx.$
   *En posant*

   $$t = \sqrt{x+1} \Rightarrow dt = \frac{1}{2\sqrt{x+1}}dx \Rightarrow dx = 2\sqrt{x+1}dt \Rightarrow dx = 2tdt,$$

*on obtient*

$$\begin{aligned}
\int \frac{1}{1+\sqrt{x+1}}dx &= \int \frac{1}{1+t}2tdt \\
&= 2\int \frac{t}{1+t}dt \\
&= 2\int \frac{t+1-1}{1+t}dt \\
&= 2\int (\frac{t+1}{1+t}+\frac{-1}{1+t})dt \\
&= 2\int 1dt - 2\int \frac{1}{1+t}dt \\
&= 2t - 2\ln \mid 1+t \mid +c,\ c\in\mathbb{R} \\
&= 2\sqrt{x+1} - 2\ln \mid 1+\sqrt{x+1} \mid +c,\ c\in\mathbb{R}.
\end{aligned}$$

*(à la fin on a remplacé* $t=\sqrt{x+1}$*).*

*(c)* $\int \frac{3}{2x(\ln x)^2}dx.$
*En posant*

$$t=\ln x \Rightarrow dt = \frac{1}{x}dx,$$

*on obtient*

$$\begin{aligned}
\int \frac{3}{2x(\ln x)^2}dx &= \frac{3}{2}\int \frac{1}{(\ln x)^2}\frac{1}{x}dx \\
&= \frac{3}{2}\int \frac{1}{t^2}dt \\
&= \frac{3}{2}\int t^{-2}dt \\
&= \frac{3}{2}\frac{t^{-2+1}}{-2+1}dt \\
&= \frac{-3}{2}\frac{1}{t}+c,\ c\in\mathbb{R} \\
&= \frac{-3}{2\ln x}+c,\ c\in\mathbb{R}.
\end{aligned}$$

*(à la fin on a remplacé* $t=\ln x$*).*

**Exercice 1.3.** *Intégrer les fractions rationnelles suivantes :*

1. $\int \frac{x^3+4x^2+6x-3}{x^2+2x+1}dx$.
*1-ère étape : effectuer la division euclidienne :*

$$\frac{x^3+4x^2+6x-3}{x^2+2x+1} = x+2+(\frac{x-5}{x^2+2x+1}).$$

*2-ème étape : d'écomposer* $(\frac{x-5}{x^2+2x+1})$ *en fractions simples :*

$$\begin{aligned}\frac{x-5}{x^2+2x+1} &= (\frac{x-5}{(x+1)^2}\\ &= \frac{A}{(x+1)}+\frac{B}{(x+1)^2}\\ &= \frac{A(x+1)+B}{(x+1)^2}\\ &= \frac{Ax+(A+B)}{(x+1)^2}\\ &= \frac{1x+(-5)}{(x+1)^2},\end{aligned}$$

*par identification, on obtient :*

$$\begin{cases} A=1 \\ A+B=-5 \end{cases} \quad \Rightarrow A=1, B=-6,$$

*d'où*

$$\frac{x-5}{(x+1)^2} = \frac{1}{x+1}+\frac{-6}{(x+1)^2}.$$

*Alors,*

$$\frac{x^3+4x^2+6x-3}{x^2+2x+1} = x+2+(\frac{x-5}{x^2+2x+1}) = x+2+(\frac{1}{x+1}+\frac{-6}{(x+1)^2}).$$

*3-ème étape : intégrer :*

$$\begin{aligned}\int \frac{x^3+4x^2+6x-3}{x^2+2x+1}dx &= \int (x+2+\frac{x-5}{x^2+2x+1})dx\\ &= \int (x+2+\frac{1}{x+1}+\frac{-6}{(x+1)^2})dx\\ &= \int (x+2)dx+\int \frac{1}{x+1}dx-6\int \frac{1}{(x+1)^2}dx\\ &= \frac{x^2}{2}+2x+\ln\mid x+1\mid -6\frac{-1}{(x+1)}+c,\ c\in\mathbb{R}\\ &= \frac{x^2}{2}+2x+\ln\mid x+1\mid +\frac{6}{(x+1)}+c,\ c\in\mathbb{R}.\end{aligned}$$

