Université A. Mira de Bejaia
Faculté de la Technologie
Département 2° Année S.T
Module : RDM

30 / 05 / 2010

# *Epreuve de Moyenne Durée N°01*

**(02 Heures)**

**Exercice 1**

Pour la figure ci-dessous, les corps A et B ne sont pas solidaires (il n y a pas de collage entre les deux corps A et B)

Déterminer la charge critique « q » (charge limite) que doit appliquer le corps B sur le corps A pour le rompre par cisaillement ?

On donne :

- Contrainte admissible de cisaillement $[\tau] = 100 N/mm^2$.
- Dimensions : a = 100 mm, e = 10 mm

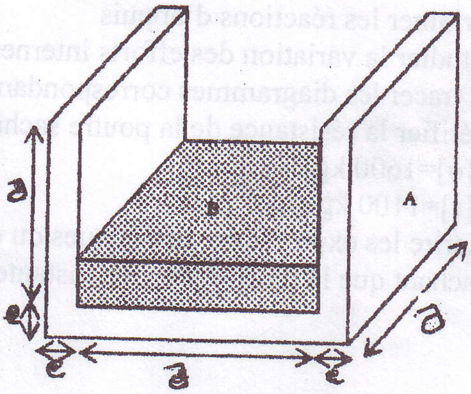



*Figure 1*

**Exercice 2 :**

Trois tiges métalliques (1,2 et 3) soutiennent une membrure rigide BCD. Avant l'application de la charge P, la membrure reste horizontale. Tous les joints sont du type rotule. Si on applique une charge P de 72 KN, calculer :

a) La contrainte normale qui s'exerce dans chacune des tiges
b) Le déplacement du point où est appliquée la force P

Données : $A_1 = 960\ mm^2$, $A_2 = A3 = 640\ mm^2$.
$E_1 = 105000$ MPa, $E_2 = 210000$ MPa et $E_3 = 70000$ MPa
$L_1 = 1512$mm, $L_2 = 2016$mm et $L_3 = 672$mm
a= 900 mm , b = 600 mm

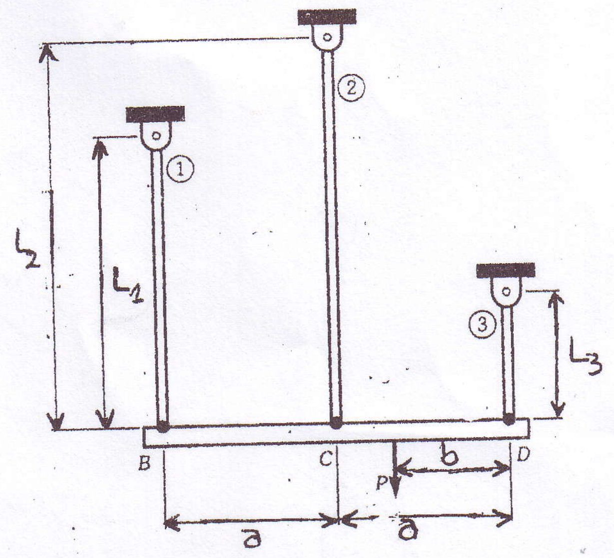



Figure 2

**Exercice 3**

Soit la poutre AE encastrée en A et simplement appuyée en D, soumise à Trois types de chargement, sa section droite à la forme indiquée selon la figure ci-dessous (figure 3) :

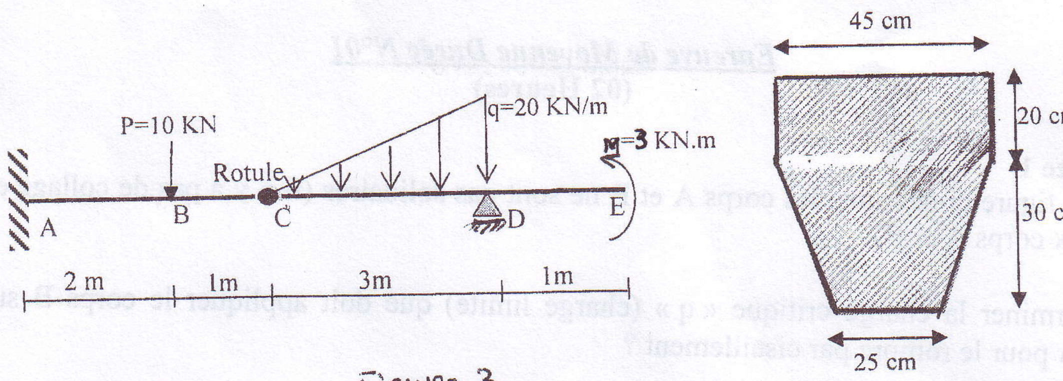


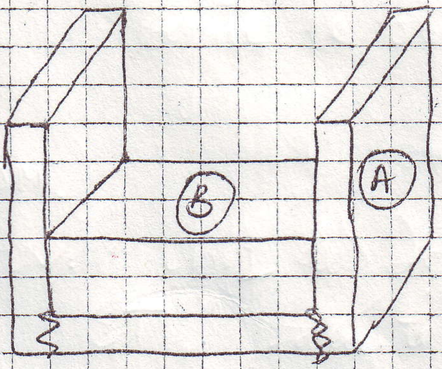

Figure 3

1) Déterminer les réactions d'appuis
2) Etudier la variation des efforts internes (efforts tranchants et moments fléchissant) et tracer les diagrammes correspondants.
3) Vérifier la résistance de la poutre sachant que les contraintes admissibles sont:
   $[\sigma]=1600\ kg/cm^2$
   $[\tau]=1100\ kg/cm^2$
4) Ecrire les expressions analytiques du déplacement V(x), et de la rotation θ(x) sachant que la rigidité EI est constante.

