Exercise 1

Which of the following expressions are propositions? In the case of a proposition , say whether it is true or false :

❶ $\sqrt{3}$ is an irrational number.

❷ The integer n divides 12.

❸ $\forall n \in \mathbb{N},\ n+1=5$.

❹ $\exists n \in \mathbb{N},\ n+1=5$.

❺ 25 is a multiple of 5 and 2 divides 7.

❻ 25 is a multiple of 5 or 2 divides 7.

Exercise 2

Let P, Q be propositions. Give the truth table of these propositions.

❶ $(P \Longrightarrow Q) \wedge (\overline{P} \Longrightarrow Q)$.

❷ $(\overline{P \Longrightarrow Q}) \iff (P \wedge \overline{Q})$.

❸ $(P \Longrightarrow Q) \iff (\overline{Q} \Longrightarrow \overline{P})$.

Exercise 3

Let f and g be two functions of $\mathbb{R}$ in $\mathbb{R}$, write in terms of quantifiers the following expressions :

❶ f never equals zero.

❷ f is even.

❸ f is bounded.

❹ f is strictly increasing function.

❺ f less than g.

Exercise 4

Show which of the following propositions are true and which are false, then give their negation :

❶ $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x+y>0$.

❷ $(\exists x \in \mathbb{R}, x+1=0) \wedge (\exists x \in \mathbb{R}, x+2=0)$.

❸ $\exists x \in \mathbb{R}, (x+1=0 \wedge x+2=0)$.

❹ $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x+y>0$.

Exercise 5

❶ Using the proof by contradiction prove that $\sqrt{2}$ is not a rational number.

❷ Prove by induction : $\forall n \in \mathbb{N}^*, 1+2^3+3^3+\cdots+n^3=\frac{n^2(n+1)^2}{4}$.

❸ By contrapositive, prove that $[(n^2-1)$ is not divisible by $8] \Longrightarrow (n$ is even$)$.

❹ Let a be an integer. Prove by cases : 2 divides $a(a+1)$