*2.* $\int \frac{x-1}{x^2+x+1}dx$. *On a*

$$\begin{aligned}
\int \frac{x-1}{x^2+x+1}dx &= \frac{1}{2}\int \frac{2x-2}{x^2+x+1}dx \\
&= \frac{1}{2}\int \frac{2x+1-3}{x^2+x+1}dx \\
&= \frac{1}{2}\int (\frac{2x+1}{x^2+x+1} + \frac{-3}{x^2+x+1})dx \\
&= \frac{1}{2}\int (\frac{2x+1}{x^2+x+1})dx + \frac{1}{2}\int (\frac{-3}{x^2+x+1})dx \\
&= \frac{1}{2}\int (\frac{2x+1}{x^2+x+1})dx - \frac{3}{2}\int (\frac{1}{x^2+x+1})dx \\
&= \frac{1}{2}\ln \mid x^2+x+1 \mid -\frac{3}{2}\int (\frac{1}{x^2+x+1})dx
\end{aligned}$$

*On calcule l'intégrale* $\int \frac{1}{x^2+x+1}dx$,

$$x^2+x+1 = x^2+2\frac{1}{2}x+\frac{1}{4}-\frac{1}{4}+1 = (x+\frac{1}{2})^2+\frac{3}{4}$$

$$= (x+\frac{1}{2})^2+\frac{3}{4} = \frac{3}{4}(\frac{(\frac{2x+1}{2})^2}{\frac{3}{4}}+1)$$

$$= \frac{3}{4}((\frac{2x+1}{\sqrt{3}})^2+1)$$

*on pose*

$$t = \frac{2x+1}{\sqrt{3}} \Rightarrow dt = \frac{2}{\sqrt{3}}dx \Rightarrow \frac{\sqrt{3}}{2}dt = dx,$$

*puis*

$$\begin{aligned}
\int \frac{1}{x^2+x+1}dx &= \int \frac{1}{\frac{3}{4}((\frac{2x+1}{\sqrt{3}})^2+1)}dx \\
&= \frac{4}{3}\int \frac{1}{((\frac{2x+1}{\sqrt{3}})^2+1)}dx \\
&= \frac{4}{3}\int \frac{1}{(t^2+1)}\frac{\sqrt{3}}{2}dt \\
&= \frac{2}{\sqrt{3}}\int \frac{1}{(t^2+1)}dt \\
&= \frac{2}{\sqrt{3}}\arctan(t)+c,\ \ c\in\mathbb{R} \\
&= \frac{2}{\sqrt{3}}\arctan(\frac{2x+1}{\sqrt{3}})+c,\ \ c\in\mathbb{R}.
\end{aligned}$$

*à la fin*

$$\begin{aligned}
\int \frac{x-1}{x^2+x+1}dx &= \frac{1}{2}\ln\mid x^2+x+1\mid -\frac{3}{2}\int (\frac{1}{x^2+x+1})dx \\
&= \frac{1}{2}\ln\mid x^2+x+1\mid -\frac{3}{2}\frac{2}{\sqrt{3}}\arctan(\frac{2x+1}{\sqrt{3}})+c,\ \ c\in\mathbb{R} \\
&= \frac{1}{2}\ln\mid x^2+x+1\mid -\sqrt{3}\arctan(\frac{2x+1}{\sqrt{3}})+c,\ \ c\in\mathbb{R}.
\end{aligned}$$

3. $\int_4^6 \frac{7x+6}{x^2-x-6}dx$.
*On calcule d'abord* $\int \frac{7x+6}{x^2-x-6}dx$.
*On décompose d'abord* $\frac{7x+6}{x^2-x-6}$ *en éléments simples :*

$$\begin{aligned}
\frac{7x+6}{x^2-x-6} &= \frac{7x+6}{(x+2)(x-3)} \\
&= \frac{A}{x+2}+\frac{B}{x-3} \\
&= \frac{A(x-3)+B(x+2)}{(x+2)(x-3)} \\
&= \frac{Ax-3A+Bx+2B}{(x-1)(x+1)} \\
&= \frac{(A+B)x+(-3A+2B)}{(x+1)(x-1)} \\
&= \frac{7x+6}{x^2-1}
\end{aligned}$$

*par identification, on obtient :*

$$\begin{cases} (A+B)=7 \\ (-3A+2B)=6 \end{cases} \Rightarrow \begin{cases} 3A+3B=21 \\ (-3A+2B)=6 \end{cases} \Rightarrow A=\frac{8}{5}, B=\frac{27}{5},$$

*d'où*

$$\frac{7x+6}{x^2-x-6}=\frac{7x+6}{(x+2)(x-3)}=\frac{\frac{8}{5}}{x+2}+\frac{\frac{27}{5}}{x-3}=\frac{8}{5(x+2)}+\frac{27}{5(x-3)}.$$

