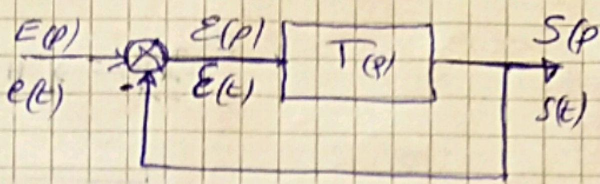
Exo1

1/ Entrée $\begin{cases} e(t) : \\ E(p) \end{cases}$



Sortie $\begin{cases} S(t) \\ S(p) \end{cases}$

Erreur $\begin{cases} \varepsilon(p) \\ \varepsilon(t) \end{cases}$

2/ FTBO $T(p)$

FTBF $G(p) = \dfrac{T(p)}{1+T(p)}$

3/ Les relations qui lient $S(p)$ à $S(t)$ et $\varepsilon(p)$ à $\varepsilon(t)$ en régime permanent.

* $\lim_{t\to\infty} s(t) = \lim_{p\to 0} p\, S(p)$

* $\lim_{t\to\infty} \varepsilon(t) = \lim_{p \to 0} p\, \varepsilon(p)$

4/ Les performances recherchées dans un asservissement : - stabilité
- précision, - rapidité, - robustesse.

Exo2 :

Un système est régie par l'équation différentielle suivante

$$\frac{d^2 s(t)}{dt^2} + 3 \frac{d\, s(t)}{dt} + 2 s(t) = e(t) \quad , \quad s(t) : \text{la sortie}, \ e(t) : \text{L'entrée}.$$

1/ Application de la transformée de Laplace

$$\xrightarrow{TL} p^2 S(p) + 3p S(p) + 2 S(p) = E(p) \Rightarrow \frac{S(p)}{E(p)} = \frac{1}{p^2+3p+2} = \frac{1}{(p+1)(p+2)}$$

2/ Calcul des pôles et des zéros de $T(p)$

$$T(p) = k \frac{(p+z_0)(p+z_1)}{(p+p_1)(p+p_2)} \quad , \quad \frac{1}{(p+1)(p+2)} \Rightarrow T(p) \text{ ne possède pas}$$

de zéro, $p_1 = -1$, $p_2 = -2$ deux pôles

3/ $S(p) = E(p) \cdot T(p) \Rightarrow$ ~~$S(p)$~~ $e(t) = t \rightarrow E(p) = 1/p^2$

$$S(p) = \frac{1}{p^2} \cdot \frac{1}{(p+2)(p+1)} \quad , \quad S(p) = \frac{Ap+B}{p^2} + \frac{C}{p+1} + \frac{D}{p+2}$$

$$S(p) = -\frac{3}{4}\frac{1}{p} + \frac{1}{2}\frac{1}{p^2} + \frac{1}{p+1} - \frac{1}{4}\frac{1}{p+2}$$

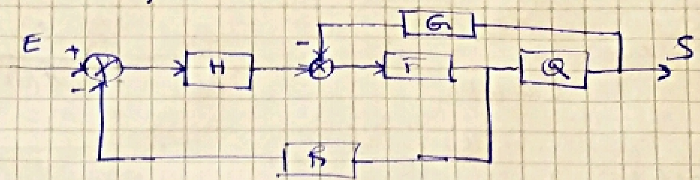
$$\Rightarrow S(t) = -\frac{3}{4} + \frac{1}{2}t + e^{-t} - \frac{1}{4}e^{-2t}$$

