Exercise 1 Determine whether the relations defined below are reflexives, symmetrics, antisymmetrics, transitives.

1. Let $\mathscr{R}$ be the relation given by $a\mathscr{R}b \iff a = |b|$, for all $a, b \in \mathbb{Z}$.
2. Let S be the relation given by $aSb \iff a+b$ is even, for all $a, b \in \mathbb{Z}$.

Exercise 2 Let $\mathscr{R}$ be the relation on the set of ordered pairs of real numbers, $\mathbb{R} \times \mathbb{R}$, such that

$$(x,y)\mathscr{R}(u,v) \iff x^2+y^2 = u^2+v^2.$$

❶ Show that $\mathscr{R}$ is an equivalence relation.

❷ Find the equivalence class of $(0,0)$ and $(0,1)$, deduce the equivalence class of $(1,0)$.Interpret these equivalence classes geometrically.

❸ Find the quotient set $(\mathbb{R}^2)/\mathscr{R}$.

Exercise 3 We define the following relation S on $\mathbb{N}^*$

$$\boxed{\forall n, m, \in \mathbb{N}^*,\ nSm \iff m \text{ divides } n}$$

❶ Verify that $6S2$ and $5S1$.

❷ Show that the relation S is a partial order relation on $\mathbb{N}^*$.

❸ Let $A = \left\{2,3,4\right\}$ be a subset of $\mathbb{N}^*$

(a) What are the minimal elements? least element? maximal elements? the greatest element?

(b) What are the bounds of A (upper bound and lower bound)?

Homework

Exercise 1 Determine whether the relation $\mathscr{R}$ on the set of all integers is reflexive, symmetric, anti-symmetric and transitive. Where $x\mathscr{R}y$ if and only if

❶ $x \neq y$

❷ $xy \geqslant 1$

❸ $x = y+1$ or $x = y-1$

❹ $x \equiv y (mod 7)$

❺ $x + y = 7$

Exercise 2 Let $\mathscr{R}$ be the relation on the set of ordered pairs of positive integers such that

$$\boxed{\forall (a,b), (c,d) \in \mathbb{Z}^+ \times \mathbb{Z}^+, \quad (a,b)\mathscr{R}(c,d) \iff ad = bc.}$$

Show that $\mathscr{R}$ is an equivalence relation.

Exercise 3 Let $\mathscr{R}$ be a relation on the set $\mathbb{N}^*$

$$\boxed{\forall a, b \in \mathbb{N}^*,\ a\mathscr{R}b \iff \exists k \in \mathbb{N} : \frac{b}{a} = 2^k}$$

1. Show that $\mathscr{R}$ is an order relation on $\mathbb{N}^*$.
2. Decide whether $\mathscr{R}$ is a totally order relation on $\mathbb{N}^*$. Why?

Remark 1. *For the first exercise of homewok, choose only two relations.*