*Bonne chance*

Corrigé de l'EMD N°01 RDM

Exo 1 03 pts




0,25

La charge critique "q" pour que le corps A se rompe ? il faut que :

$\tau = \frac{F}{S_c} > [\tau]$ 0,50

$\Rightarrow F > [\tau] . S_c$ ... (1) 0,25

on $F = q . a^2$ 0,5

$S_c = (e . a) \times 2$ 0,5

(1) $\Rightarrow q a^2 > [\tau] . 2 e a$ 0,25

$\Rightarrow q > \frac{2e}{a} [\tau] = \frac{2 . 10}{100} . 100 = 20 \ N/mm^2$ 0,25

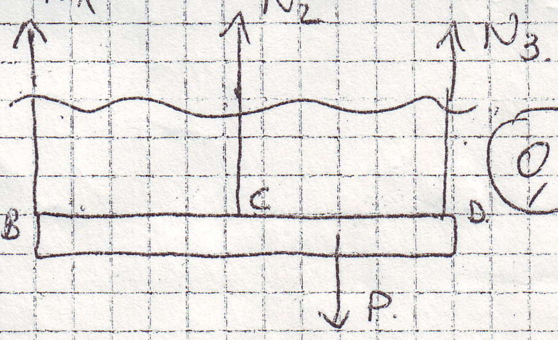
on prend $\boxed{q = 20 \ N/mm^2}$ 0,5

Exo 2 05 pts

1) Les contraintes normales dans les tiges

$\Delta = 3 - 2 = 1$

$\Rightarrow$ système hyperstatique




0,25

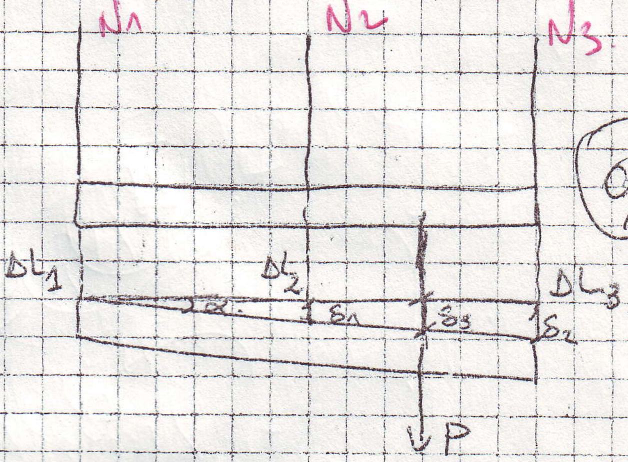
$\begin{cases} \Sigma F_x = 0 \\ \Sigma M/B = 0 \end{cases} \Rightarrow$

$\Sigma F_x = 0 \Rightarrow N_1 + N_2 + N_3 = P$ (0,25)

$\Sigma M/B = 0 \Rightarrow 2a\,N_3 + a\,N_2 = P(2a - b)$ (0,25)

on cherche une 3ème équation (équation de déformation)

Après déformation :




(0,5)

on a :

$$\delta_2\ tg\alpha = \frac{\delta_2}{2a} = \frac{\delta_1}{a}$$ (0,25)

$$\delta_2 = \Delta L_3 - \Delta L_1$$

$$\delta_1 = \Delta L_2 - \Delta L_1$$

$$\Rightarrow \frac{\Delta L_3 - \Delta L_1}{2\cancel{a}} = \frac{\Delta L_2 - \Delta L_1}{\cancel{a}}$$

$$\Rightarrow \Delta L_3 - \cancel{\Delta L_1} = 2\Delta L_2 - \cancel{2}\Delta L_1$$

$$\Rightarrow \Delta L_3 = 2\Delta L_2 - \Delta L_1$$ (0,25)

on a

$$\sigma = E\varepsilon = E\frac{\Delta L}{L}$$

$$\frac{N}{A} = E\frac{\Delta L}{L} \Rightarrow \Delta L = \frac{N.L}{E.A}$$

$$\Rightarrow \frac{N_3 . L_3}{E_3\,A_3} = 2\frac{N_2 L_2}{E_2 A_2} - \frac{N_1 L_1}{E_1 A_1}$$ (0,25)

<u>AN</u>

$$\begin{cases} N_1 + N_2 + N_3 = 72\times10^3 \\ 1800\,N_3 + 900\,N_2 = 864\times10^5 \\ 1{,}5\times10^{-5}\,N_3 = 3\times10^{-5}\,N_2 - 1{,}5\times10^{-5}\,N_1 \end{cases}$$ (0,25) (0,25) (0,25)

$$\Rightarrow \begin{cases} N_1 = 12\ kN \\ N_2 = 24\ kN \\ N_3 = 36\ kN \end{cases}$$ (0,25) (0,25) (0,25)

$$\Rightarrow \begin{cases} \sigma_1 = \dfrac{N_1}{A_1} = \dfrac{12000}{960} = 12{,}5\ MPa & 0{,}25 \\ \sigma_2 = \dfrac{N_2}{A_2} = \dfrac{24000}{640} = 37{,}5\ MPa & 0{,}25 \\ \sigma_3 = \dfrac{N_3}{A_3} = \dfrac{36000}{640} = 56{,}25\ MPa & 0{,}25 \end{cases}$$

2) Le deplacement du pt ou est appliquée la force P.