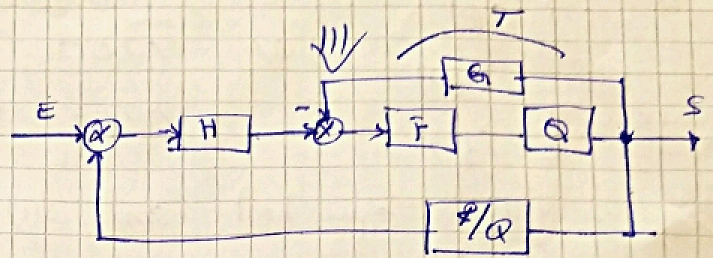
4/ $e(t) = 1 \Rightarrow E(p) = 1/p$, $S(p) = \dfrac{1}{p} \cdot \dfrac{1}{(p+1)(p+2)} \Rightarrow S(p) = \dfrac{1}{2}\dfrac{1}{p} - \dfrac{1}{p+1} + \dfrac{1}{2}\dfrac{1}{(p+2)}$

$$\Rightarrow S(t) = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}$$

$$S(\infty) = \lim_{t\to\infty} s(t) = \lim_{p\to 0} p\, S(p) = 1/2$$

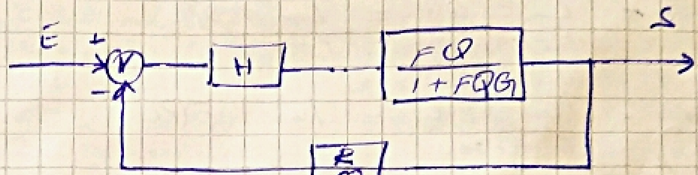
Exo3 : On considère le schéma fonctionnel suivant :





$$T = \frac{F.Q}{1 + FQ.G}$$

$$FTBF = \frac{S(p)}{E(p)} = \frac{HFQ/_{1+FQG}}{1 + \frac{HFQ}{1+FQG} \cdot \frac{R}{Q}}$$



$$FTBF = \frac{HFQ}{1 + FQG + HFR}$$

$$FTBF = \frac{HFQ}{1 + F(QG + HR)}$$

$F = Q = G = R = 2$, $H = \frac{10}{p+1}$

$$FTBF = \frac{40/_{p+1}}{1 + 8 + \frac{40}{p+1}} = \frac{40/_{p+1}}{9 + \frac{40}{p+1}} = \frac{40}{9p + 49} \quad , \quad FTBF = \frac{40}{9p+49}$$

2/ entrée échelon : $e(t) = 1 \Rightarrow E(p) = 1/p$

$s(t) = ?$ $\quad S(p) = E(p) \cdot \frac{40}{9p+49} \Rightarrow S(p)\ \frac{1}{p} \cdot \frac{40}{9p+49}$

$$S(p) = \frac{A}{p} + \frac{B}{9p+49} \Rightarrow Bp + 9Ap + 49A = 40$$

$B + 9A = 0$

$49A = 40 \Rightarrow A = \frac{40}{49} \Rightarrow B = -9 \cdot \frac{40}{49}$

$$S(p) = \frac{40}{49} \cdot \frac{1}{p} - 9 \cdot \frac{40}{49} \cdot \frac{1}{9p+49}$$

$$\Rightarrow S(p) = \frac{40}{49} \frac{1}{p} - \frac{40}{49} \cdot \frac{1}{p + 49/9} \Rightarrow s(t) = \frac{40}{49}\left(1 - e^{-\frac{49}{9}t}\right) u(t)$$

erreur en régime permanent $\varepsilon(\infty) = e(\infty) - s(\infty)$, $e(\infty) = 1$

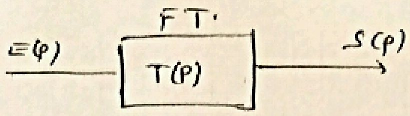
$$s(\infty) = \lim_{t \to \infty} s(t) = \lim_{t \to \infty} \frac{40}{49}\left(1 - e^{-49/9\, t}\right) = \frac{40}{49}$$

$$\varepsilon = 1 - \frac{40}{49} = \frac{49 - 40}{49} = 9/40 = 0{,}225$$

3/ le temps de réponse à 95% de $s(\infty)$ associé à cette entrée

$$s(t_r) = \frac{40}{49}\left(1 - e^{-49/9\, t_r}\right) = 0{,}95 \cdot \frac{40}{49} \Rightarrow 0{,}05 = e^{-49/9\, t_r} \Rightarrow \boxed{t_r = 0{,}55\ (s)}$$

