Partial Discharge Pulse Propagation in Shielded Power Cable and Implications for Detection Sensitivity

Key Words: partial discharge (PD), shielded power cable, high frequency propagation.

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Closed form solutions for PD propagation in shielded power cable are presented for the case of attenuation which is proportional to frequency, as is typical for such cable.

Introduction

In 1982, Boggs and Stone [1] defined the fundamental limits to the electrical detection of corona and partial discharge (PD), i.e., wideband detection of a PD-induced pulse in the presence of thermal noise. This paper treated the effect of frequency-dependent attenuation in shielded power cable in that context. However, most of the plots in that paper were the result of numerical computations. In the same year, Stone and Boggs [2] set out a theory for high-frequency attenuation of shielded power cable. They showed good agreement between attenuation predicted from measured material properties and measured, high-frequency attenuation of shielded power cable. Since 1982, measurements of high-frequency cable attenuation have been reported by a number of authors for a variety of cables. In addition, software tools have become available that facilitate an analytic solution for the parameters of interest. This article summarizes the theory for PD propagation in shielded power cable for both symmetric (Gaussian) and asymmetric PD-pulse waveforms, based on the assumption that the attenuation constant (dB/m or Nepers/m) of the cable is proportional to frequency. This appears to be the most complete possible analytic exposition of PD attenuation in shielded-power cable, which has obvious applications to field PD measurements of such cable.

PD Pulse Shape

PD pulses tend to be either very narrow, symmetric pulses, which can be modeled as Gaussian with a pulse width in the nanosecond range or somewhat broader pulses that are asymmetric with a pulse width of a few tens of nanoseconds as the measured waveform shown in Figure 1 [3]. The case of Gaussian pulses is much easier to treat as a Gaussian is mathematically simple. However, we have been able to treat the case of asymmetric PD pulses by representing them as a sum of Gaussian pulses.

PD Pulse Propagation

A. Gaussian Pulse

For the case of a nanosecond-wide Gaussian PD pulse, the pulse is generated by a Gaussian current propagating in the conductor:

$$I(t) = I_o \exp\left(-\frac{t^2}{2\sigma^2}\right) \tag{1}$$

where I_0 is the peak pulse current amplitude, and the pulse width (full width at half maximum or FWHM) is 2.36 σ. The voltage waveform on the conductor can be expressed as:

$$V(t) = \frac{Q\,Zc}{2\sigma\sqrt{2\pi}} \exp\left(-\frac{t^2}{2\sigma^2}\right) \tag{2}$$

where Q is the partial discharge magnitude in Coulombs and Z_C is the cable characteristic impedance.

As is well-known, the Fourier transform of a Gaussian is a Gaussian. Therefore, we can transform the Gaussian PD pulse in the time domain into a Gaussian voltage amplitude distribution in the frequency domain, and then multiply by the attenuation $\exp(-\alpha\omega L)$ in the frequency domain, where α is the attenuation constant, ω is the radial frequency, and L is the cable length. Transforming into the time domain we can obtain an analytic expression that was not realized in 1982. The equation obtained is difficult, but not impossible, to evaluate. However, the equation is easy to evaluate for the peak PD pulse amplitude as a function of distance propagated, which is given in (3):

$$V_p(L) = \frac{QZc}{2\sigma\sqrt{2\pi}}\left(\exp\left(\frac{\alpha^2L^2}{2\sigma^2}\right)\operatorname{erfc}\left(\frac{\alpha L}{\sigma\sqrt{2}}\right)\right) \quad (3)$$

As the pulse propagates down the cable, it loses high-frequency energy, resulting in a reduction in the optimum bandwidth for detection, which can be taken as roughly the –6 dB bandwidth [1]. We can determine the –6 dB bandwidth (Hz) as:

$$BW(L) = \frac{-\alpha L + \sqrt{\alpha^2L^2 + 2\sigma^2\ln(2)}}{2\pi\sigma^2} \quad (4)$$

so that the –6 dB bandwidth for the 1.2 ns FWHM Gaussian pulse at inception is about 370 MHz. This decreases to about 20 MHz after propagating roughly 100 m down the cable, which explains why typical detection bandwidths are ~20 MHz.