<u>on a.</u>

$$tg\,\alpha = \frac{\delta_1}{a} = \frac{\delta_2}{2a} = \frac{\delta_3}{2a-b} \quad 0{,}25$$

$$\Rightarrow \delta_3 = \frac{\delta_1}{a}(2a-b).$$

$$\Rightarrow \delta_1 = \Delta L_2 - \Delta L_1$$

$$\Delta L_1 = \frac{N_1 L_1}{E_1 A_1} = \frac{12000 \cdot 1512}{105000 \cdot 960} = 0{,}18\ mm.$$

$$\Delta L_2 = \frac{N_2 L_2}{E_2 A_2} = \frac{24000 \times 2016}{210000 \times 640} = 0{,}36\ mm.$$

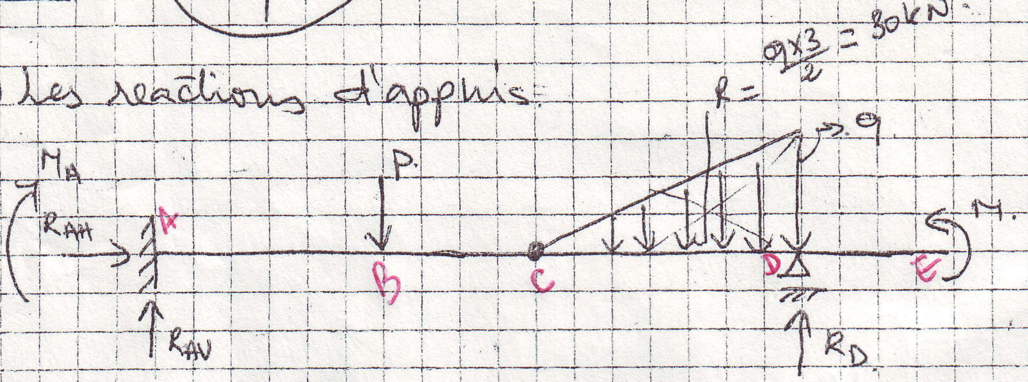
$$\Rightarrow \delta_1 = 0{,}36 - 0{,}18 = 0{,}18\ mm \quad 0{,}25$$

$$\Rightarrow \delta_3 = \frac{0{,}18}{900}(1800-600) = 0{,}24$$

$$\Rightarrow \Delta L_p = \Delta L_1 + \delta_3 = 0{,}18 + 0{,}24 = 0{,}42\ mm \quad 0{,}25$$

Exo3 (12 pts)

1) Les réactions d'appuis:

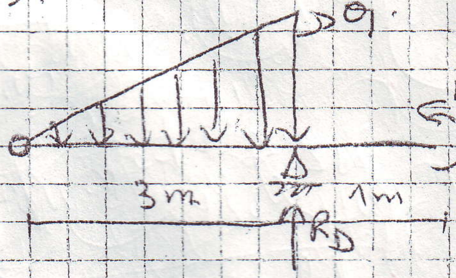



$\Sigma F_x = 0 \Rightarrow R_{AH} = 0$

$\Sigma F_y = 0 \Rightarrow R_{AV} + R_D - 10 - \frac{20 \times 3}{2} = 0$

$\Rightarrow R_{AV} + R_D = 40 kN \ldots\ldots (1)$

$\Sigma M/c_{droite} = \Sigma M/c_{gauche} = 0$ (0,25)




$\Rightarrow M + R_D \times 3 - R \cdot \frac{2}{3} \times 3 = 0$

$\Rightarrow R_D = 19 kN$ (0,25)

$\Sigma M/A = 0 \Rightarrow M + (R_D \times 6) - R\left(\frac{2}{3} \times 3 + 3\right) - P \times 2 - M_A = 0$

$\Rightarrow M_A = 3 + (19 \times 6) - (30 \times 5) - (10 \times 2)$

$= -53 kN.m$ (0,25)

$\Sigma M/D = 0 \Rightarrow M + R\left(\frac{1}{3} \times 3\right) + P(4) - (R_{AV} \times 6) - M_A = 0$

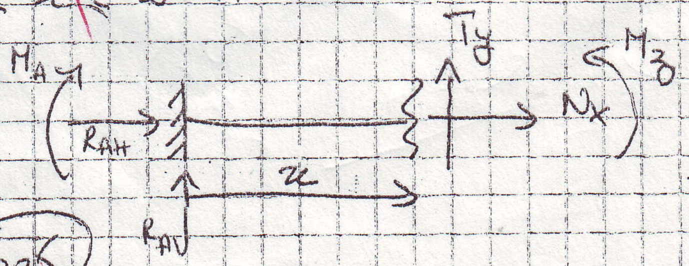
$\Rightarrow R_{AV} = \frac{3 + 30 + 40 + 53}{6} = 21 kN$ (0,25)

