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First year of mathematics	**Algebra 1**	**2024-2025**
Tutorial Series N°1	**Mathematical logic**	

Exercise 1

Which of the following expressions are propositions? In the case of a proposition , say whether it is true or false :

1. $\sqrt{3}$ is a irrational number.
2. The integer n divides 12.
3. $\forall n \in \mathbb{N}$, $n+1=5$.
4. $\exists n \in \mathbb{N}$, $n+1=5$.
5. 25 is a multiple of 5 and 2 divides 7.
6. 25 is a multiple of 5 or 2 divides 7.

Exercise 2

Let P, Q and R be propositions. Give the truth table of these propositions.

1. $(P \Longrightarrow Q) \wedge (\overline{P} \Longrightarrow Q)$.
2. $(\overline{P \Longrightarrow Q}) \iff (P \wedge \overline{Q})$.
3. $(P \Longrightarrow Q) \iff (\overline{Q} \Longrightarrow \overline{P})$.

Exercise 3

Let f and g be two functions of $\mathbb{R}$ in $\mathbb{R}$, write in terms of quantifiers the following expressions :

1. f never equals zero.
2. f is even.
3. f is bounded.
4. f is strictly increasing function.
5. f less than g.

Exercise 4

Show which of the following propositions are true and which are false, then give their negation :

1. $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x+y>0$.
2. $(\exists x \in \mathbb{R}, x+1=0) \wedge (\exists x \in \mathbb{R}, x+2=0)$.
3. $\exists x \in \mathbb{R}, (x+1=0 \wedge x+2=0)$.
4. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x+y>0$.

Exercise 5

1. Using the proof by contradiction prove that $\sqrt{2}$ is not a rational number.
2. Prove by induction : $\forall n \in \mathbb{N}^*, 1+2^3+3^3+\cdots+n^3 = \frac{n^2(n+1)^2}{4}$.
3. By contrapositive, prove that $[(n^2-1)$ is not divisible by $8] \Longrightarrow (n$ is even$)$.
4. Let a be an integer. Prove by cases : 2 divides $a(a+1)$