fiche TD n° 2 / 2022 - 2023

Exercice 1 :

I la glace se réchauffe

$Q_1 = m_{glace} \, C_{p\,glace} \, (T_0 - T_{-20}) = 10 \times 2,1 (0 + 20) = 420 J$

$$-20 \xrightarrow{Q_1} 0°C \xrightarrow{Q_2} 0°C \xrightarrow{Q_3} 100°C \xrightarrow{Q_4} 100°C$$

. Q_2 : quantité de chaleur pour fondre la glace

$Q_2 = m L_f = 10 \times 334,4 = 3344 J$

$Q_3 = m_{eau} \, C_{p\,eau} \, (T_{100} - T_0) = 10 \times 4,18 (100) = 4180 J$

$Q_4 = n L_v = \frac{10}{18} \times 40500 = 22500 J$

$Q_T = Q_1 + Q_2 + Q_3 + Q_4 = 420 + 3344 + 4180 + 22500 = 30444 J$

II $\sum Q_i = 0 \Rightarrow Q_{cal} + Q_{eau} = 4375$

1) $\mu_{H_2O} \, C_{H_2O} (T_{e1} - T_1) + m_{H_2O} \, C_{p\,H_2O} (T_{e1} - T_1) = 4375$

$$\mu_{H_2O} = \frac{4375 - m_{H_2O} \, C_{p\,H_2O} (T_{e1} - T_1)}{C_{p\,H_2O} (T_{e1} - T_1)} =$$

$$\mu_{H_2O} = \frac{4375 - 50 \times 1 (45 - 20)}{1 \times (45 - 20)} = 125 g$$

2) Cp de l'acier

$\sum Q_i = 0 \Rightarrow Q_{cal} + Q_{H_2O} + Q_{Acier} = 0$

$Q_{acier} = -(Q_{cal} + Q_{H_2O}) = -4375$

$$Q_{acier} = m_{acier} C_{p\,acier} (T_{e1} - T_1) = -4375$$

$$C_{p\,acier} = \frac{-4375}{m_{acier}(T_{e1} - T_1)} = \frac{-4375}{50 \times (45-90)} = 1,94 \frac{cal}{g°C}$$

* Calcul de la chaleur latente de fusion de la glace.

$$Q_{cal} + Q_{H_2O} + Q_{Acier} + Q_{glace} = 0$$

$$Q_{glace} = m_g L_f + m_g C_{p\,H_2O} (T_{e2} - T_f)$$

$$\Rightarrow \mu_{H_2O} C_{p\,H_2O} (T_{e2} - T_{e1}) + m_{H_2O} C_{p\,H_2O} (T_{e2} - T_{e1}) + m_{acier} C_{p\,acier} (T_{e2} - T_{e1})$$

$$+ m_g L_f + C_{glace}\, m_{glace} (T_{e2} - T_f) = 0$$

$$\Rightarrow L_f = \frac{-[(T_{e2} - T_{e1})[\mu_{H_2O} C_{p\,H_2O} + m_{H_2O} C_{H_2O} + m_{Acier} C_{Acier}] + m_g C_{eau} (T_{e2} - T_f)}{m_g}$$

$$\Rightarrow L_f = \frac{-[(11,39-45)(125 \times 1 + 50 \times 1 + 50 \times 1,94) + 100 \times 1 (11,39 - 0)}{100}$$

$$L_f = 80 \frac{cal}{g}$$

Exo2 :

1) Etat 1 $\longrightarrow$ Etat 2

$\begin{cases} V_1 = 1 m^3 \\ P_1 = 1 atm \\ T_1 = 273,15 \end{cases}$ $\qquad$ $\begin{cases} V_2 \\ P_2 = 20 \text{ bars} \\ T_2 = T_1. \end{cases}$

a) $V_2 = ?$

$PV = cte$: loi de Boyle-Mariotte

$$P_1 V_1 = P_2 V_2 \Rightarrow V_2 = \frac{P_1 V_1}{P_2} \Rightarrow V_2 = \frac{1,013 \times 10^5 \times 1}{20 \times 1,013\, 10^5} = 0,05\, m^3$$

$$\Rightarrow V_2 = 50 l$$

b) $W = -\int P_{ext}\, dV$; $P_{ext} = P_{gaz} = \frac{nRT}{V}$ (transformation reversible)