The waveform after attenuation is a complicated function. We have not been able to obtain an analytic solution for the pulse width as a function of distance, although a numerical solution is possible. The pulse width can be approximated by assuming that the pulse after frequency-dependent attenuation is Gaussian. In this approximation, the pulse FWHM as a function of distance propagated is given by

$$PW(L) = \frac{2.36\sigma}{\exp\left(\frac{\alpha^2L^2}{2\sigma^2}\right)\operatorname{erfc}\left(\frac{\alpha L}{\sigma\sqrt{2}}\right)} \quad (5)$$

and the energy in the pulse is given by:

$$W(L) = \frac{Q^2Zc}{4\sigma\sqrt{\pi}}\left[\exp\left(\frac{\alpha^2L^2}{2\sigma^2}\right)\operatorname{erfc}\left(\frac{\alpha L}{\sigma\sqrt{2}}\right)\right] \quad (6)$$

B. Asymmetric PD Pulse

In order to carry out the above analysis for the asymmetric waveform of Figure 1, we create a waveform from a sum of Gaussians. Because the above analysis can be carried out for each Gaussian component of the waveform, the problem can be solved. As high frequencies are attenuated more rapidly than low frequencies, the pulse will become increasingly symmetric as a function of distance propagated.

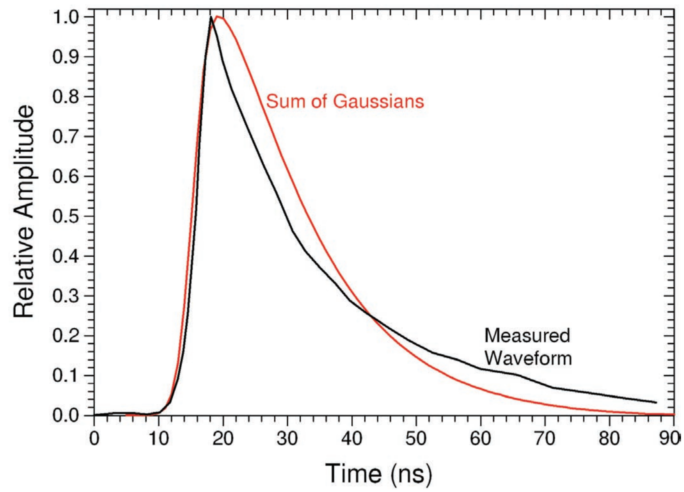

Figure 1. Measured asymmetric PD waveform provided by Prof. Lemke along with the sum of Gaussians used to simulate the measured waveform [3].

The measured PD waveform of Figure 1 is, therefore, modeled as a sum of Gaussians, a plot of which is also shown in Figure 1:

$$I(t) = I_0\sum_{i=1}^{60}\left(A_0(i)\exp\left(-\frac{(t-t_0(i))^2}{2\sigma_0(i)^2}\right)\right) \text{ where}$$

$$A_0(i) = \frac{0.2245}{\sqrt{\exp\left(\frac{i}{5}\right)}},\ \sigma_0(i) = 10^{-9}\frac{\exp\left(i^{0.25}\right)}{\sqrt{2}},\ t_0(i) = \frac{i+5000}{10^9} \quad (7)$$

which results in a peak current of I_0, peak voltage $V_0 = I_0Zc/2$, and a charge of $Q = 2.15\text{x}10^{-8}\, I_0$.

The waveform can be computed as a function of distance propagated by noting that, for each Gaussian in the sum, we can take a Fourier transform, multiply by the cable attenuation, and transform back into the time domain, as was developed for a single Gaussian PD pulse above. Thus, the waveform after propagating a distance L is just the sum of the resulting waveforms after this process. As noted above, the resulting solution is difficult to evaluate as it consists of the sum of products of very large numbers multiplied by very small numbers. We can develop an approximate solution by assuming that the attenuated Gaussian components remain Gaussian, which results in an approximate waveform given by (10):