Vérification :

(1) $\Rightarrow 21 + 19 = 40 kN$ (Vérifiée)

2) Les efforts internes : Il y a 04 tronçons

1er tronçon AB : $0 \leq x \leq 2m$




$\Sigma F_x = 0 \Rightarrow N_x = 0$

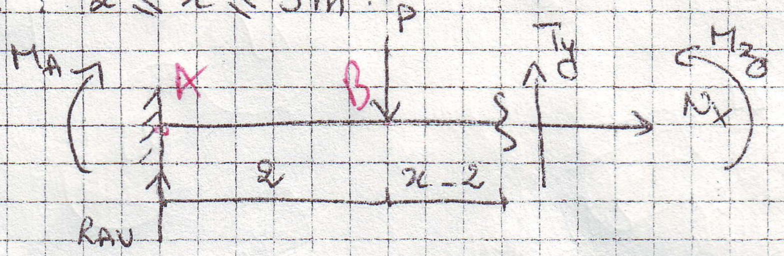
$\Sigma F_y = 0 \Rightarrow T_y = -R_{AV}$

$\Rightarrow \boxed{T_y = -21 kN}$ (0,25)

$\Sigma M/i = 0 \Rightarrow M_z - R_{AV}.x - M_A = 0$

$\Rightarrow M_z = 21x - 53$ 0,25 $\begin{cases} \to x = 0 & M_z = -53 \text{ 0,25} \\ \to x = 2 & M_z = -11 \text{ 0,25} \end{cases}$

$2^{eme}$ tronçon BC : $2 \leq x \leq 3m$.




$\Sigma Fx = 0 \Rightarrow N_x = 0$

$\Sigma F_y = 0 \Rightarrow$

$T_y = -R_{AV} + P$

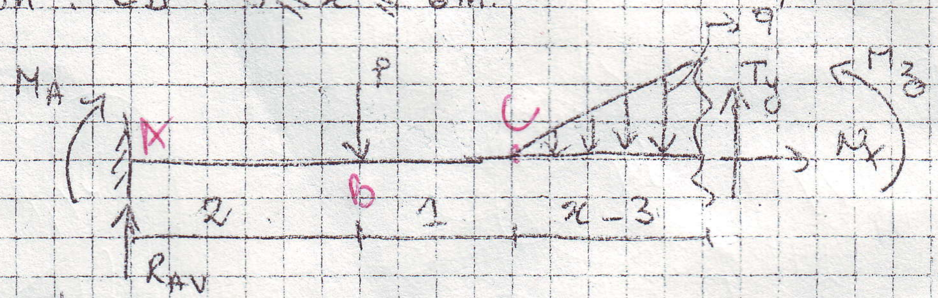
$= -21 + 10 = -11$ kN 0,25

$\Sigma M/i = 0 \Rightarrow M_z - R_{AV}x + P(x-2) - M_A = 0$

$\Rightarrow M_z = 21x - 10x + 20 - 53$

$= 11x - 33$ 0,25 $\begin{cases} \to x = 2 & M_z = -11 \text{ 0,25} \\ \to x = 3 & M_z = 0 \text{ 0,25} \end{cases}$

$3^{eme}$ tronçon : CD : $3 \leq x \leq 6m$.




$tg\alpha = \frac{20}{3} = \frac{q'}{x-3}$ 0,25

$\Rightarrow q' = \frac{20(x-3)}{3}$

$\Sigma Fx = 0 \Rightarrow N_x = 0$

$\Sigma Fy = 0 \Rightarrow T_y + R_{AV} - P - \frac{q'(x-3)}{2} = 0$

$\Rightarrow T_y = -11 + \frac{10(x-3)^2}{3}$ 0,25 $\begin{cases} \to x = 3 & T_y^{max} = -11 \text{ 0,25} \\ \to x = 6 & T_y = 19 \text{ 0,25} \\ \to x = 4{,}82 & T_y = 0 \end{cases}$

$\Sigma M/i = 0 \Rightarrow M_z = 21x - P(x-2) + M_A - \frac{10}{3}(x-3)^2 \cdot \frac{(x-3)}{3}$

