Corrigé de la Série N° 02

Exercice 1

1. Etat (1) $\xrightarrow[\text{adiabatique}]{\text{Compression réversible}}$ Etat (2)

$P_1 = 10^5$ Pa — $Q = 0$ — $P_2 = 10^6$ Pa

$V_1 = 10$ L — V_2

$T_1 = 300$ K — T_2

(a) Calcul de V_2 et T_2

* $P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \Rightarrow V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{1/\gamma} = 1{,}94$ L

* $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 578$ K

(b) Calcul de W_{12}

$$W_{12} = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1} = 2350 \text{ J}$$

2. Etat (1) $\xrightarrow{\text{Compression réversible isotherme}}$ Etat (3)

$P_1 = 10^5$ Pa — $P_3 = 10^6$ Pa

$V_1 = 10$ L — V_3

$T_1 = 300$ K — $T_3 = T_1 = 300$ K

Calcul de V_3 et W_{13}

* $P_1 V_1 = P_3 V_3 \Rightarrow V_3 = \frac{P_1 V_1}{P_3} = 1$ L

* $W_{13} = -nRT_1 \ln\left(\frac{V_3}{V_1}\right)$

$W_{13} = -P_1 V_1 \ln\left(\frac{V_3}{V_1}\right) = 2303$ J

Exercice (2)

1. Température d'équilibre ($T_{éq}$)

- corps froid ($T_1 = 18°C$) { calorimètre ; $m_1 = 100\,g$ eau
- corps chaud ($T_2 = 60°C$) { $m_2 = 80\,g$ eau

$Q_{cédée} = m_2 c_{eau} (T_{éq} - T_2)$

$Q_{reçue} = \left(C_{calorimètre} + m_1 c_{eau} \right) (T_{éq} - T_1) = m_1 c_{eau} (T_{éq} - T_1)$

$Q_{cédée} + Q_{reçue} = 0 \Rightarrow (m_1 + m_2) c_{eau} T_{éq} = c_{eau} (m_1 T_1 + m_2 T_2)$

$$T_{éq} = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} = 36,67\,°C$$

2. capacité calorifique du calorimètre

$T_{éq} = 35,9°C$

$Q_{cédée} = m_2 c_{eau} (T'_{éq} - T_2)$

$Q_{reçue} = C_{calorimètre} (T'_{éq} - T_1) + m_1 c_{eau} (T'_{éq} - T_1)$

$Q_{cédée} + Q_{reçue} = 0 \Rightarrow$

$$C_{cal} = \frac{-m_2 c_{eau} (T'_{éq} - T_2) - m_1 c_{eau} (T'_{éq} - T_1)}{T_1 - T_1}$$

$$C_{cal} = \frac{-4,18 \left[80 \times (-24,1) + 100 (17,9) \right]}{17,9} = 32,23\,J/K$$

La masse en eau équivalente :

$$m = \frac{C_{cal}}{c_{eau}} = \frac{32,23}{4,18} = 7,71\,g$$

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Exercice (2) (Suite)

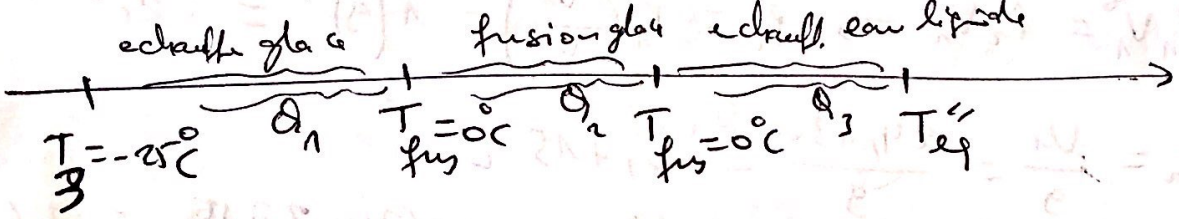
3. Température d'équilibre $T''_{eq} > 0$ (la glace fond totalement)

