A. Mira-Bejaia university	**Faculty of Exact Sciences**	**Department of Mathematics**
First year of mathematics	**Algebra 1**	**2024-2025**
Tutorial Series N°3	**Functions**	

Exercise 1 Let $f:\mathbb{R}\longrightarrow\mathbb{R}$, $g:\mathbb{R}\longrightarrow\mathbb{R}$ be the functions defined by $f(x)=4x-3$, $g(x)=4x^2-7x+3$.
Find $(f+g)(x)$, $(f+g)(2)$, , $(fg)(x)$, $(fg)(0)$ and $(f\circ g)(x)$, $(g\circ f)(x)$.

Exercise 2

❶ Let $f:E\longrightarrow F$ be a constant function $f(x)=y_0$, for all $x\in E$. Determine $f^{-1}(B)$ for $B\subset F$?

❷ Let $f:\mathbb{R}\longrightarrow\mathbb{R}$, and $g:\mathbb{R}\longrightarrow\mathbb{R}$ be the functions given by $f(x)=x^2$ and $g(x)=(x-1)^2$.

(a) Determine $f(\{-1,1\})$, $f([-1,1])$, $g([0,2])$.

(b) Determine $f^{-1}(\{1\})$, $f^{-1}([0,1])$, $f^{-1}([-1,1])$, $g^{-1}(\left\{-1\right\})$, $g^{-1}([-1,1])$.

❸ Let $f:]0,+\infty[\longrightarrow]0,+\infty[$ be the function given by $f(x)=\frac{1}{x}$. Determine $f^{-1}(]0,1[)$, and $f^{-1}([1,+\infty[)$?

Exercise 3 Let $f:E\longrightarrow F$ be a function. Let A and B be two subsets of E, and let C and D be two subsets of F. Prove that

❶ $f(A\cup B)=f(A)\cup f(B)$.

❷ $f^{-1}(C_F(C))=C_E(f^{-1}(C))$

❸ If $C\subset D$, then $f^{-1}(C)\subset f^{-1}(D)$.

Exercise 4 Let $f:E\longrightarrow F$ be a function. Show that

❶ $\forall A\in\mathscr{P}(E)$, $A\subset f^{-1}(f(A))$. Give an example of a function f and a subset $A\subset E$, such that $f^{-1}(f(A))\subsetneqq A$

❷ $\forall B\in\mathscr{P}(F), f(f^{-1})(B)\subset B$. Give an example of a function f and a subset $B\subset F$, such that $B\subsetneqq f(f^{-1}(B))$.

Exercise 5 Let f be a function E defined by

$$f:\mathbb{R}\longrightarrow\mathbb{R}$$
$$x\mapsto f(x)=\frac{2x}{1+x^2}.$$

❶ Determine $f^{-1}\left(\left\{\frac{1}{2}\right\}\right)$, $f^{-1}\left(\left\{2\right\}\right)$. Is f injective? Surjective?

❷ For which $y\in\mathbb{R}$ the equation $f(x)=y$ has solutions in $\mathbb{R}$? Show that $f(\mathbb{R})=[-1,1]$.

❹ Show that the function g defined by

$$g:[-1,1]\longrightarrow[-1,1]$$
$$x\mapsto g(x)=f(x).$$

is bijective and find its inverse g^{-1}.

Exercise 1 (Homework) Let $f(x)=\frac{1}{x^2}$, $x\neq 0$, $x\in\mathbb{R}$

❶ Determine the direct image $f(E)$ where $E=\left\{x\in\mathbb{R}:1\leqslant x\leqslant 2\right\}$.

❷ Determine the inverse image $f^{-1}(G)$, where $G=\left\{x\in\mathbb{R}:1\leqslant x\leqslant 4\right\}$.

Exercise 2(Homework) Let $f:E\longrightarrow F$ and $g:F\longrightarrow G$ be a functions. Let $H\subset G$. Show that

$$(g\circ f)^{-1}(H)=f^{-1}(g^{-1}(H)).$$