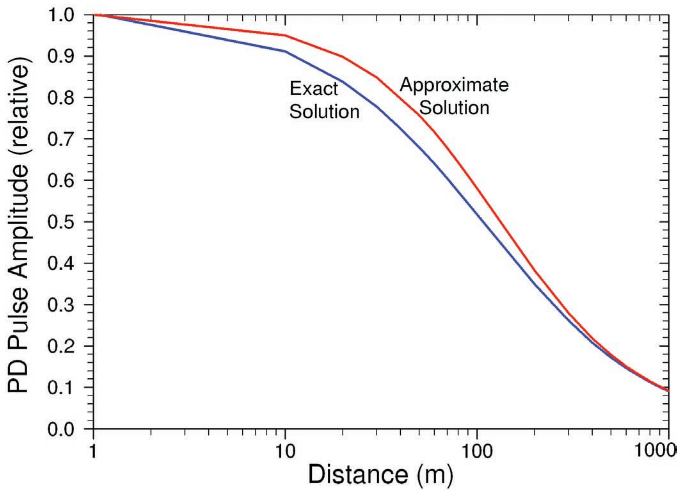

Figure 2. Asymmetric PD pulse amplitude as a function of distance propagated for the exact and approximate solutions with an attenuation constant of 5x10^-11 s/m. The approximate solution appears to be accurate to within 10% over the full range of distance.

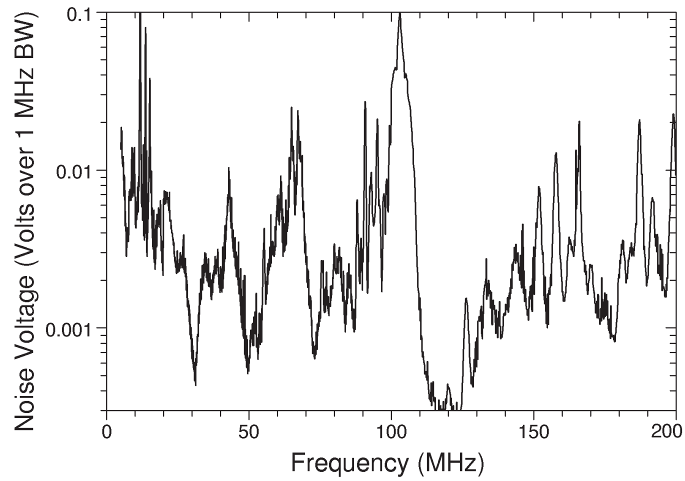

Figure 3. Measured average noise spectrum on cables in urban/industrial environments after correction for PD coupler response and for gain in the measurement electronics. Over the frequency range up to 50 MHz, an average noise floor of about 3 mV seems reasonable. Given a 1 MHz measurement bandwidth, this corresponds to 3 μV/Hz½.

$$\sigma_i(L) = \frac{\sigma_0(i)}{\exp\left(\frac{\alpha^2 L^2}{2\sigma_0(i)^2}\right)\operatorname{erfc}\left(\frac{\alpha L}{\sigma_0(i)\sqrt{2}}\right)} \quad (8)$$

$$A_i(L) = A_0(i)\left(\exp\left(\frac{\alpha^2 L^2}{2\sigma_0(i)^2}\right)\operatorname{erfc}\left(\frac{\alpha L}{\sigma_0(i)\sqrt{2}}\right)\right) \quad (9)$$

$$V(t,L) = V_0 \sum_{i=1}^{60}\left(A_i(L)\exp\left(-\frac{(t-t_0(i))^2}{2\sigma_i(L)^2}\right)\right). \quad (10)$$

Figure 2 shows the pulse amplitude as a function of distance propagated for the asymmetric pulse based on the exact solution and the approximation of (10). The approximate solution of (10) appears to be accurate to within 10% over the full range of distance, which is adequate for the purposes of PD detection.

This process can be applied to any waveform that can be written as a sum of Gaussians. And essentially, any waveform can be written as a sum of Gaussians, from triangular waveforms to rectangular pulses. Thus, this approach provides a very general formulation for propagation of an essentially arbitrary pulse on a shielded cable subject to the assumptions given above, the most important of which is the form of the frequency dependent attenuation.