Corps chaud ($T_1 = 18°C$) { calorimètre + $m_1 = 100\,g$ eau

Corps froid ($T_3 = -10°C$) { $m_3 = 20\,g$ glace

$$Q_{cédée} = (m_1 c_{eau} + C)(T''_{eq} - T_1)$$

la glace fond complètement; elle passe par 03 étapes;



$$Q_{reçue} = Q_1 + Q_2 + Q_3 = m_3 c_g (T_{fus} - T_3) + m_3 L_{fus} + m_3 c_e (T''_{eq} - T_{fus})$$

$$Q_{cédée} + Q_{reçue} = 0 \Rightarrow$$

$$(m_1 c_{eau} + C) T''_{eq} + m_3 c_e T''_{eq} = (m_1 c_e + C) T_1 - m_3 c_g (T_{fus} - T_3) - m_3 L_{fus} + m_3 c_e T_{fus}$$

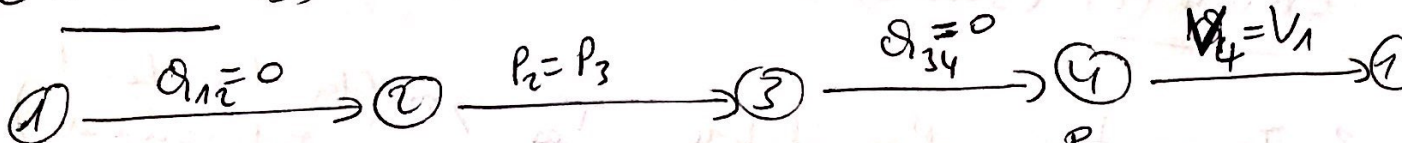
($T_{fus} = 0$ et $T_3 = 0$)

$$T''_{eq} = \frac{(m_1 c_e + C) T_1 + m_3 c_g T_3 - m_3 L_{fus}}{(m_1 + m_3) c_e + C}$$

$$T''_{eq} = \frac{(418 + 32{,}23) 18 + 20 \cdot 2{,}11 (-10) - 20 \cdot 334}{120 \cdot 4{,}18 + 32{,}23} = \frac{1004{,}14}{533{,}83}$$

$$T''_{eq} = 1{,}88\,°C$$

Exercice (3)



1	2	3	4
$P_1 = 1$ atm	P_2	$P_3 = P_2$	P_4
V_1	V_2	V_3	$V_4 = V_1$
$T_1 = 298$ K	T_2	T_3	T_4

1. Calcul des paramètres $V_1, P_2, V_2, T_2, P_3, V_3, T_3, V_4, P_4, T_4$

* $P_1 V_1 = n R T_1 \Rightarrow V_1 = \frac{n R T_1}{P_1} = \frac{1 \cdot 0{,}082 \cdot 298}{1} = 24{,}436$ L
* $P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 (9)^{1{,}4} = 21{,}674$ atm
* $V_2 = \frac{V_1}{9} = \frac{24{,}436}{9} = 2{,}715$ L
* $P_2 V_2 = n R T_2 \Rightarrow T_2 = \frac{P_2 V_2}{n R} = \frac{21{,}674 \cdot 2{,}715}{1 \cdot 0{,}082} = 717{,}621$ K
* $P_3 = P_2 = 21{,}674$ atm
* $V_3 = \frac{V_4}{3} = \frac{V_1}{3} = \frac{24{,}436}{3} = 8{,}145$ L
* $T_3 = \frac{P_3 V_3}{n R} = \frac{21{,}674 \times 8{,}145}{1 \cdot 0{,}082} = 2152{,}863$ K
* $V_4 = V_1 = 24{,}436$ L
* $P_3 V_3^{\gamma} = P_4 V_4^{\gamma} \Rightarrow P_4 = P_3 \left(\frac{V_3}{V_4}\right)^{\gamma} = 21{,}674 \left(\frac{1}{3}\right)^{1{,}4}$

$P_4 = 4{,}656$ atm

* $T_4 = \frac{P_4 V_4}{n R} = \frac{4{,}656 \times 24{,}436}{1 \cdot 0{,}082} = 1387{,}488$ K

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2. Calcul de Q, W, ΔU et ΔH

* Etat ① ⟶ ②

$Q_{12} = 0$; $\Delta U_{12} = W_{12} + Q_{12}$

$\Delta U = W_{12} = \frac{nR(T_2 - T_1)}{\gamma - 1} = \frac{1 \cdot 8{,}314 (717{,}621 - 298)}{0{,}4} = 8721{,}822\ J$

$\Delta H_{12} = \frac{nR\gamma(T_2 - T_1)}{\gamma - 1} = \gamma \cdot \Delta U_{12} = 1{,}4 (8721{,}822) = 12210{,}551\ J$