$= 21x - 10x + 20 - 53 - \frac{10}{9}(x-3)^3$

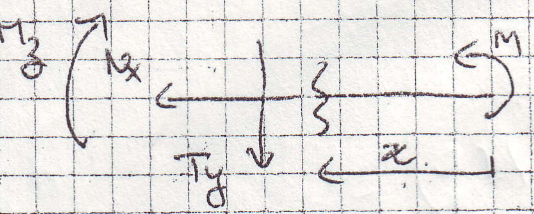
$= 11x - 33 - \frac{10}{9}(x-3)^3$ 0,25

$x = 3$ $M_z = 0$ 0,25

$x = 6$ $M_z = 3$ kN.m 0,25

$x = 4,82$ $M_z^{max} = 13,32$ kN.m

4ème tronçon : ED : $0 \leq x \leq 1$m

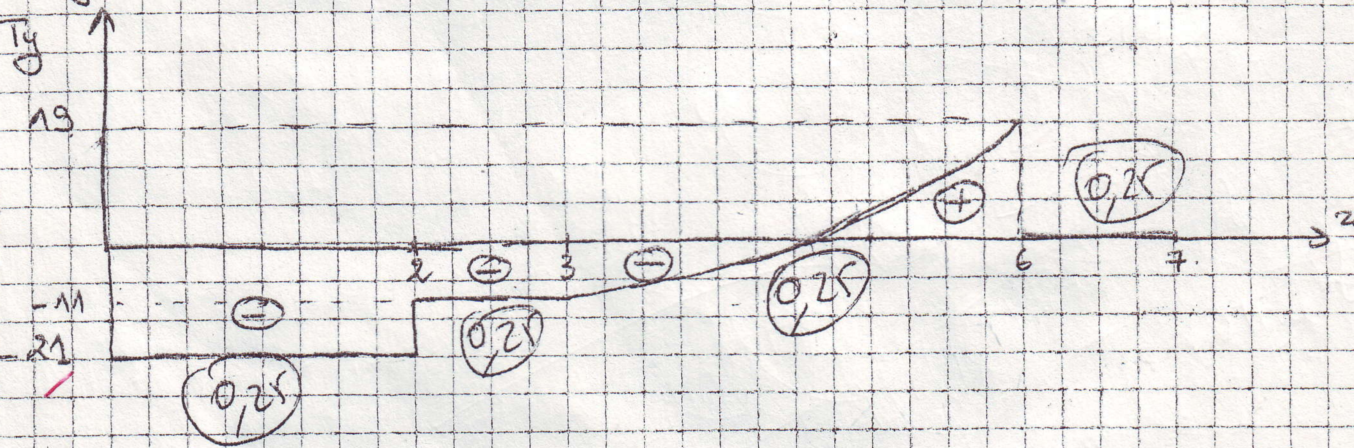


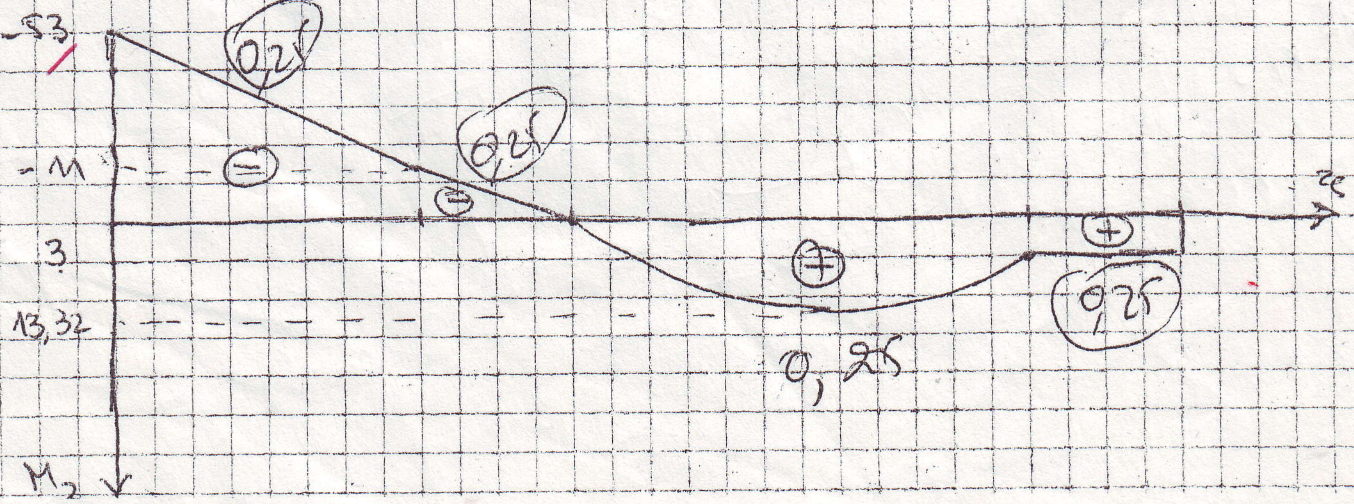

$\Sigma Fx = 0 \Rightarrow N_x = 0$

$\Sigma Fy = 0 \Rightarrow T_y = 0$ 0,25

$\Sigma M/i = 0 \Rightarrow M_z = 3$ kN.m 0,25

* Diagrammes



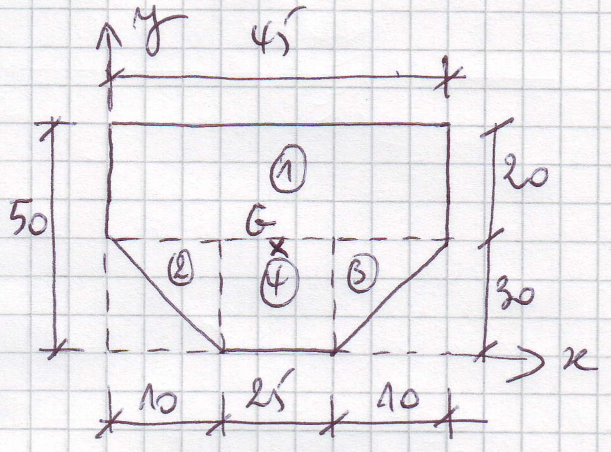



3) Vérification de la résistance

3.1) Sections dangereuses :