Measured Noise on the Cable

The spectral distribution of noise on the cable during measurement of partial discharge varies, depending on the local environment. However, we can obtain an idea of the typical noise by averaging over a number of locations and removing the coherent noise sources (radio stations, etc.) and focus on the broad background noise floor. Figure 3 shows such data corrected for the PD coupler response and for the gain in the measurement electronics. These data are measured with a 1 MHz bandwidth. An examination of the data in the range up to 50 MHz suggests a noise floor in the range of 3 mV, which implies a noise voltage density of about 3 μV/Hz½. We will analyze the PD detection sensitivity in terms of an assumed background noise floor of this form.

Table 1: Measured Attenuation Constants

Voltage	Dielectric	Ground	Attenuation Constant, α	Reference
15 kV	PILC, singe phase	Pb	2.17E-10	[6]
10 kV	PILC, 3 cond	Pb	1.1 to 1.5E-10	[7]
15 kV	EPR1	Cu Tape	4.78E-11	[6]
15 kV	EPR2	Cu Tape	1.64E-10	[6]
15 kV	EPR2	10 Neut Wire	9.17E-11	[6]
15 kV	EPR3	Cu Tape	5.10E-11	[6]
15 kV	EPR3	6 Neut Wire	8.76E-11	[6]
15 kV	EPR3	10 Neut Wire	5.50E-11	[6]
15 kV	EPR3	16 Neut Wire	3.93E-11	[6]
46 kV	EPR	Cu Tape	7.33E-11	[2]
28 kV	XLPE	Conc Neut	4.35E-11	[2]
15 kV	TR-XLPE	Cu Tape	3.28E-11	[6]
15 kV	TR-XLPE	10 Neut Wire	6.14E-11	[6]
24-138	XLPE		4.40E-11	[8]

Measured Data for High-Frequency Attenuation

For solid dielectric cable, which is the main subject of PD measurements in distribution systems, high frequency attenuation is caused mainly by the shield layers [2]–[5]. Although the properties of cable shields can vary widely, they typically have conductivity in the range of 0.1 to 1 S/m and a relative dielectric constant in the MHz range of a few hundred to a few thousand. Most importantly, attenuation can be modeled as proportional to frequency. This allows an analytic solution for PD pulse attenuation as a function of distance, as discussed below.

Table 1 shows the attenuation constant as determined from measurements on cables. For XLPE cable, the measured attenuation constant varies from 3.28×10^{-11} Nepers-s/m to 6.14×10^{-11} Nepers-s/m, when the attenuation at frequency ω is given by $\exp(-\alpha\omega L)$, where α is the attenuation constant in Nepers-s/m, ω is the radial frequency, and L is the distance propagated. The attenuation constant, α, at a given frequency can be converted to decibels per meter by decibels per meter = $[20 \log_{10}(e)]\, 2\pi f \alpha = [8.68]\, 2\pi f \alpha$, where f is the frequency in hertz.

Veen [7] provides many measurements of PILC cable, but because such cable is not our focus, we have provided only the range of his measurements in Table 1. Interestingly, some of the earliest attenuation measurements from 1982 [2] and most recent measurements [8] for XLPE cable are in very good agreement, which suggests that the substantial improvements to cable shields over the last 25 years have not resulted in major changes to their dielectric properties.

The measured attenuation data of Table 1 provide a basis for evaluating the range of PD detection sensitivity that is likely to be achieved under field conditions for jacketed cable, which is in good condition. The attenuation of unjacketed cable can vary greatly as a result of intermittent contact between the neutral wires and the ground shield. Based on the data of Table 1, we will use an attenuation constant of 4×10^{-11} s/m as a typical value. But PD detection sensitivity will be evaluated as a function of the product of the attenuation constant, α, times the distance propagated, L, so that these figures can be used to estimate the PD pulse magnitude and bandwidth for arbitrary attenuation constants.