$W = -\int nRT \frac{dV}{V}$, avec $T = T_1 = T_2 = cte$

$W = -nRT_1 \ln \frac{V_2}{V_1} = -P_1V_1 \ln \frac{V_2}{V_1}$

$W = 1013\,10^5 \times 1 \ln \frac{1}{0,05} = 303,5\, KJ$.

on applique le 1er principe $\Delta U = W + Q$

compression isotherme $\Delta U = 0$ $(\Delta T = 0)$

$W + Q = 0 \Rightarrow Q = -W \Rightarrow Q = -303,5\, KJ$.

2) Etat ② $\longrightarrow$ Etat ③

$\begin{cases} P_2 \\ V_1 \\ T_2 \end{cases}$ $\begin{cases} P_3 \\ V_3 \\ T_3 \end{cases}$

a) $V_3 = ?$ $T_3 = ?$

$PV^\gamma = cte$ (loi de laplace)

$P_2V_2^\gamma = P_3V_3^\gamma \Rightarrow \frac{P_2}{P_3} = \left(\frac{V_3}{V_2}\right)^\gamma \Rightarrow V_3 = V_2\left(\frac{P_2}{P_3}\right)^{1/\gamma}$

$\Rightarrow V_3 = 0,05\left(\frac{20}{1}\right)^{1/1,4} \Rightarrow V_3 = 0,425\, m^3 = 425\, l$.

$P_3V_3 = nRT_3 \Rightarrow T_3 = \frac{P_3V_3}{nR} = \frac{P_3V_3\, T_1}{P_1V_1}$

$T_3 = \frac{1,013\,10^5 \times 0,425 \times 273}{1,013\,10^5 \times 1} = 116,025°C = -156,97°K$

$nRT_1 = P_1V_1 \Rightarrow nR = \frac{P_1V_1}{T_1}$

b) $W = -\int_{V_2}^{V_3} P_{ext}\, dV$; $P_{ext} = P_{gaz}$

$PV^{\gamma} = cte \Rightarrow P = \frac{cte}{V^{\gamma}}$ $\left(PV^{\gamma} = P_2 V_2^{\gamma} = P_3 V_3^{\gamma} = cte \right.$

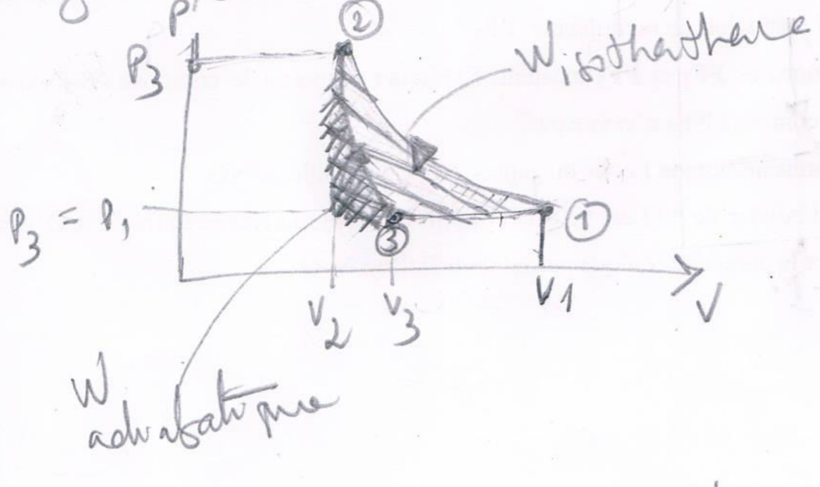
$$W = -\int_{V_2}^{V_3} P_2 V_2^{\gamma} \frac{dV}{V^{\gamma}} = -P_2 V_2^{\gamma} \int_{V_2}^{V_3} V^{-\gamma} dV$$

$$W = -P_2 V_2^{\gamma} \left(\frac{1}{1-\gamma}\right) V^{1-\gamma} \Big]_{V_2}^{V_3} = -P_2 V_2^{\gamma} \left[\left(\frac{1}{1-\gamma}\right)\left(V_2^{1-\gamma} - V_2^{1-\gamma}\right)\right]$$

$$W = 20 \cdot \times 1{,}013 10^5 (0{,}05)^{1,4} = \frac{1}{(1-1{,}4)}\left[(0{,}05)^{1-1,4} - (0{,}425)^{1-1,4}\right]$$