I $\left| \begin{array}{l} M_z^{max} = -53 \text{ kN.m} \\ T_y^{max} = -21 \text{ kN} \end{array} \right.$

II $\left| \begin{array}{l} M_z = 13,32 \text{ kN.m} \\ T_y = 0 \text{ kN} \end{array} \right.$




3.2/- Calcul de $y^+_{max}$ et $y^-_{max}$

$$x_G = 22,50 \text{ cm}.$$

le calcul de $y_G$ se résume dans le tableau :

| Section | $A_i$ ($cm^2$) | $y_{Gi}$ (cm) | $A_i y_i$ ($cm^3$) |
|---|---|---|---|
| 1 | 900 | 40 | 36000. |
| 2 | 150 | 20 | 3000 |
| 3 | 150 | 20 | 3000. |
| 4 | 750 | 15 | 11250. |
| $\Sigma$ | 1950 | x | 53250. |

$$y_G = 27,30 \text{ (cm)}.$$

0,25

3.3/- Calcul de $I_{Gz}$ : $I_{Gz} = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$.

$$(I_z)_1 = I_z + (y_G - y_{G1})^2 A_1 = \frac{45 \cdot 20^3}{12} + (40 - 27,30)^2 \times 900 = 175161,0 \text{ cm}^4$$

01

$$(I_z)_2 = (I_z)_3 = \frac{10 \cdot 30^3}{12} + (27,30 - 20)^2 \times 150 = 15.493,50 \text{ cm}^4.$$

$$(I_z)_4 = \frac{25 \cdot 30^3}{12} + (27,30 - 15)^2 \times 750 = 169.717,50 \text{ cm}^4.$$

$$I_{Gz} = 375.865,50 \text{ cm}^4.$$

Vérifications :

Section I : Flexion simple.

$\left\{ \begin{array}{l} M_z^{max} = -53 \text{ kN.m} \\ T_y = -21 \text{ kN} \end{array} \right. \quad \left. \begin{array}{l} y^+_{max} = 22,70 \text{ cm} \\ y^-_{max} = 27,30 \text{ cm} \end{array} \right\}$ 0,25

$$\sigma^{+}_{max} = \frac{M_z^{max}}{I_{Gz}} \cdot y^{+}_{max} = \frac{53 \times 10^4}{375865{,}50} \times 22{,}70 = 32{,}00 \ Kg/cm^2$$

0,25

$$\sigma^{-}_{max} = \frac{M_z^{max}}{I_{Gz}} \cdot y^{\ominus}_{max} = \frac{53 \times 10^4}{375.865{,}50} \times 27{,}30 = 38{,}48 \ Kg/cm^2.$$

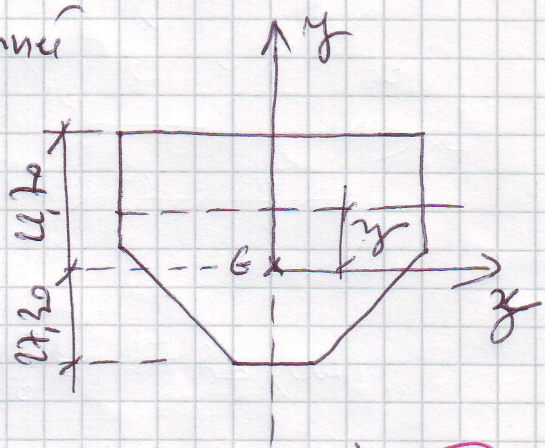
0,25

$$\mathrm{Max}\left(\sigma^{+}_{max}, \sigma^{-}_{max}\right) = 38{,}48 \ Kg/cm^2 < [\sigma] = 1600 \frac{Kg}{cm^2}$$

0,25 $\Rightarrow$ Condition de résistance Vérifiée.

La Contrainte de cisaillement est donnée Par :

$$\tau = \frac{T_y^{max} \cdot S_z^{*}}{b \cdot I_{Gz}}.$$




$$S_z^{*} = (22{,}70 - y) \cdot 45 \times \left(\frac{22{,}70 - y}{2} + y\right)$$

$$= 45 \times (22{,}70 - y)\left(\frac{22{,}70 + y}{2}\right) = 22{,}50 \cdot \left(515{,}29 - y^2\right)$$ 0,25

$$\frac{d\tau}{dy} = 0 \Rightarrow y = 0 \Rightarrow \left(S_z^{*}\right)_{max} = 22{,}50 \times 515{,}29$$

$$= 11.594{,}02 \ (cm^3).$$

$$\tau^{max} = \frac{21 \times 10^2}{45 \times 375865{,}50} \times 11594{,}02 = 1{,}44 \ \frac{Kgf}{cm^2} < [\tau].$$

0,25

$$\sqrt{\sigma^2 + 3\tau^2} = \sqrt{38{,}48^2 + 3 \times (1{,}44)^2} = 38{,}57 \frac{Kg}{cm^2}$$