Implications for PD Detection

A. Gaussian Pulse

As noted in detail in [9], a matched filter is very difficult to implement for detection of PD propagating on a cable, as changes in the waveform with distance propagated require a bank of filters, each of which is near optimum for a range of distances propagated. In the case of a Gaussian PD signal propagating in white noise, a rectangular filter of optimum bandwidth comes very close (within about 1 dB) of a matched filter [1]. Thus in this case, the change in the –6 dB bandwidth of the pulse gives an indication of the optimum detection bandwidth.

The signal-to-noise ratio for a Gaussian pulse detected in white noise is simply *W/S*, where *W* is the energy in the pulse, and *S* is the spectral noise power density (W/Hz). Based on the above discussion, the spectral noise power density for a noise voltage V_n (V/Hz$^{1/2}$) is given by $V_n^2/4Zc$, where Zc is the characteristic impedance of the cable. An approximate formula for the energy in the PD pulse as a function of distance propagated has been given in (6). The signal-to-noise ratio as a function of distance propagated for the ideal case, i.e., a matched filter (or equivalently, cross correlation with the known signal waveform) therefore can be approximated by:

$$SN(L) = \frac{Q^2 Zc^2}{4\sigma V_n^2 \sqrt{\pi}} \left[\exp\left(\frac{\alpha^2 L^2}{2\sigma^2} \right) \operatorname{erfc}\left(\frac{\alpha L}{\sigma\sqrt{2}} \right) \right] \quad (11)$$

We can assume that σ is about 5×10^{-10} s, which corresponds to a pulse width of about 1.2 ns. Thus given a value of α, which is likely to range between 3×10^{-11} and 1.5×10^{-10} s/m for solid dielectric cable, one can plot the signal-to-noise ratio for any value of V_n (V/Hz$^{1/2}$). Figure 4 plots the signal to noise ratio for a 1 pC PD pulse ($Q = 10^{-12}$ C) for a background noise level of 1 μV/Hz$^{1/2}$, and a characteristic impedance of 30 Ω as a function of the product, αL.

We can compute the PD detection sensitivity based on a 10 dB signal to noise ratio as a function of distance propagated by using (6) for the energy in the pulse and the above value for *S*. This results in the "matched filter" detection sensitivity shown in

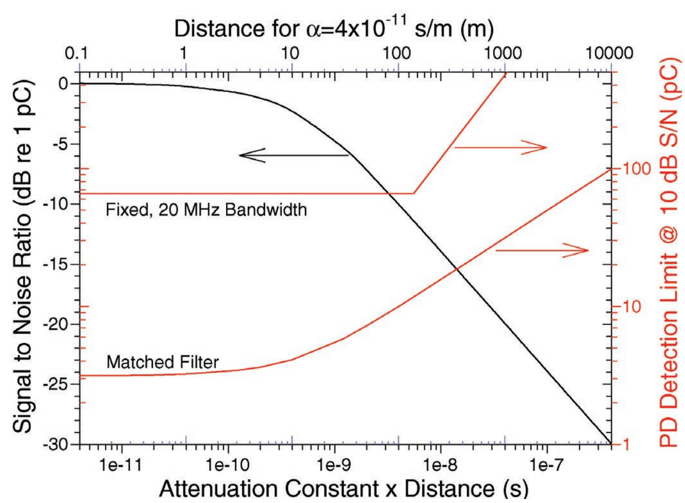

Figure 4. Signal-to-noise ratio and PD detection sensitivity for 10 dB signal to noise ratio for an assumed noise voltage spectral density of 1 μV/√Hz$^{1/2}$. The bottom axis is the product of the attenuation constant (s/m) times the distance propagated (m), while the top axis shows distance (m) for an assumed attenuation constant of 4×10^{-11} s/m, which, according to Table 1, is a typical value for XLPE cable. The PD detection limit (pC) varies linearly with the voltage spectral noise density. Thus the PD detection limit is very sensitive to the background noise. The PD detection limit is shown for the case of a matched filter, for which the bandwidth is a function of the distance propagated, and for a fixed 20 MHz bandwidth, both for a 10 dB S/N ratio.