Exo 5



6

1/ $T\dfrac{ds(t)}{dt} + s(t) = K\,e(t)$

$\underline{TL} \longrightarrow T\,p\,S(p) + S(p) = K\,E(p) \Rightarrow (Tp+1)\,S(p) = K\,E(p) \Rightarrow \dfrac{S(p)}{E(p)} = \dfrac{K}{Tp+1}$

$$T(p) = \frac{K}{1+Tp}$$

2/ La valeur finale de $s(t)$ en utilisant le théorème de la valeur finale

$s(\infty) = \lim\limits_{t\to\infty} s(t) = \lim\limits_{p\to 0} p\,S(p) \Rightarrow$ $S(p) = T(p).E(p)$, $e(t) = 1$

$S(p) = \dfrac{K}{1+Tp}\cdot\dfrac{1}{p} \Rightarrow s(\infty) = \lim\limits_{p\to 0} \cancel{p}\,\dfrac{K}{1+Tp}\cdot\dfrac{1}{\cancel{p}} = K.$

3/ Calcul de $s(t)$: $S(p) = \dfrac{K}{1+Tp}\cdot\dfrac{1}{p} = \dfrac{A}{p} + \dfrac{B}{1+Tp}$

$A = p.\,S(p)\Big|_{p=0} = \dfrac{K}{1+Tp}\Big|_{p=0} = K.$, $B = (1+Tp)\,S(p)\Big|_{p=-1/T} = -\dfrac{KT}{\cancel{T}}$

$S(p) = \dfrac{K}{p} - \dfrac{KT}{1+Tp} \Rightarrow S(p) = \dfrac{K}{p} - \dfrac{K}{\frac{1}{T}+p}$ $\quad / u(t) = 1$

$\Rightarrow s(t) = \left(K - K e^{-\frac{t}{T}}\right) u(t) \Rightarrow s(t) = K\left(1 - e^{-\frac{t}{T}}\right).u(t)$

$s(\infty) = \lim\limits_{t\to\infty} s(t) = \lim\limits_{t\to\infty} K\left(1 - \underbrace{e^{-t/T}}_{\to 0}\right) = K.$

4/ Calcule de t_r : temps de réponse.

forme canonique $\dfrac{S(p)}{E(p)} = \dfrac{K_0.\,\omega_0^2}{p^2 + 2\xi\omega_0 p + \omega_0^2}$

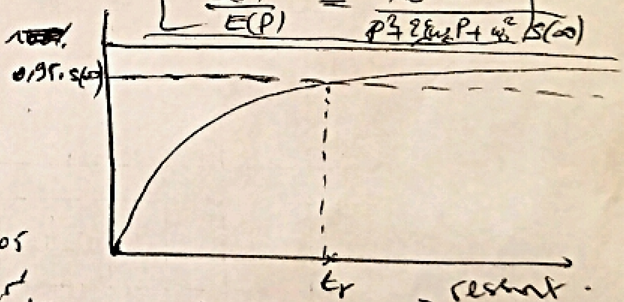
$s(t_r) = 0{,}95\; s(\infty)$

$\Rightarrow \cancel{K}\left(1 - e^{-t_r/T}\right) = 0{,}95.\,\cancel{K}$

$\Rightarrow 1 - e^{-t_r/T} = 0{,}95$

$\Rightarrow (-)\,e^{-t_r/T} = (-)\,0{,}05 \Rightarrow -\dfrac{t_r}{T} = \ln 0{,}05$

$\Rightarrow \boxed{t_r = 3T}$



Exo 6 : PFD $\Rightarrow \sum \vec{F}_{ext} = m\vec{\gamma}$

amortisseur = (ressort + vérin)