$W = -145{,}66 \, Kj$

le gaz travaille moins qu'avant, la surface située sous la courbe adiabatique est plus faible que la surface située sous l'isotherme



EXO 3 :

$A \xrightarrow{\text{Compression isotherme}} B \xrightarrow{\text{échauffement isochore}} C \xrightarrow{\text{détente adiab}} A$

échauffement isochore $V = cte$ $(V_B = V_C)$

A: $\begin{cases} P_A = 1 bar \\ V_A = ? \\ T_A = 300 K \end{cases}$

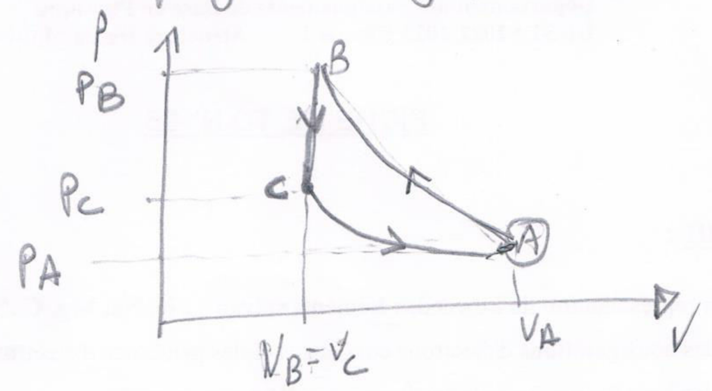
B: $\begin{cases} P_B \\ V_B \\ T_B = T_A \end{cases}$

C: $\begin{cases} P_C \\ V_C = V_B \\ T_C \end{cases}$

A: $\begin{cases} P_A \\ T_A \\ V_A \end{cases}$

$PV^{\gamma} = cte$

Diagramme de Clapeyron ($P = f(V)$) qualitativement



Etat (A)

$P_A V_A = nRT_A \Rightarrow V_A = \frac{nRT_A}{V_A}$

$\Rightarrow V_A = \frac{1 \times 8,314 \times 300}{10^5} = 0,0249\ m^3 = 24,9\ l$

Etat (B) isotherme $T_B = T_A = 300°K$

$\frac{V_A}{V_B} = 10 \Rightarrow V_B = \frac{V_A}{10} = 2,49\ l$

$P_B V_B = nRT_B \Rightarrow P_B = \frac{nRT_B}{V_B}$

$P_B = \frac{1 \times 8,314 \times 300}{2,49 \times 10^{-3}} = 9,9768\ 10^5\ pas \simeq \boxed{10\ bars}$

Etat (C) isochore $V_C = V_B = 2,49\ l$

$P_C = ?\quad T_C = ?$

C → A adiabatique reversible $PV^\gamma = cte$

$P_A V_A^\gamma = P_C V_C^\gamma \Rightarrow P_C = P_A \left(\frac{V_A}{V_C}\right)^\gamma$

$P_C = 1 \times \left(\frac{24,9}{2,49}\right)^{1,4} = 25,1\ bars$

$P_C V_C = nRT_C \Rightarrow T_C = \frac{P_C V_C}{nR} = \frac{25,1 \times 10^5 \times 2,49\ 10^{-3}}{1 \times 8,314} = 751,7°K$

ou $T_A V_A^{\gamma-1} = T_C V_C^{\gamma-1} \Rightarrow T_C = T_A \left(\frac{V_A}{V_C}\right)^{\gamma-1} = 300 \left(\frac{24,9}{2,49}\right)^{1,4-1} = 751,7°K$

