

Exemple 1.9. Soit l'ensemble $B = [-1, 3]$ et

$$\begin{aligned} f: \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto f(x) = x^2 - 1. \end{aligned}$$

On a

$$\begin{aligned} f^{-1}(B) &= \{x \in \mathbb{R} / f(x) \in B\} \\ &= \{x \in \mathbb{R} / f(x) \in [-1, 3]\} \\ &= \{x \in \mathbb{R} / -1 \leq f(x) \leq 3\} \\ &= \{x \in \mathbb{R} / -1 \leq f(x)\} \cap \{x \in \mathbb{R} / f(x) \leq 3\} \\ &= \{x \in \mathbb{R} / -1 \leq x^2 - 1\} \cap \{x \in \mathbb{R} / x^2 - 1 \leq 3\} \\ &= \{x \in \mathbb{R} / x^2 \geq 0\} \cap \{x \in \mathbb{R} / x^2 - 4 \leq 0\} \\ &= \mathbb{R} \cap [-2, 2] \\ &= [-2, 2]. \end{aligned}$$