1/ $\Rightarrow \vec{F}(t) + \vec{F}_k + \vec{F}_f = m\vec{\gamma}$

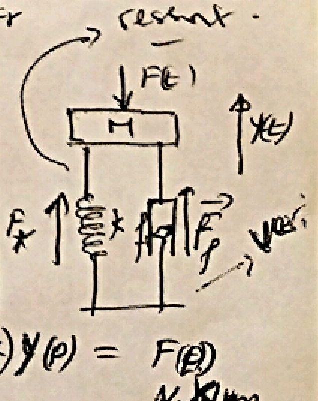
$\Rightarrow -F(t) + F_k + F_f = -m\gamma$

$\Rightarrow F_k + F_f + m\gamma = F(t)$

$F_k = K\,y(t)$

$F_f = f\,v = f\dfrac{dy(t)}{dt}$

$\gamma = \dfrac{d^2y(t)}{dt^2}$



$\Rightarrow K\,y(t) + f\dfrac{dy(t)}{dt} + m\dfrac{d^2y(t)}{dt^2} = F(t) \xrightarrow{TL} (mp^2 + fp + k)\,Y(p) = F(p)$

$F(t) \to$ entrée
$y(t) \to$ sortie | fonction de transfert $\dfrac{Y(p)}{F(p)} = \dfrac{1}{mp^2 + fp + K} = \dfrac{\frac{1}{K}\cdot\frac{K}{m}}{p^2 + \frac{f}{m}p + K/m}$

$\Rightarrow \omega_0^2 = \dfrac{K}{m} \Rightarrow \omega_0 = \sqrt{\dfrac{K}{M}}$, $\xi = \dfrac{f}{2\sqrt{Km}}$, ω_0 : pulsation propre non amortie

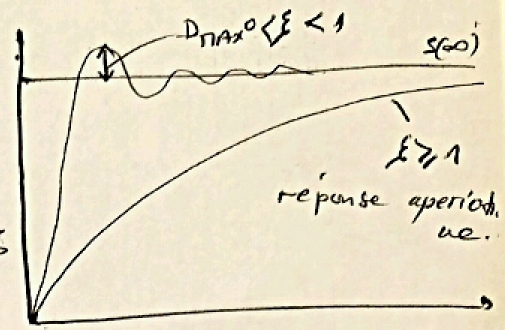
ξ : Coefficient d'amortissement

Suite exo6

2/ dimensionnement de f pour avoir une réponse oscillatoire amortie $\xi < 1$

$$\xi = \frac{f}{2\sqrt{km}}$$

$$\xi < 1 \Rightarrow \frac{f}{2\sqrt{km}} < 1 \Rightarrow \boxed{f < 2\sqrt{km}}$$

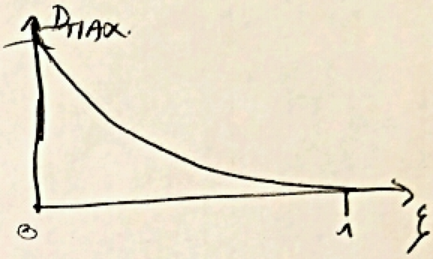


3/ Évolution de D_{MAX} en fonction des paramètres k, f et m.

$$D_{MAX} = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)$$

si $\xi \nearrow \Rightarrow D_{MAX} \searrow$, $\boxed{\xi = \frac{f}{2\sqrt{km}}}$

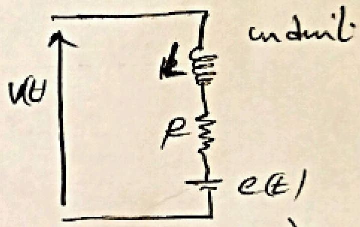
- si $f \nearrow \Rightarrow \xi \nearrow \Rightarrow D_{MAX} \searrow$
- si k et/ou $m \nearrow \Rightarrow \xi \searrow \Rightarrow D_{MAX} \nearrow$



Exo7 - 1/ L'ensemble des équations qui régit le MCC.