*Puis,*

$$\begin{aligned}
\int \frac{7x+6}{x^2-x-6}dx &= \int \frac{8}{5(x+2)}+\frac{27}{5(x-3)}dx \\
&= \int \frac{8}{5(x+2)}dx+\int \frac{27}{5(x-3)}dx \\
&= \frac{8}{5}\int \frac{1}{(x+2)}dx+\frac{27}{5}\int \frac{1}{(x-3)}dx \\
&= \frac{8}{5}\ln\mid x+2\mid+\frac{27}{5}\ln\mid x-3\mid+c,\ \ c\in\mathbb{R}.
\end{aligned}$$

*Donc*

$$\begin{aligned}
\int_4^6 \frac{7x+6}{x^2-x-6}dx &= [\frac{8}{5}\ln\mid x+2\mid+\frac{27}{5}\ln\mid x-3\mid]_4^6 \\
&= [\frac{8}{5}\ln\mid 6+2\mid+\frac{27}{5}\ln\mid 6-3\mid]-[\frac{8}{5}\ln\mid 4+2\mid+\frac{27}{5}\ln\mid 4-3\mid] \\
&= \frac{8}{5}\ln\mid 8\mid+\frac{27}{5}\ln\mid 3\mid-\frac{8}{5}\ln\mid 6\mid-\frac{27}{5}\ln\mid 1\mid \\
&= \frac{8}{5}\ln(8)+\frac{27}{5}\ln(3)-\frac{8}{5}\ln(6) \\
&= \frac{8}{5}\ln(4*2)+\frac{27}{5}\ln(3)-\frac{8}{5}\ln(3*2) \\
&= \frac{8}{5}\ln(4)+\frac{8}{5}\ln(2)+\frac{27}{5}\ln(3)-\frac{8}{5}\ln(3)-\frac{8}{5}\ln(2) \\
&= \frac{8}{5}\ln(4)+\frac{27}{5}\ln(3)-\frac{8}{5}\ln(3).
\end{aligned}$$

**Exercice 1.4.**

1. *Intégrer les fonctions trigonométriques suivantes :*

(a) $\int \sin^2 x \cos^2 x dx$.
*On utilise les deux formulles suivantes :*

$$\cos(2x) = 2\cos^2(x) - 1 \Rightarrow \cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

*et*

$$\cos(2x) = 1 - 2\sin^2(x) \Rightarrow \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

*d'où*

$$\begin{aligned}
\sin^2 x \cos^2 x &= (\frac{1}{2} - \frac{1}{2}\cos(2x))(\frac{1}{2} + \frac{1}{2}\cos(2x)) \\
&= \frac{1}{2}(1 - \cos(2x))\frac{1}{2}(1 + \cos(2x)) \\
&= \frac{1}{4}(1 - \cos^2(2x)) \\
&= \frac{1}{4}(1 - (\frac{1}{2} + \frac{1}{2}\cos(4x))) \\
&= \frac{1}{4}(1 - \frac{1}{2} - \frac{1}{2}\cos(4x)) \\
&= \frac{1}{4}(\frac{1}{2} - \frac{1}{2}\cos(4x)) \\
&= \frac{1}{8}(1 - \cos(4x)),
\end{aligned}$$

*donc*

$$\begin{aligned}
\int \sin^2 x \cos^2 x dx &= \int \frac{1}{8}(1 - \cos(4x))dx \\
&= \frac{1}{8}\int (1 - \cos(4x))dx \\
&= \frac{1}{8}(x - \frac{1}{4}\sin(4x)) + c,\ c \in \mathbb{R} \\
&= \frac{1}{8}x - \frac{1}{32}\sin(4x) + c,\ c \in \mathbb{R}.
\end{aligned}$$

(b) $\int \sin(3x)\cos(4x)dx$.