Figure 4. The effective signal bandwidth was presented in Figure 5. The PD detection sensitivity of Figure 4 is based on PD detection using a matched filter (or equivalently cross correlation) or some other technique, such as recording a large quantity of data and extracting the PD signals using wavelet analysis, which does not require triggering on the first PD pulse above the noise. In the case of PD detection based on triggering on the first PD pulse above the noise, the detection sensitivity will be degraded somewhat, as the effective peak noise is about four times the RMS noise voltage for the measurement bandwidth. Thus for a 1 pC PD pulse which originates with a pulse width of 1.2 ns, the optimum detection (–6 dB) bandwidth at a detection distance of 100 m is about 20 MHz and the peak voltage amplitude is about 1 mV (Figure 6).

For the assumed noise voltage spectral density of 1 $\mu V/Hz^{1/2}$ over a bandwidth of 20 MHz, the effective peak noise voltage will be about 18 mV. Thus, the PD detection sensitivity based on triggering on the PD pulse above the noise will be in the range of 25 pC, which compares to about 12 pC in Figure 4 based on extracting the PD signal from the noise using more sophisticated signal processing. PD detection is normally carried out with a fixed PD detection bandwidth in the range of 20 MHz, as the distance of the PD source from the detector is not known. At small distances from the PD source, the PD detection sensitivity using a fixed PD detection bandwidth suffers because the detection bandwidth is smaller than optimum, so that useful signal is "thrown away". For large distances from the PD source, sensitivity suffers because the detection bandwidth is greater than optimum, so that noise is being detected without a useful signal. As a result, for short distances, the PD detection sensitivity is constant and limited by the detection bandwidth. For large distances, the PD detection sensitivity degrades more rapidly than the theoretical optimum (matched filter). An estimate of the PD detection sensitivity for a fixed bandwidth of 20 MHz and the conditions discussed in the caption is shown in Figure 4.

B. Asymmetric Pulse

As can be seen from Figures 5 and 6, beyond about 100 m propagation distance, the Gaussian and asymmetric pulses differ little in amplitude or bandwidth. The bandwidth of the asymmetric PD pulse is very close to the typical measurement bandwidth of field PD detection/location systems, about 20 MHz. As a result, the PD detection sensitivity for the asymmetric pulse based on a matched filter or similar detection scheme will be approximately that for the fixed 20 MHz bandwidth in Figure 4 for PD propagation distances up to about 100 m. Beyond 100 m, the PD detection sensitivity should follow the "matched filter" line in Figure 4. For the case of fixed 20 MHz bandwidth, the PD detection sensitivity will be roughly the same as for the narrow Gaussian pulse, as the fixed 20 MHz bandwidth essentially turns the narrow pulse into one with a width and amplitude similar to the asymmetric pulse.

C. Summary

The above analysis provides a basis for estimating the PD detection sensitivity as a function of distance, background noise, and cable attenuation constant, that, based on Table 1, can vary from about $3x10^{-11}$ s/m to $2x10^{-10}$ s/m. This roughly order of magnitude variation in attenuation constant translates into an order of magnitude variation in the distance over which a PD pulse can be detected, as the attenuation constant enters all the equations as a product with distance. The measured data for XLPE cable (both TR-XLPE and XLPE) vary from $3.3x10^{-11}$ to $6.15x10^{-11}$ s/m,

