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First year of mathematics	**Algebra 2**	**2024-2025**
Tutorial Series N°1	**Vector spaces**	

Exercise 1

❶ On $E=\mathbb{R}^{*+}\times\mathbb{R}$, we define two binary operations by :

$$\oplus : (x,y)\oplus(x',y')=(xx',y+y')$$
$$\times : \lambda\times(x,y)=(x^{\lambda},\lambda y).$$

Show that $(E,\oplus,\times)$ is a vector space over $\mathbb{R}$.

❷ We define, on E, another binary operation by :

$$\otimes : \lambda\otimes(x,y)=(x,0),\quad \forall\lambda\in\mathbb{R},$$

Is $(E,\oplus,\otimes)$ a vector space over $\mathbb{R}$?.

Exercise 2

❶ We consider the $\mathbb{R}$ - vector space $(\mathbb{R}^2,+,\cdot)$ such that $(x,y)+(x',y')=(x+x',y+y')$, $\lambda\cdot(x,y)=(\lambda x,\lambda y)$. Which of the following sets are vector subspaces ? Support your answer.

(a) $E_1=\{(x,y)\in\mathbb{R}^2 : 2x+y=0\}$

(b) $E_2=\{(x,y)\in\mathbb{R}^2 : x+y=1\}$

(c) $E_3=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\leqslant 1\}$

❷ Is $H=\{f\in\mathscr{F}(\mathbb{R},\mathbb{R}) : f \text{ is increasing}\}$ a vector subspace of $\mathscr{F}(\mathbb{R},\mathbb{R})$ $\Big(\mathscr{F}(\mathbb{R},\mathbb{R}),+,\cdot)$ is the vector space of functions from $\mathbb{R}$ in $\mathbb{R}$ with binary operations defined by $(f+g)(x)=f(x)+g(x)$, $(\alpha\cdot f)(x)=\alpha f(x)\Big)$.

Exercise 3

Show that $\mathbb{R}^3$ is spanned by $\{u_1=(1,0,1),\ u_2=(1,1,0),\ u_3=(0,1,1)\}$.
Let $w_1=(1,-2,3)$, $w_2=(1,1,1)$ be vectors in $\mathbb{R}^3$.

❶ Is w_1 in $\langle u_1,\ u_2\rangle$?

❷ Is w_2 in $\langle u_1,\ u_2\rangle$?

❸ Is $\{u_1,\ u_2,\ u_3\}$ linearly independent?

Exercise 4

Let $F_1=\{(x,y,z)\in\mathbb{R}^3 : x+y+z=0\}$ and $F_2=\langle v_1,\ v_2,\ v_3\rangle$ be two subsets of $\mathbb{R}^3$, such that $v_1=(1,1,2)$, $v_2=(1,2,3)$, $v_3=(0,2,1)$.

❶ Prove that F_1,F_2 are vector subspaces of $\mathbb{R}^3$.

❷ Find a basis for the subspaces F_1, F_2, $F_1\cap F_2$, F_1+F_2

❸ Are F_1 and F_2 a direct sum in $\mathbb{R}^3$? if not, determine a complement (or a supplement) to F_1 in $\mathbb{R}^3$.

Homework

Let $E=\mathscr{F}(\mathbb{R},\mathbb{R})$ be set of functions from $\mathbb{R}$ in $\mathbb{R}$.

❶ Show that $(E,+,\cdot)$ is a vector space over $\mathbb{R}$ together with two ordinary operations

$$(f+g)(x)=f(x)+g(x),\ \forall f,\ g\in E.$$
$$(\alpha\cdot f)(x)=\alpha f(x),\ \forall\alpha\in\mathbb{R} \text{ and } \forall f\in E.$$

❷ We denote by F the vector subspace of even functions and G the vector subspace of odd functions. Show that F and G are complementary in E.