*On a,*

$$\begin{aligned}
\int \sin(3x)\cos(4x)dx &= \int \frac{1}{2}(\sin((3+4)x)+\sin((3-4)x))dx \\
&= \int \frac{1}{2}(\sin(7x)+\sin(-x))dx \\
&= \int \frac{1}{2}\sin(7x)dx + \int \frac{1}{2}\sin(-x)dx \\
&= \frac{-1}{14}\cos(7x)dx + \frac{1}{2}\cos(-x) + c,\ c \in \mathbb{R} \\
&= \frac{-1}{14}\cos(7x)dx + \frac{1}{2}\cos(x) + c,\ c \in \mathbb{R}.\ (\cos(-x)=\cos(x)).
\end{aligned}$$

(c) $\int_0^{3\pi} \cos^2 x \sin^3 x dx$.
*On calcule d'abord* $\int \cos^2 x \sin^3 x dx$.
*On a,*

$$\begin{aligned}
\int \sin^3 x \cos^2 x dx &= \int \sin^2 x \cos^2 x \sin x dx \\
&= \int (1-\cos^2 x)\cos^2 x \sin x dx,
\end{aligned}$$

*on pose* $t = \cos x$ *d'où* $dt = -\sin x dx$, *on obtient*

$$\begin{aligned}
\int (1-\cos^2 x)\cos^2 x \sin x dx &= \int -(1-\cos^2 x)\cos^2 x(-\sin x)dx \\
&= \int -(1-t^2)t^2 dt \\
&= \int (t^4 - t^2)dt \\
&= \frac{t^5}{5} - \frac{t^3}{3} + c,\ c \in \mathbb{R} \\
&= \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + c,\ c \in \mathbb{R}.
\end{aligned}$$

*Et*

$$\begin{aligned}\int_0^{3\pi} \cos^2 x \sin^3 x dx &= [\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3}]_0^{3\pi} \\ &= [\frac{\cos^5(3\pi)}{5} - \frac{\cos^3(3\pi)}{3}] - [\frac{\cos^5 0}{5} - \frac{\cos^3 0}{3}] \\ &= \frac{\cos^5(\pi)}{5} - \frac{\cos^3(\pi)}{3} - \frac{\cos^5 0}{5} + \frac{\cos^3 0}{3} \\ &= \frac{-1}{5} - \frac{-1}{3} - \frac{1}{5} + \frac{1}{3} \\ &= \frac{-2}{5} + \frac{2}{3} = \frac{4}{15}.\end{aligned}$$

2. *Considérons les primitives suivantes :*

$$I = \int \frac{\sin x}{\sin x + \cos x} dx \ \ et \ \ J = \int \frac{\cos x}{\sin x + \cos x} dx.$$

   (a) *Calculer $I + J$ et $I - J$.*
   *Alors,*

$$\begin{aligned}I + J &= \int \frac{\sin x}{\sin x + \cos x} dx + \int \frac{\cos x}{\sin x + \cos x} dx \\ &= \int (\frac{\sin x}{\sin x + \cos x} + \frac{\cos x}{\sin x + \cos x}) dx \\ &= \int \frac{\sin x + \cos x}{\sin x + \cos x} dx \\ &= \int 1 dx \\ &= x + c_1, \ \ c_1 \in \mathbb{R}.\end{aligned}$$

*Et*

$$\begin{aligned}I - J &= \int \frac{\sin x}{\sin x + \cos x} dx - \int \frac{\cos x}{\sin x + \cos x} dx \\ &= \int (\frac{\sin x}{\sin x + \cos x} - \frac{\cos x}{\sin x + \cos x}) dx \\ &= \int \frac{\sin x - \cos x}{\sin x + \cos x} dx \\ &= - \int \frac{-\sin x + \cos x}{\sin x + \cos x} dx \\ &= -\ln \mid \sin x + \cos x \mid + c_2, \ \ c_2 \in \mathbb{R}.\end{aligned}$$

(b) *En déduire $I$ et $J$.*
*On a,*

$$\begin{cases} I+J=x \\ I-J=-\ln|\sin x+\cos x| \end{cases} \Rightarrow \begin{cases} I=x-J \\ 2I=x-\ln|\sin x+\cos x| \end{cases}$$

$$\Rightarrow \begin{cases} J=x-I \\ I=\frac{x-\ln|\sin x+\cos x|}{2} \end{cases}$$

$$\Rightarrow \begin{cases} I=\frac{x}{2}-\frac{\ln|\sin x+\cos x|}{2} \\ J=x-(\frac{x}{2}-\frac{\ln|\sin x+\cos x|}{2}) \end{cases}$$

$$\Rightarrow \begin{cases} I=\frac{x}{2}-\frac{\ln|\sin x+\cos x|}{2}+c_3, \;\; c_3\in\mathbb{R} \\ J=\frac{x}{2}+\frac{\ln|\sin x+\cos x|}{2}+c_4, \;\; c_4\in\mathbb{R}. \end{cases}$$