* Etat ② ⟶ ③

$Q_{23} = \frac{n\gamma R}{\gamma - 1}(T_3 - T_2) = \frac{1 \cdot 1{,}4 \cdot 8{,}314}{0{,}4}(2152{,}863 - 717{,}621)$

$Q_{23} = 41764{,}107\ J$

$\Delta U_{23} = \frac{nR}{\gamma - 1}(T_3 - T_2) = 29831{,}505\ J$

$W_{23} = \Delta U_{23} - Q_{23} = -11932{,}602\ J$

$\Delta H_{23} = \gamma\, \Delta U_{23} = 41764{,}107\ J$

* Etat ③ ⟶ ④

$Q_{34} = 0$

$\Delta U_{34} = W_{34} = \frac{nR(T_4 - T_3)}{\gamma - 1} = \frac{8{,}314 (1387{,}488 - 2152{,}863)}{0{,}4}$

$\Delta U_{34} = W_{34} = -15908{,}319\ J$

$\Delta H_{34} = \gamma \cdot \Delta U_{34} = -22271{,}647\ J$

* Etat ④ ⟶ ①

$W_{41} = 0$

$\Delta U_{41} = Q_{41} = \frac{nR}{\gamma - 1}(T_1 - T_4) = \frac{8{,}314}{0{,}4}(298 - 1387{,}488)$

$\Delta U_{41} = Q_{41} = -22645,008$ J

$\Delta H_{41} = \gamma \Delta U_{41} = -31703,011$ J

* Sur le cycle :

$Q_{cycle} = \sum Q = 0 + 41764,107 + 0 - 22645,008$

$Q_{cycle} = 19119,099$ J

$W_{cycle} = \sum W = 8721,822 - 11932,602 - 15908,319 + 0$

$W_{cycle} = -19119,099$ J

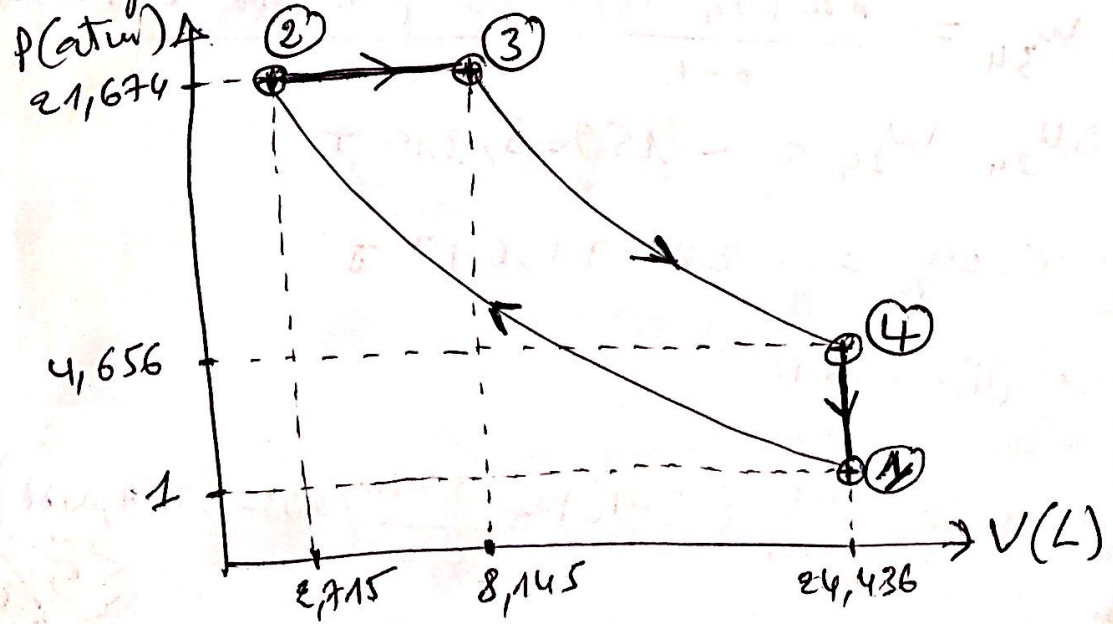
$\Delta U_{cycle} = \sum \Delta U = 0$

$\Delta U_{cycle} = W_{cycle} + Q_{cycle} = 0$

$\Delta H_{cycle} = \sum \Delta H = 0$

$\Delta H_{cycle} = \gamma . \Delta U_{cycle} = 0$

3. Diagramme de Clapeyron



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