0,25

$$38{,}57 \ Kg/cm^2 \leq [\sigma] = 1600 \frac{Kg}{cm^2}$$

$\Rightarrow$ critère Vérifié

Section II :

$$M_z^{max} = 13{,}32 \text{ KN.m} , \quad y^+_{max} = 27{,}30 \text{ cm}$$
$$y^-_{max} = 22{,}70 \text{ cm.}$$
0,25

$$\sigma^+_{max} = \frac{13{,}32 \times 10^4}{375\,865{,}50} \times 27{,}30 = 9{,}67 \text{ Kg/cm}^2$$

$$\sigma^-_{max} = \frac{13{,}32 \times 10^4}{375\,865{,}50} \times 22{,}70 = 8{,}04 \frac{\text{Kg}}{\text{cm}^2}$$

$$\text{Max}\left(\sigma^+_{max}, \sigma^-_{max}\right) = 9{,}67 \frac{\text{Kg}}{\text{cm}^2} < [\sigma] = 1600 \frac{\text{Kg}}{\text{cm}^2}$$ 0,25

La résistance de la poutre est vérifiée. 0,25

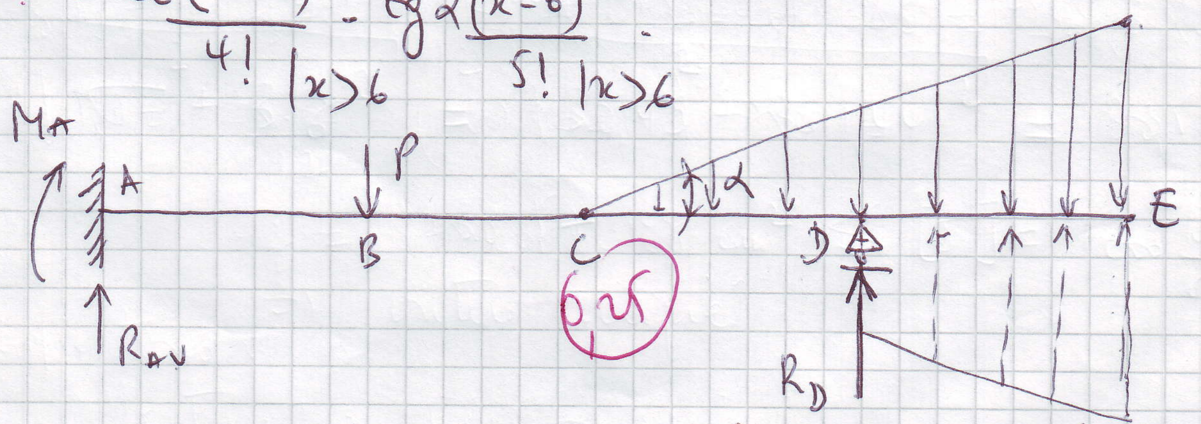
4- $v(x)$ et $\theta(x)$ :

Conditions initiales : $\left.\begin{matrix} V_0 = 0 \\ \theta_0 = 0 \end{matrix}\right\}$ encastrement 0,25

$T_0 = -R_{AV} = -21$ KN. ; $tg\alpha = \frac{20}{3} \simeq 6{,}67$

$M_0 = -M_A = 53$ KN.m.

$$EI\,v(x) = -\frac{21x^3}{3!} + \frac{53 \cdot x^2}{2!} + \frac{10}{3!}(x-2)^3\Big|_{x>2} + tg\alpha \frac{(x-3)^5}{5!}\Big|_{x>3} - \frac{19(x-6)^3}{3!}\Big|_{x>6}$$
0,25
$$- \frac{20(x-6)^4}{4!}\Big|_{x>6} - tg\alpha \frac{(x-6)^5}{5!}\Big|_{x>6} .$$




$$\Rightarrow \quad EI\,\theta(x) = -\frac{21x^2}{2!} + 53x + \frac{10}{2!}(x-2)^2\Big|_{x>2} + tg\alpha \frac{(x-3)^4}{4!} - \frac{19(x-6)^2}{2!}\Big|_{x>6}$$
0,25
$$- \frac{20(x-6)^3}{3!}\Big|_{x>6} - tg\alpha \frac{(x-6)^4}{4!}\Big|_{x>6} .$$

FIN

3) Verification de la resistance

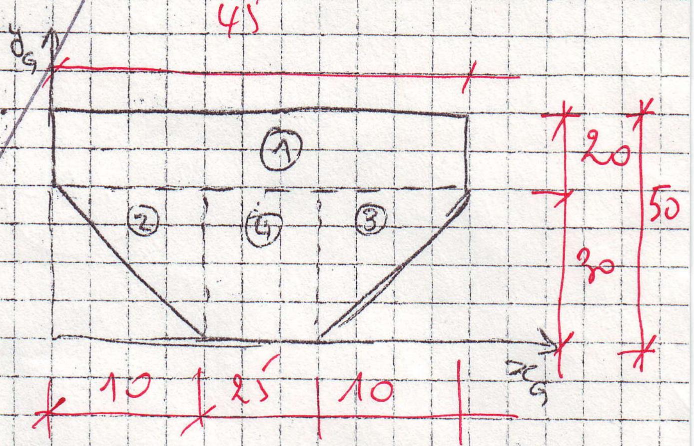
1) Sections dangereuses

I $\begin{cases} M_z^{max} = -53 \text{ KN.m} \\ T_y^{max} = -21 \text{ KN} \end{cases}$

