A. Mira-Bejaia university	**Faculty of Exact Sciences**	**Department of Mathematics**
First year of mathematics	**Algebra 2**	**2024-2025**
Tutorial Series N°2	**Linear maps**	

Exercise 1

Determine whether the following functions f_i, $(i = 1, \ldots, 4)$, are linear :

$$f_1 : \mathbb{R}^3 \longrightarrow \mathbb{R}^3 \qquad (x, y, z) \mapsto f_1(x, y, z) = (y, x, 0)$$

$$f_2 : \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \qquad (x, y) \mapsto f_2(x, y) = (2x + y, y + 1)$$

$$f_3 : \mathbb{R} \longrightarrow \mathbb{R} \qquad x \mapsto f_3(x) = \sin(x)$$

$$f_4 : \mathbb{R}^2 \longrightarrow \mathbb{R}^3 \qquad (x, y) \mapsto f_4(x, y) = (xy, x, y)$$

Exercise 2

Let f be function defined from $\mathbb{R}^3$ to $\mathbb{R}^4$ by :

$$f(x, y, z) = (x + y + z, x - y - z, 3x - z, y + 2z)$$

1. Show that f is a linear map.
2. Find $\ker(f)$ and Imf , and determine their dimensions. Is f bijective?

Exercise 3

We denote $B = \{e_1, e_2, e_3\}$ the canonical basis of $\mathbb{R}^3$ and f the endomorphism in $\mathbb{R}^3$ defined by :

$$f(e_1) = -2e_1 + 2e_3, f(e_2) = 3e_2, f(e_3) = -4e_1 + 4e_3$$

1. Let $u = (x, y, z) \in \mathbb{R}^3$. Calculate $f(u)$.
2. Determine a basis of $\ker(f)$.
3. Is f injective? can it be surjective? Why?
4. Determine a basis of $Im(f)$. Deduce the rank of f.
5. Show that $\mathbb{R}^3 = \ker(f) \oplus Im(f)$.

Homework

Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear map such that $f(1,1) = (2,2)$ and $f(2,0) = (0,0)$

1. Compute $f(2,2)$, $f(3,1)$ and $f(a,b)$, for all $a, b \in \mathbb{R}$.
2. Find $\ker(f)$ and Imf, and determine their dimensions. Is f bijective?