## 2 Exercice supplémentaire (à la maison)

**Exercice 2.1.** *Trouver les primitives suivantes :*

1. $\int (x+1)\sqrt{x^2+2x+5}dx$.
*On a,*

$$\begin{aligned} \int (x+1)\sqrt{x^2+2x+5}dx &= \frac{1}{2}\int (2x+2)(x^2+2x+5)^{\frac{1}{2}}dx \\ &= \frac{1}{2}\frac{(x^2+2x+5)^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c, \;\; c\in\mathbb{R} \\ &= \frac{1}{3}(x^2+2x+5)^{\frac{3}{2}}+c, \;\; c\in\mathbb{R}. \end{aligned}$$

*L'utilisation de* $\int f'(x)f^n(x)dx=\frac{f^{n+1}(x)}{n+1}+c$.

2. $\int \sin x e^{\cos x}dx$.
*On a,*

$$\begin{aligned} \int \sin x e^{\cos x}dx &= -\int -\sin x e^{\cos x}dx \\ &= -e^{\cos x}+c, \;\; c\in\mathbb{R}. \end{aligned}$$

*L'utilisation de* $\int f'(x)e^{f(x)}dx=e^{f(x)}+c$.

*3.* $I = \int \sin(2x)e^{3x}dx$.
*On pose*

$$u(x) = \sin(2x) \quad \Rightarrow \quad u'(x) = 2\cos(2x),$$
$$v'(x) = e^{3x} \quad \Rightarrow \quad v(x) = \frac{1}{3}e^{3x},$$

*et l'on intègre par parties, ce qui donne :*

$$\begin{aligned} I &= \sin(2x)\frac{1}{3}e^{3x} - \int 2\cos(2x)\frac{1}{3}e^{3x}dx \\ &= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{3}\int \cos(2x)e^{3x}dx \end{aligned}$$

*Le calcule de* $\int e^x \cos(x)dx$ *:*
*on pose*

$$u(x) = \cos(2x) \quad \Rightarrow \quad u'(x) = -2\sin(2x),$$
$$v'(x) = e^{3x} \quad \Rightarrow \quad v(x) = \frac{1}{3}e^{3x},$$

*et l'on intègre une deuxième fois par parties, ce qui donne :*

$$\begin{aligned} \int \cos(2x)e^{3x}dx &= \cos(2x)\frac{1}{3}e^{3x} - \int -2\sin(2x)\frac{1}{3}e^{3x}dx \\ &= \frac{1}{3}\cos(2x)e^{3x} + \frac{2}{3}\int \sin(2x)e^{3x}dx \end{aligned}$$

*d'où,*

$$\begin{aligned}
I &= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{3}\int \cos(2x)e^{3x}dx \\
&= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{3}(\frac{1}{3}\cos(2x)e^{3x} + \frac{2}{3}\int \sin(2x)e^{3x}dx) \\
&= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{9}\cos(2x)e^{3x} - \frac{4}{9}\int \sin(2x)e^{3x}dx \\
&= \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{9}\cos(2x)e^{3x} - \frac{4}{9}I \\
&\Rightarrow I + \frac{4}{9}I = \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{9}\cos(2x)e^{3x} \\
&\Rightarrow \frac{13}{9}I = \frac{1}{3}\sin(2x)e^{3x} - \frac{2}{9}\cos(2x)e^{3x} \\
&\Rightarrow I = \frac{9}{13}(\frac{1}{3}\sin(2x)e^{3x} - \frac{2}{9}\cos(2x)e^{3x}) + c,\ c \in \mathbb{R} \\
&\Rightarrow I = \frac{3}{13}\sin(2x)e^{3x} - \frac{2}{13}\cos(2x)e^{3x} + c,\ c \in \mathbb{R}.
\end{aligned}$$

*4.* $\int \frac{\sin x}{\cos^4 x}dx$.
*On a,*

$$\begin{aligned}
\int \frac{\sin x}{\cos^4 x}dx &= \int \sin x \cos^{-4} x dx \\
&= -\int -\sin x \cos^{-4} x dx \\
&= -\frac{\cos^{-4+1} x}{-4+1} + c,\ c \in \mathbb{R} \\
&= -\frac{\cos^{-3} x}{-3} + c,\ c \in \mathbb{R} \\
&= \frac{1}{3\cos^3 x} + c,\ c \in \mathbb{R}.
\end{aligned}$$