- Équation électrique :

$$v(t) = R\,i(t) + L\frac{di(t)}{dt} + e(t) \quad —— ①$$



- Équations électromécaniques.

$$e(t) = k_1\,\Omega(t) \quad —— ②$$

$$\Gamma_m(t) = k_2\,i(t) \quad —— ③$$

- Équation mécanique → (moment d'inertie) → (coeff de freinage)

$$\Gamma_m(t) - \Gamma_r(t) = J\frac{d\Omega(t)}{dt} + f\,\Omega(t) \quad —— ④$$

Tensions induit

2/ ① $\xrightarrow{TL}$ $V(p) - E(p) = (R + Lp)I(p) \Rightarrow I(p) = \frac{V(p) - E(p)}{R + Lp} = \frac{1}{R+Lp}\cdot(V(p) - E(p))$

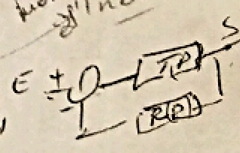
$\Rightarrow \boxed{H_1(p) = \frac{1}{R+Lp}}$

③ $\xrightarrow{TL}$ $\Gamma_m(p) = k_2 I(p) \Rightarrow \boxed{H_2(p) = k_2}$

④ $\xrightarrow{TL}$ $\Gamma_m(p) - \Gamma_r(p) = (Jp + f)\Omega(p)$

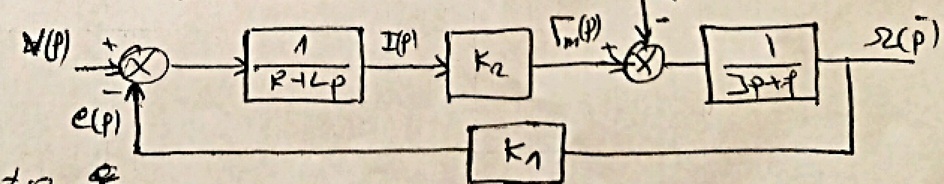
$\Rightarrow \Omega(p) = \frac{1}{Jp+f}\cdot(\Gamma_m(p) - \Gamma_r(p)) \Rightarrow \boxed{H_3(p) = \frac{1}{Jp+f}}$