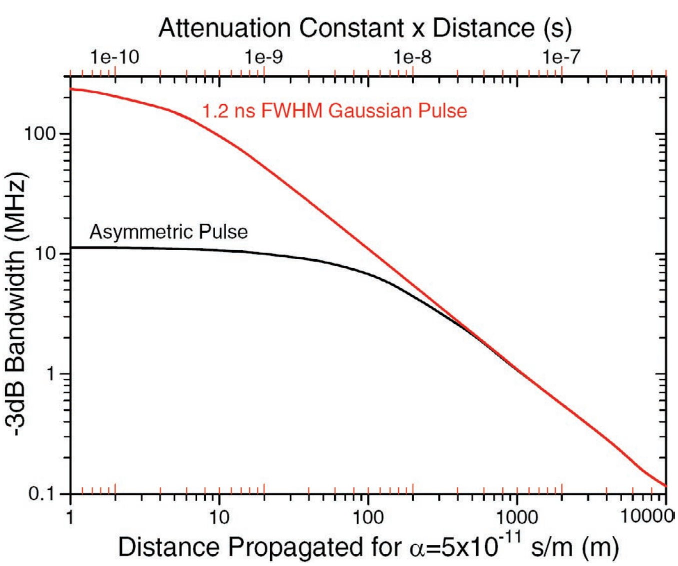

Figure 5. –3 dB bandwidth as a function of distance propagated for a 1.2 ns (FWHM) Gaussian PD pulse and an asymmetric PD pulse propagating on a cable with a 30 Ω characteristic impedance and attenuation constant of $5x10^{-11}$ s/m. Beyond a few hundred meters, the bandwidths merge.

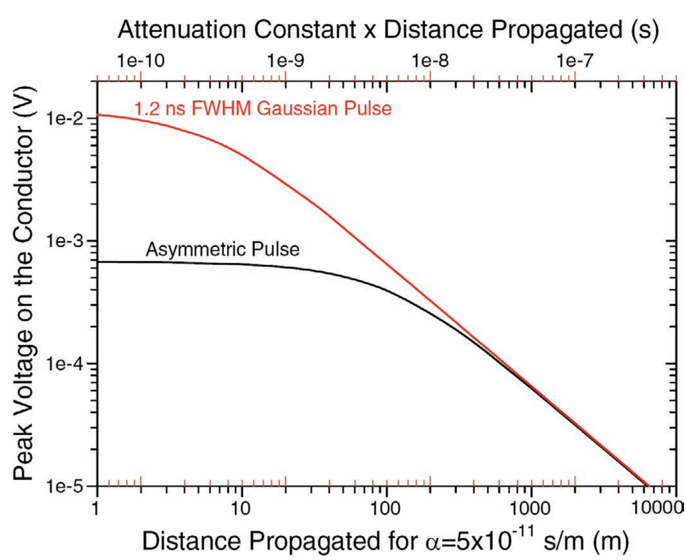

Figure 6. Peak voltage on the conductor for a 1 pc Gaussian and asymmetric PD pulse propagating on a cable with a 30 Ω characteristic impedance.

although the attenuation could increase if the number of neutral wires were reduced. Thus XLPE cable probably can vary from about $3x10^{-11}$ s/m to $9x10^{-11}$ s/m. EPR cables are likely to have somewhat greater attenuation, and the measured data range from $3.9x10^{-11}$ to $1.6x10^{-10}$ s/m. The attenuation of PILC cable is even greater. Cables with Cu tape shields tend to have greater attenuation than those with concentric neutrals, and the measured data are for new cables. With field aging of the Cu tape, the conduction across the laps is likely to decrease, resulting in increased attenuation and decreased propagation velocity. The spiraling of the return current down the Cu tape will also cause an axial magnetic field that will induce eddy current in the conductor, resulting in additional attenuation.

All of the data presented are for jacketed cables, in which the metallic ground shield is held against the ground shield semicon by the jacket. The attenuation of unjacketed cables is difficult to assess or predict, as the concentric neutral wires will make only intermittent contact with the ground shield, and the attenuation will depend on both the frequency of such contact and the average distance (capacitance) of the neutral wires to the ground shield.

Conclusions

A general analytic approach has been developed for predicting the propagation characteristics of PD pulses of arbitrary shape, as well as the partial discharge detection sensitivity that can be achieved for such PD pulse waveforms. The approach has been applied to two typical waveforms, a narrow Gaussian pulse, and a slower, asymmetric pulse. Graphs are presented in a generic form in terms of the product of distance propagated and attenuation constant for an easily scaled background noise, so that the PD propagation characteristics and detection sensitivity can be determined for any parameters of interest.

For both narrow Gaussian and broader asymmetric PD pulses, the PD detection sensitivity under relatively noisy field conditions is likely to be in the range of 30 to 100 pC at 1000 m depending on the method of PD detection and based on a background voltage spectral noise density of 1 $\mu V/\sqrt{Hz}$. Because the PD detection limit (pC) scales linearly with the voltage spectral noise density, the PD detection sensitivity achieved in the field is very sensitive to ambient noise conditions.

Acknowledgments

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