*L'utilisation de* $\int f'(x)f^n(x)dx = \frac{f^{n+1}(x)}{n+1} + c$.
*On calcule aussi par changement de variable :*
*en posant*

$$t = \cos x \Rightarrow dt = -\sin x dx,$$

*on obtient*

$$\begin{aligned}
\int \frac{\sin x}{\cos^4 x}dx &= \int \frac{1}{\cos^4 x}\sin x dx \\
&= -\int \frac{1}{\cos^4 x}(-\sin x)dx \\
&= -\int \frac{1}{t^4}dt \\
&= -\int t^{-4}dt \\
&= -\frac{t^{-4+1}}{-4+1} + c,\ c \in \mathbb{R} \\
&= -\frac{t^{-3}}{-3} + c,\ c \in \mathbb{R} \\
&= \frac{1}{3t^3} + c,\ c \in \mathbb{R} \\
&= \frac{1}{3\cos^3 x} + c,\ c \in \mathbb{R}.
\end{aligned}$$

*(à la fin on a remplacé $t = \cos x$).*

5. $\int \cos x \sin^4 x dx$.
*On a,*

$$\begin{aligned}
\int \cos x \sin^4 x dx &= \frac{\sin^{4+1} x}{4+1} + c,\ c \in \mathbb{R} \\
&= \frac{\sin^5 x}{5} + c,\ c \in \mathbb{R}.
\end{aligned}$$

*L'utilisation de $\int f'(x)f^n(x)dx = \frac{f^{n+1}(x)}{n+1} + c$.*
*On calcule aussi par changement de variable :*
*en posant*

$$t = \sin x \Rightarrow dt = \cos x dx,$$

*on obtient*

$$\begin{aligned}\int \cos x \sin^4 x dx &= \int \sin^4 x \cos x dx \\ &= \int t^4 dt \\ &= \frac{t^5}{5} + c,\ c \in \mathbb{R} \\ &= \frac{\sin^5}{5} + c,\ c \in \mathbb{R}.\end{aligned}$$

*(à la fin on a remplacé* $t = \sin x$*).*

6. $\int \frac{\cos x}{1-\sin^2 x} dx.$
*en posant*

$$t = \sin x \Rightarrow dt = \cos x dx,$$

*on obtient*

$$\begin{aligned}\int \frac{\cos x}{1-\sin^2 x} dx &= \int \frac{1}{1-\sin^2 x} \cos x dx \\ &= \int \frac{1}{1-t^2} dt\end{aligned}$$

*On décompose d'abord* $\frac{1}{1-t^2}$ *en éléments simples :*

$$\begin{aligned}\frac{1}{1-t^2} &= \frac{1}{(1-t)(1+t)} \\ &= \frac{A}{1-t} + \frac{B}{1+t} \\ &= \frac{A(1+t)+B(1-t)}{(1-t)(1+t)} \\ &= \frac{A+At+B-Bt}{(1-t)(1+t)} \\ &= \frac{(A-B)t+(A+B)}{(1-t)(1+t)} \\ &= \frac{0t+1}{(1-t)(1+t)}\end{aligned}$$

*par identification, on obtient :*

$$\begin{cases}(A-B)=0 \\ (A+B)=1\end{cases} \Rightarrow \begin{cases}A=B= \\ (A+B)=1\end{cases} \Rightarrow A = \frac{1}{2}, B = \frac{1}{2},$$

*d'où*

$$\frac{1}{1-t^2} = \frac{1}{(1-t)(1+t)} = \frac{\frac{1}{2}}{1-t} + \frac{\frac{1}{2}}{1+t} = \frac{1}{2(1-t)} + \frac{1}{2(1+t)}.$$

*Puis,*

$$\begin{aligned}
\int \frac{1}{1-t^2}dx &= \int (\frac{1}{2(1-t)} + \frac{1}{2(1+t)})dx \\
&= \int \frac{1}{2(1-t)}dx + \int \frac{1}{2(1+t)}dx \\
&= \frac{-1}{2}\ln \mid 1-t \mid + \frac{1}{2}\ln \mid 1+t \mid + c, \;\; c \in \mathbb{R} \\
&= \frac{-1}{2}\ln \mid 1-\sin x \mid + \frac{1}{2}\ln \mid 1+\sin x \mid + c, \;\; c \in \mathbb{R}
\end{aligned}$$

*(à la fin on a remplacé* $t = \sin x$*). Donc*

$$\int \frac{\cos x}{1-\sin^2 x}dx = \frac{-1}{2}\ln \mid 1-\sin x \mid + \frac{1}{2}\ln \mid 1+\sin x \mid + c, \;\; c \in \mathbb{R}.$$