	A	B	C
P(bar)	1	10	25,1
V(l)	24,9	2,49	2,49
T°K	300	300	751,7

3) $\underline{A \to B}$ compression isotherme

$$W_{AB} = -\int P_{ext}\, dV = -\int_{V_A}^{V_B} nRT_A \frac{dV}{V} = -nRT_A \ln\frac{V_B}{V_A} = nRT_A \ln\frac{P_B}{P_A}$$

$$W_{AB} = 1 \times 8{,}314 \times 300 \ln\frac{10}{1} = +5743{,}1\,J$$

$$\Delta U = 0 \ (T = cte) \Rightarrow W_{AB} + Q_{AB} = 0 \Rightarrow Q_{AB} = -W_{AB} = -5743{,}1\,J$$

il s'agit d'une compression isotherme, le système reçoit du travail $W > 0$

$\underline{B \to C}$ échauffement isochore ($V = cte$)

$$W = -\int P_{ext}\, dV \quad (dV = 0) \Rightarrow \boxed{W_{BC} = 0}$$

$$\Delta U_{BC} = Q_{BC}$$

$$\Delta U_{BC} = \int_{T_B}^{T_C} n C_V\, dT = n C_V (T_C - T_B)$$

$$\begin{cases} C_p - C_V = R \\ \dfrac{C_p}{C_V} = \gamma \end{cases} \Rightarrow \begin{cases} C_V = \dfrac{R}{\gamma - 1} \\ C_p = \dfrac{\gamma}{\gamma - 1} R \end{cases}$$

$$\Rightarrow \Delta U = \frac{nR}{\gamma - 1}(T_C - T_B)$$

$$\Delta U_{BC} = Q_{BC} = \frac{1 . 8{,}314}{1{,}4 - 1}(751{,}7 - 300) = 9388{,}58\,J$$

C → A détente adiabatique

$Q_{CA} = 0$

$\Delta U_{CA} = W_{CA}$

$\Delta U_{CA} = W_{CA} = \int_{T_C}^{T_A} n\,C_v\,dT = n\,C_v(T_A - T_C)$

$\Delta U_{CA} = \frac{nR}{\gamma - 1}(T_A - T_C)$.

$\Delta U_{CA} = \frac{1 \times 8,314}{1,4 - 1}(300 - 751,7)$

$\Delta U_{CA} = W_{CA} = -9\,388,58\,J$

Il s'agit d'une détente, le gaz fournit du travail au milieu extérieur. $W < 0$

Calcul de ΔH. ; $\Delta H = n\,C_p\,dT$

$A \longrightarrow B \quad \Delta H_{AB} = 0 \quad (T = cte)$

$B \longrightarrow C \quad \Delta H_{BC} = \int_{T_B}^{T_C} n\,C_p\,dT = nR\frac{\gamma}{\gamma - 1}(T_C - T_B)$

$\Delta H_{BC} = 1 \times 8,314 \times \frac{1,4}{1,4 - 1}(751,7 - 300) = 13144\,J$

$C \longrightarrow A : \quad \Delta H_{CA} = \int_{T_C}^{T_A} n\,C_p\,dT = nR\frac{\gamma}{\gamma - 1}(T_A - T_C)$

$\Delta H = 1 \times 8,314 \times \frac{1,4}{1,4 - 1} = (300 - 751,7) = -13144\,J$

	A → B	B → C	C → A
W (J)	5743,1	0	−9388,58
Q (J)	−5743,1	9388,58	0
ΔU (J)	0	9388,58	−9388,58
ΔH (J)	0	13144	−13144

$Q_{cycle} = Q_{AB} + Q_{BC} + Q_{CA}$
$-5743,1 + 9388,58 + 0 = +3645,48 J$

$W_{cycle} = W_{AB} + W_{BC} + W_{CA}$
$= 574,1 + 0 - 9388,58 = -3645,48 J$

$\Delta U_{cycle} = Q_{cycle} + W_{cycle} = +3645,48 - 3645,48 = 0$

$W_{cycle} = -3645,48 J < 0$ Il s'agit d'un cycle moteur : le système fournit du travail au milieu extérieur