moment d'inertie

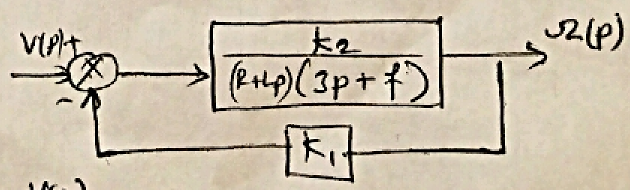


② $\xrightarrow{TL}$ $E(p) = k_1\Omega(p) \Rightarrow \boxed{H_4(p) = k_1}$

$$H(p)_{BF} = \frac{T(p)}{1 + T(p)R(p)}$$



3 a : $\frac{\Omega(p)}{V(p)} = ?$ $V(p) \neq 0$, $\Gamma_r(p) = 0$



$$\frac{\Omega(p)}{V(p)} = \frac{k_2}{(R+Lp)(Jp+f) + k_1k_2}$$

$\Gamma_r(p) = 0$ $\Omega(p) = \frac{k_2}{\ldots} \cdot V(p)$

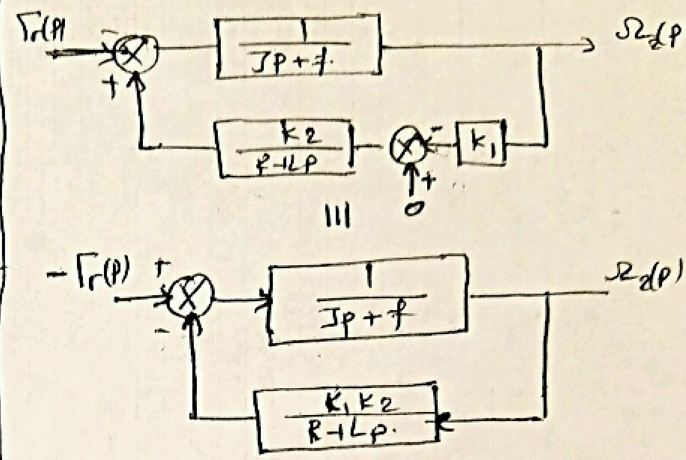
Suite exo 7

3.b $\Gamma_r \neq 0$, $V(p) = 0$ $\quad \dfrac{\Omega_2(p)}{\Gamma_r(p)} = ?$

$$\frac{\Omega_2(p)}{-\Gamma_r(p)} = \frac{R+Lp}{(R+Lp)(Jp+f)+K_1K_2}$$

$$\Rightarrow \frac{\Omega_2(p)}{\Gamma_r(p)} = -\frac{R+Lp}{(R+Lp)(Jp+f)+K_1K_2}$$

$$\Rightarrow \Omega_2(p) = -\frac{R+Lp}{(R+Lp)(Jp+f)+K_1K_2} \cdot \Gamma_r(p)$$



3.c $\Gamma_r(p) \neq 0$, $V(p) \neq 0$ $\quad \Omega(p) = f_1(H_i(p)) \cdot V(p) + f_2(H_i(p))\, \Gamma_r(p)$

$$\Omega(p) = \Omega_1(p) + \Omega_2(p) = \frac{K_2}{(R+Lp)(Jp+f)+K_1K_2} V(p) - \frac{R+Lp}{(R+Lp)(Jp+f)+K_1K_2} \cdot \Gamma_r(p)$$

$$\boxed{\Omega(p) = \frac{H_1(p) \cdot H_2(p) \cdot H_3(p)}{1 + H_1(p) H_2(p) H_3(p) H_4(p)} V(p) - \frac{H_3(p)}{1 + H_1(p) \cdot H_2(p) \cdot H_3(p) H_4(p)} \Gamma_r(p)}$$

4/ $\Gamma_r(p) = 0$. Calcule de FTBO.

$$FTBO(p) = T(p) = H_1 H_2 \cdot H_3 \cdot H_4 = \frac{K_1 K_2}{(R+Lp)(Jp+f)} = \frac{e(p)}{V(p)}$$

$$FTBF(p) = H(p) = \frac{H_1 H_2 H_3}{1 + H_1 H_2 H_3 H_4}$$

$$H(p) = \frac{K_2}{(R+Lp)(Jp+f)+K_1K_2} = \frac{\Omega(p)}{V(p)}$$

5/ La relation qui lie la vitesse de rotation $\Omega(p)$ à la position $\theta(p)$

$$\Omega(t) = \frac{d\,\theta(t)}{dt} \xrightarrow{TL} \Omega(p) = p\,\theta(p).$$

EXO 8

1/ ~~les équations~~ les équations qui regissent le fonctionnement des elements du système (en supposant $\Gamma_r = 0$).

- potentiomètre (1) d'entrée: $\left.\begin{array}{l}\theta_e(p) \longrightarrow v_e(p) \\ 2\pi \longrightarrow E\end{array}\right| \Rightarrow V_e(p) = \frac{E}{2\pi} \cdot \theta_e(p)$,
- Le potentiomètre de sortie (2): $\left.\begin{array}{l}\theta_s(p) \longrightarrow v_s(p) \\ 2\pi \longrightarrow E\end{array}\right| \Rightarrow V_s(p) = \frac{E}{2\pi}\theta_s(p)$
- Amplificateur: $V_a(p) = (V_e(p) - V_s(p))\,A$.
- Moteur MCC. $\dfrac{\Omega(p)}{V_a(p)} = \dfrac{K_2}{(R+Lp)(Jp+f)+K_1K_2}$, $\Omega(p) = p\,\theta_s(p)$

$$\Rightarrow \frac{\theta_s(p)}{V_a(p)} = \frac{K_2}{(R+Lp)(Jp+f)+K_1K_2} \cdot \frac{1}{p}.$$

Exo 7