II $\begin{cases} M_z = 13{,}32 \text{ KN.m} \\ T_y = 0 \end{cases}$

2) Calcul $y^+_{max}$, $y^-_{max}$

$x_G = 22{,}5 \text{ cm}$




| Section | $A_i$ | $y_{G_i}$ | $A_i y_{G_i}$ |
|---|---|---|---|
| 1 | 900 | 40 | 36000 |
| 2 | ~~200~~ 150 | 20 | ~~2000~~ 3000 |
| 3 | ~~200~~ 150 | 20 | ~~2000~~ 3000 |
| 4 | 750 | 15 | 11250 |
| Σ | ~~1850~~ 1950 | | ~~51250~~ 53250 |

$\Rightarrow y_G = \cancel{27{,}70}$ cm 0,25

27,30 cm

~~2) Section~~

3) Calcul $I_z$

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$$

$$(I_z)_1 = I_{z_{G_1}} + (y_G - y_{G_1})^2 A_1$$

$$= \frac{45(20)^3}{12} + (40 - \cancel{27{,}7})^2 \cdot 900 = \cancel{166161} \text{ cm}^4$$

27,30 — 175161,0 cm⁴

1

$$(I_z)_2 = (I_z)_3 = \frac{10(30)^3}{36} + (\cancel{27{,}7} - 20)^2 \cdot \cancel{100} = \cancel{13429} \text{ cm}^4$$

27,30 — 150 — 15493,50

$$(I_z)_4 = \frac{25(30)^3}{\ldots} + (\cancel{27{,}7} - 15)^2 \cdot 750 = \cancel{1772{,}11}\ 5\ldots$$

27,30

$\Rightarrow I_z = \cancel{370236{,}5}\ cm^4$
$375.865{,}5\ cm^4$

Verification

- Section I : Flexion simple

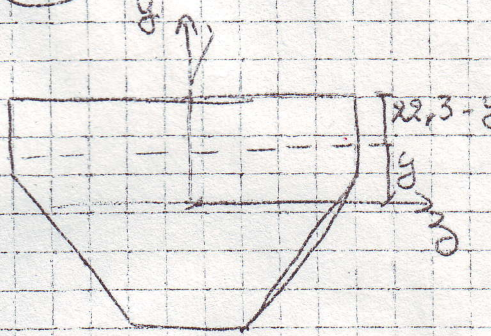
$$\left\{\begin{array}{l} M_z^{max} = -53\ KN.m \\ T_y = -21\ KN \end{array}\right. \qquad \left.\begin{array}{l} y^+_{max} = \cancel{22{,}3}\ 22{,}70\ cm \\ y^-_{max} = \cancel{27{,}7}\ 27{,}30\ cm \end{array}\right\} \quad 0{,}25$$

$$\sigma^+_{max} = \frac{53 \times 10^4}{\cancel{370236{,}5}\ 375865{,}5} (\cancel{22{,}3}\ 22{,}70) = \cancel{3192{,}28}\ 32{,}00\ kg/cm^2 \quad 0{,}25$$

$$\sigma^-_{max} = \frac{53 \times 10^4}{\cancel{370236{,}5}\ 375865{,}50} (\cancel{27{,}7}\ 27{,}30) = \cancel{39{,}65{,}30}\ 38{,}48\ kg/cm^2 \quad 0{,}25$$

$$\Rightarrow \max(\sigma^+_{max}, \sigma^-_{max}) = \cancel{39{,}65{,}30}\ 38{,}48\ kg/cm^2 \leq [\sigma] = 1600\ kg/cm^2 \quad 0{,}25$$

$$\tau = \frac{T_y^{max}\, S_z^*}{b.\, I_z}$$




$$S_z^* = (22{,}3 - y).45 \times \left(\frac{22{,}3 - 8}{2} + y\right)$$

$$= 45\,(22{,}\cancel{3} - y)\left(\frac{22{,}\cancel{7} + y}{2}\right) \quad 0{,}25$$

$$= 22{,}5\,(\cancel{497{,}29}\ 515{,}29 - y^2) = 22{,}5\,(515{,}29 - y^2)$$

$$\Rightarrow \tau = \frac{T_y^{max} \cdot 22{,}5\,(497{,}29 - y^2)}{45 \cdot 370236{,}5}$$

$$\frac{d\tau}{dy} = 0 \Rightarrow y = 0 \Rightarrow \quad S_z^* = 22{,}5 \times 515{,}29 = 11594{,}02$$

$$\tau^{max} = \frac{21 \times 10^2 \cdot \cancel{11189{,}025}\ 11594{,}02}{\cancel{45 \times 370236{,}5}\ 45 \quad 375865{,}5} \simeq \cancel{1{,}46}\ 1{,}44\ kg/cm^2 < [\tau] = 1100 \quad 0{,}25$$

$$\sqrt{(\cancel{39{,}65})^2 + 3\,(\cancel{1{,}46})^2} = \cancel{39{,}73}\ 38{,}57\ kg/cm^2 \leq [\sigma] = 1600\ kg/cm^2 \quad 0{,}25$$
$38{,}49 \qquad 1{,}44$